LECTURES ON THE THERMODYNAMIC LIMIT FOR COULOMB SYSTEMS

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1. INTRODUCTION

These lecture notes present an outline of the proof of the existence of the thermodynamic limit for Coulomb systems. A brief statement of the main results has appeared previously, [9], and the full work will appear shortly, [11]. What we have tried to do in these notes is to present the ideas and methods used in constructing this proof while leaving out most of the details of the analysis. In some places, such as section 3, we treat only the simplest kind of Coulomb system: two species of charged particles (one positive and one negative) whose only interaction is through the Coulomb potential. In other places we simply state various lemmas and theorems without proof.

The basic pre-requirement for the existence of a thermodynamic limit for Coulomb systems is the Dyson-Lenard Theorem, [1], which gives a lower bound to the energy of a system of charged particles. It is therefore very fortunate that the proof of this theorem is presented in a particularly nice form, in Professor Lenard's lectures which are included in this volume.

Statement of the Problem

Statistical Mechanics as developed by Gibbs and others rests on the hypothesis that equilibrium properties of matter can be completely described in terms of a phase-space average, or partition function, $Z = \mathrm{Tr}\{\exp(-\beta H)\}$, with H the Hamiltonian and β the reciprocal temperature. It was realized early that there were grave difficulties in justifying this assumption in terms of basic microscopic dynamics. These questions, which involve the time evolution of macroscopic systems, have still not been satisfactorily resolved, but the great success of equilibrium statistical mechanics in offering qualitative and quantitative equilibrium explanations for such varied phenomena as superconductivity, specific heats of crystals, chemical equilibrium constants, etc., have left little doubt about the essential correctness of the partition function method. However, since Z cannot be evaluated explicitly for any reasonable physical Hamiltonian H, comparison with experiment always involves some

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