A Transit Time Model of Space Charge and Its Comparison to Experimental Data

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Abstract
A simple one-dimensional account of space charge in an anode-cathode (AK) gap is based on a transit time model. Two limiting cases are treated: thermionic and field emission. Each case is compared to experimental thermionic and field emission data in the literature and the correspondence is shown to be good.

Keywords
Fowler-Nordheim Equation; Richardson Equation; Child-Langmuir Equation; Space Charge; Transit time

INTRODUCTION

Space charge forces are increasingly important in the performance of high brightness electron sources. Thermionic, field, and photoemission sources all contain variations ($e.g.$ geometric field enhancement, non-uniform surface coatings, etc.) with length scales difficult for Particle-in-Cell codes to capture. In these instances, the space charge forces near the cathode can have an effect, particularly when the electron velocities are small compared to the relativistic velocities that they ultimately acquire. The present work presents simple theoretical models that may be adapted to analyze the consequences of space charge on the emission characteristics.

TRANSIT TIME MODEL

In equilibrium, the one dimensional (1D) current density J past an emission barrier is constant and given by the product of unit charge q, a supply function $f_o(k)$ and a transmission probability $D(k)$, where k is the wave number. When $k > k_o \equiv (2mV_o)^{1/2}/\hbar$ (where V_o is the barrier maximum) then thermionic emission (over the barrier) occurs, and conversely, when $k < k_o$, field emission (tunneling) occurs. In the vacuum between anode and cathode, a sheet charge density $\sigma(x)$ is defined by

$$\sigma(x) = \int_0^x \rho\left(x'\right) dx' \tag{1}$$

in terms of which a "transit time" τ is defined by

$$\tau \equiv q\sigma(D)/J \tag{2}$$

where the anode-cathode separation is D. An integration of Poisson's Equation then demonstrates that

$$V_a/D \equiv F_o = F + qN_\tau J\tau/\varepsilon_o \tag{3}$$

where N_τ is a constant of order unity and F is the field at the surface of the cathode or emission site. The self-consistent solution of Eq. 3 finds the value of $J(F,T)$ compatible with F and $\tau(F)$ but when F vanishes, $J \to J_{CL}$. In the limit of a space-charge-free AK gap, the ballistic transit time τ_o is given by $\tau_o = \sqrt{2mD/F_o}$, characteristic of motion in a constant $F_o = V_a/D$.[1] In contrast, for 2D and 3D charge distributions, the field across the AK gap varies as the inverse distance to a power and therefore a characteristic time τ is difficult to define there.

CURRENT DENSITY

The factor J is either thermionic (Richardson-Laue-Dushman Eq.), field emission (Fowler-Nordheim Eq.), or space charge limited (Child-Langmuir Eq.), as those limits are adequate to the analysis of the experimental data below. J_{FN} and J_{RLD} are based on the image charge barrier $V(x) = \mu + \Phi - Fx - Q/x$, where μ is the chemical potential (aka the Fermi energy), Φ is the work function, F (as in Eq. 3) is the field at the surface, Q/x is the image charge potential, $Q = 0.36$ eV-nm, and $\phi \equiv \Phi - \sqrt{4QF}$ is the Shottky barrier lowered work function. The J's are as follows:

$$J_{RLD} = A_{RLD}T^2 \exp(-\phi/k_BT) \tag{4}$$

$$J_{FN}(F) = A\frac{F^2}{\Phi t(y_o)^2}\left(\frac{\Phi^2 e^6}{4QF}\right)^\nu \exp\left(-B\frac{\Phi^{3/2}}{F}\right) \tag{5}$$

$$J_{CL}\left(\varphi_a, D\right) = \frac{4\varepsilon_o}{9}\sqrt{\frac{2q}{m}}\frac{\varphi_a^{3/2}}{D^2} \tag{6}$$

where $\nu = 2BQ/3\sqrt{\Phi} \approx 0.77281$ for copper, $A_{RLD} = 120.17$ Amp /K^2 cm^2, $A = 1.5414 \times 10^{-6}$ Amp/eV and $B = 6.8309$ 1/nm-eV$^{1/2}$. Additionally, the Forbes approximation to the elliptical integral function $v(y)$ normally encountered in J_{FN} has been implicitly used [2], and $t(y_o) \approx 1 + (1/6e)$. The term φ_a is the potential difference (as opposed to the potential energy $V(x)$).

THERMAL EMISSION ANALYSIS

An empirical relation due to Longo [4], used in life-test models of barium dispenser cathodes, models the the transition of J from J_{RLD} to J_{CL} by

$$\frac{1}{J^\alpha} = \frac{1}{(J_{RLD})^\alpha} + \frac{1}{(J_{CL})^\alpha} \tag{7}$$

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[1] we append the unit charge q on to the electric field to obtain F, which is therefore a force rather than a "field"

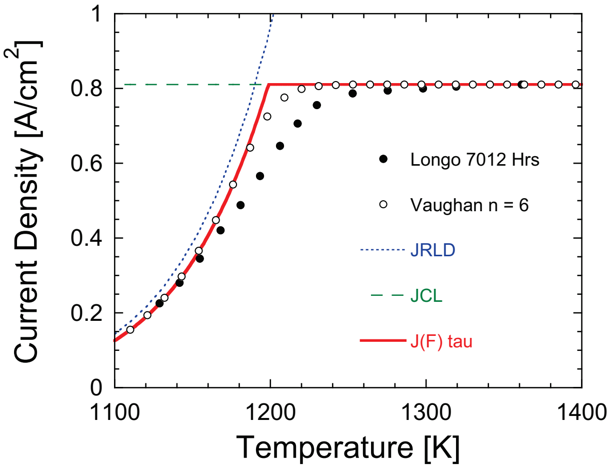

Figure 1. $J(T)$ **for Ba dispenser cathode, and an** $\alpha = 6$ **curve compared to** $J = J_{RLD}$**,** J_{CL}**, and the transit time model or standard barium dispenser cathodes.**

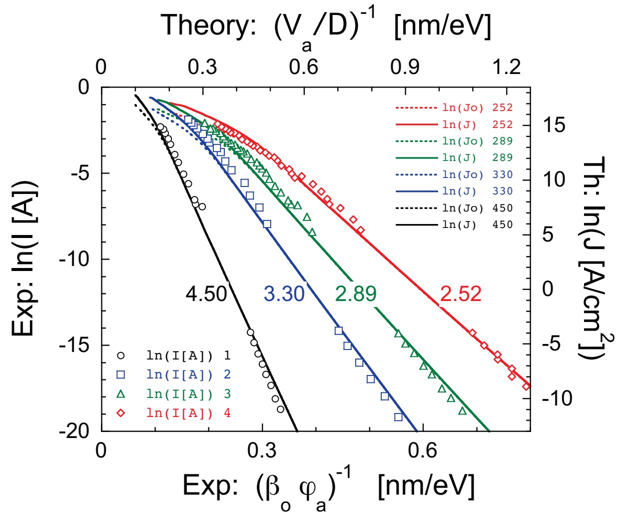

Figure 2. **Comparison of Fig. 3, Ref. [1] to theory (**$\beta_o = (4516q/$**cm)**, $D = 10\mu$**m). Numbers refer to** Φ**.**

where α was called a "shape factor" (Eq. (1) of Longo corresponds to $\alpha = 1$). Vaughan [5] asserted that "good" cathodes are characterized by $6 \leq \alpha \leq 10$ with its value serving as a fitting factor governing the smoothness of the "knee" of the transition. The choice $\alpha = 6$ is shown (labeled as "Vaughan") with Longo's data in Figure 1 along with RLD and CL curves. The line labeled "JF tau" corresponds to the self-consistent solution relating F, $J_{RLD}(F,T)$ and $\tau(F)$ with $N_\tau \equiv \int_0^D \sigma(x)dx/(D\sigma(D))$. It is clear that a space charge reduction of the field at the surface in the transit time model anticipates the shape of the data: while theory will not explain the knee region for $\alpha = 1$ (as per Longo), it does anticipate $\alpha \approx 6$ as per Vaughan.

FIELD EMISSION ANALYSIS

In a previous study [3], the effect of emitted charge on field emission was analyzed by summing over the contribution of sequentially emitted electrons and compared to the data of Barbour, *et al.* [1] for a tungsten needle on which coverings of barium were applied that successively lowered the work function of the tip. Although the onset of space charge effects in that model were anticipated, the experimental "turn-over" was more gradual than predicted theoretically. Here, the transit time model is applied to the same data, even though the application of a 1D formulation to an explicitly 3D experimental arrangement requires additional approximations and makes an exact comparison problematic: instead, the theory compares changes relative to the Ba-free condition, as in Figure 2.

Two theory lines are shown: the dashed lines correspond to using the ballistic transit time τ_o, and the solid line to using the evaluated τ as per Eq. 2. That the experimental data can be bracketed by these different theoretical models bodes well for the development of theoretical models that can be utilized by Particle-in-Cell codes which would benefit from a quantitative field emission + space charge model in the modeling of dark current, causes of intrinsic emittance and emission non-uniformity that accompany electron beam formation in particle accelerators and photoinjectors. The ballistic space charge τ_o gives reasonably accurate results, is amenable to modifications, and is possible to use because most of the behavior of the theoretical relations are a consequence of the strong variation of J_{FN}.

CONCLUSION

A transit time model of space charge was shown to be compatible experimental data reported in the literature. The details of the calculation and the methods behind the comparisons shown in the figures herein shall be presented.

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