SOME REMARKS ON THE SURFACE TENSION.

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We consider the surface tension $\beta^{-1}\tau$ for the d-dim. Ising model with nearest neighbour ferromagnetic coupling $J > 0$. τ is defined in e.g. [1] :

$$0 \leq \tau = - \lim_{L\to\infty} \frac{1}{(2L+1)^{d-1}} \lim_{M\to\infty} \log \frac{Z_{\Lambda,\pm}}{Z_{\Lambda,+}} \tag{1}$$

where Λ is a box centered at the origin of height 2M in the i_1-direction with a base of side 2L+1. The indices + and $\pm$ refer to the usual + and $\pm$ boundary conditions (b.c.).

We do the following construction: we replace the couplings J by sJ, $0 \leq s \leq 1$ for all nearest neighbour pairs $\langle ij\rangle$ crossing the plane $i_1 = -\frac{1}{2}$. By symmetry $\tau(s)$ is zero for $s = 0$ and $\tau = \tau(1)$ is equal to

$$\tau = k\int_0^1 ds\, (\rho_s^+ (\sigma_0\sigma_{-1}) - \rho_s^{\pm} (\sigma_0\sigma_{-1})), \quad \beta J = k \tag{2}$$

where ρ_s^+ ($\rho_s^{\pm}$) is the corresponding infinite volume Gibbs state with +b.c. ($\pm$b.c.) (2) is justified by correlation inequalities and using such inequalities we prove that $\tau = 0$ whenever there is no spontaneous magnetization. We can also prove that $\tau = 0$ if the spontaneous magnetization is zero for the semi-infinite system defined on $\{i \in \mathbb{Z}^d : i_1 > 0\}$ with free b.c. at $i_1 = 0$ and + b.c. elsewhere. From (2) and correlation inequalities we obtain the lower bound

$$\tau \geq 2k \int_0^1 \rho_s^+(\sigma_0)\ \rho_s^{\pm}(\sigma_0)\ ds$$

Using duality and correlation inequalities we prove that for $d=2$ $\tau(k) = m(k^*)$ where k^* is the dual temperature and m is the inverse correlation length in the state with free b.c. For $d=3$ $\tau(k) = \alpha(k^*)$ where α is the coefficient of the area law decay of the Wilson loop in the Ising gauge model [2].

For $d=3$ and k large enough we prove that $\tau - 2k$ and the correlation functions in the state $\rho^{\pm}$ are analytic in $\exp(-2k)$. Furthermore we have the Gibbs formula

$$\frac{d\tau}{dk} = \sum_{\substack{i_1=-\infty \\ i_2=i_3=0}}^{+\infty} \ \sum_{\substack{j: \\ |i-j|=1}} \left(\rho^+(\sigma_i \sigma_j) - \rho^{\pm}(\sigma_i \sigma_j) \right)$$

see [2] and [3].

References:

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