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Some effects of inflation on a firm with original cost depreciation

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We examine, in a simplified model without uncertainty, the response of a regulated firm to inflation in the cost of capital equipment. This model firm is assumed to be constrained to meet a given demand schedule, and to realize a prescribed rate of return on capital; the firm's revenue requirements include depreciation based on original cost. We explore the steady-state relationships between the firm's financial parameters (rate of return, debt ratio, payout ratio, etc.) and the rate of inflation, which follow by the use of accounting identities from the condition that inelastic demand be met; we also examine the consequences in the steady state of the additional condition that equity investors receive the return they require. We further consider a sudden jump in the rate of inflation, and show that, quite apart from considerations of capital attraction or shareholder satisfaction, this necessitates an increase in the return on capital. We show how, if the firm adjusts to increased inflation only by a gradual increase in its rate of return, holding all other parameters fixed, the necessity of meeting the given demand implies that the old shareholders, who did not foresee an increase in the rate of inflation, will, if regulation is instantaneous and exact, and the required return on equity is earned, receive a real return on their investment which is higher than they anticipated and higher than that received by the new shareholders. This effect can be traced to the use of original-cost depreciation. We then show how by adjustment of any of several parameters, i.e., the payout ratio, the debt ratio, or the balance between internal and external financing, demand can still be met while all shareholders receive exactly the real return they require. The question of whether, in the real world, such an adjustment is possible or desirable is not explored.

1. Introduction

■ The extent and manner in which regulators should take account of changing costs confronted by a regulated firm, for example under inflationary conditions, have been much discussed in the regulatory

The authors are indebted to Alvin Klevorick and Peter Rosoff for valuable criticism.

literature (see Kahn, 1970, Chapter 4, and references therein). If we think of the firm as employing two inputs, capital and labor, to produce a single output, then we can ask how the price of the output should be adjusted to reflect inflation of wages and of capital costs. We confine our attention in this paper to capital costs. For regulatory purposes, these are in turn broken down into depreciation and return on capital investment. With respect to depreciation the question is essentially whether the depreciation allowance should be based on the original cost of plant or on the replacement cost; we assume the former, in conformity with regulatory practice (see Kahn, 1970), although, as we shall see, this practice has some peculiar consequences in an inflationary environment. With respect to the return on capital investment, we assume that this investment is a mixture of debt and equity, and that a return is allowed on the debt portion equal to the embedded (not marginal) costs of the firm's debt.

Thus we are left with the question of how the regulatory body should adjust the allowed return on book equity in response to a change in the rate of inflation. We address this question in the context of a model which is strongly abstracted and simplified from a real firm subject to real regulation. Simplifying assumptions will be noted as they are made.

We characterize the regulated firm by two constraints. The first is the condition that its revenues be at all times exactly equal to revenue requirements. These include a return on capital, whose required level it is our purpose to discuss. Thus, there is no consideration of regulatory lag. The second constraint is a franchise constraint, that is, the firm is under an obligation to meet demand. In reality, demand is at least to some degree price elastic. For purposes of this analysis we assume, however, that it is perfectly inelastic.

The franchise constraint itself imposes a condition on the required return on equity, or more precisely imposes a relationship between this required return and the firm's other financial parameters. For if the firm is to meet a given demand schedule in an efficient way, it must construct a given amount of plant. It finances its construction from two kinds of sources, internal and external. Internal sources, in the model, consist of depreciation and retained earnings.[1] Depreciation is based on original cost and an estimated mean life of plant and is not subject, in our model, to adjustment in response to changes in the price level. Retained earnings are a certain fraction (the retention ratio) of earnings on equity. The financial markets impose limitations on the dividend payout ratio (i.e., on how far one can reduce dividends to finance construction), and require that a certain balance be maintained between internal and external sources of funds. Thus the franchise constraint implies a constraint on the return on equity, and hence implies certain changes on this return in response to changes in the cost of capital equipment. In particular, an increase in inflation requires an increase in rate of return, if demand is to be met. These ideas will be made more precise in what follows.

Now as adjustments in the return on equity are made, in response to changes in the price level, the equity investors are affected, and if we assume any particular share price valuation model, we can calcu-

[1] We are neglecting deferred tax reserves and the investment tax credit, although these now provide major internal sources of funds for many firms.

late how much they are affected. Of course, if investors correctly anticipate everything—changes in the rate of inflation and the adjustment of rate of return in response—they will set the stock price in such a way that they all realize precisely the return they demand under all circumstances. If, however, they do not anticipate changes in the rate of inflation, then these changes and the responsive adjustments in the return on equity may lead to unexpected gains or losses for the investors.

We shall show, using a simple stock-price model, that if the firm holds its other financial parameters (payout ratio, debt ratio, etc.) constant while the rate of return on equity is adjusted to increased inflation in such a way as to satisfy the franchise constraint, then the "old" shareholders will indeed realize unexpected gains following an unanticipated increase in the rate of inflation. They would suffer unexpected losses following a corresponding decrease. This phenomenon will turn out to be traceable to the use of a depreciation formula based on original cost in inflationary times. We show, however, that if the other financial parameters of the firm are not held constant, but are allowed to vary slightly in response to a change in the rate of inflation, while the rate of return on equity follows a correspondingly different course, then all investors can receive exactly the return they require, while the franchise constraint is still satisfied.

The antidilutionary condition, that the market price of the firm's shares not be permanently below book, is shown to imply, as usual, that the book rate of return on the firm's equity not be permanently below the investors' discount rate. This antidilutionary condition is, however, not part of the model, but must be additionally assumed.

2. Model of the regulated firm

■ As explained in the Introduction, the firm is assumed to be externally limited in two ways—by a franchise constraint and a rate of return constraint. The franchise constraint means that the firm must meet a given demand. This demand may be growing, and is assumed to be price-inelastic over the range of prices considered. To meet this demand in an efficient (cost minimizing) way, a given level of physical plant, $Z(t)$, is required. $Z(t)$ does not depend on inflation, but does depend on technological progress. From here on, we shall take $Z(t)$ as given.

With respect to the rate-of-return constraint, we assume, in accordance with the practice in most United States jurisdictions (see Bauer, 1966), that the allowed rate of return is applied to the undepreciated part of the original cost of the plant and other assets, $X(t)$, i.e. the accumulation of construction less depreciation, and not to the replacement cost of the physical plant, nor to the market value of the firm. It is also assumed for simplicity that at all times the firm collects from its customers revenues precisely equal to revenue requirements, i.e., just adequate to cover wages, depreciation, income taxes, and the required return on capital. Furthermore, since we are primarily interested here in the effects of inflation, we shall assume that the rate of return (allowed and realized) in the absence of inflation is adequate from the investors' point of view (see Sections 4 and 5).

In our model, $X(t)$ is also equal to financial capital, i.e., the accumulation of stocks, bonds and retained earnings. $X(t)$, measured in dollars, and physical plant $Z(t)$, measured in physical units, will not be proportional to each other in the presence of capital inflation even when the depreciation schedule accurately reflects retirements, as we shall assume it to do. However, $Z(t)$ and $X(t)$ are linked by the sequence of construction expenditures $C(t')$, for $t' < t$. If $p(t)$ denotes the cost of a unit of plant at time t, then for a firm started at $t = 0$ with one unit of plant

$$X(t) = f(t) + \int_0^t C(y)f(t - y)dy \tag{1}$$

$$Z(t) = f(t) + \int_0^t \frac{C(y)}{p(y)} f(t - y)dy. \tag{2}$$

In these equations we have chosen units such that

$$Z(0) = X(0) = p(0) = 1. \tag{3}$$

$f(\tau)$ denotes the life table, i.e., the fraction of a unit vintage of average plant unretired (and undepreciated) at age τ. This life table is assumed independent of vintage.

Equation (2) can be thought of as determining $C(t)$, since $Z(t)$ is given. Once $C(t)$ is known, $X(t)$ is determined by (1). For ease of computation we shall assume from now on that $Z(t)$ grows at a steady rate γ, i.e.,

$$Z(t) = e^{\gamma t}. \tag{4}$$

As we stated earlier, we assume that the firm earns the allowed rate of return, ρ, which may be time-dependent, on its total capital, X. ρ is the weighted average of the embedded interest rate, i^{em}, and the rate of return on equity, ρ_e. That is,

$$\rho = \delta i^{em} + (1 - \delta)\rho_e. \tag{5}$$

Here δ is the debt ratio. Thus, equity capital, X_e, is given by

$$X_e = (1 - \delta)X. \tag{6}$$

X_e can grow in two ways: from retained earnings and from the sale of new shares. Thus,

$$\dot{X}_e = (1 - \eta)\rho_e X_e + S, \tag{7}$$

where η is the dividend payout ratio, S is the rate of net new stock financing, and the dot denotes the time derivative: $\dot{X}_e(t) \equiv dX_e(t)/d(t)$. Defining the parameter k to be the fraction of new equity raised externally, we have $S = k\dot{X}_e$, whence

$$\frac{\dot{X}_e}{X_e} = \frac{1 - \eta}{1 - k}\rho_e. \tag{8}$$

Equation (8) is just an accounting relation and holds at every instant. If δ is constant, we have also[2]

$$\frac{\dot{X}}{X} = \frac{1 - \eta}{1 - k}\rho_e. \tag{9}$$

We are now in a position to find the rate of return which the firm

[2] If δ is increasing over a certain period, then ρ_e can be substantially less than (9) would indicate. This resembles what actually happened to many regulated firms during the recent inflationary period. This is clearly a temporary rather than a steady state expedient and because of risk considerations must be abandoned long before δ reaches unity.

must be allowed under inflationary conditions so that it can construct the plant required to meet the given demand. We do so in the next section, first for the steady state case (uniform inflation), and then for a step increase in the rate of inflation.

3. Required rate of return in the presence of inflation

■ We consider first the situation in which there is a constant inflation rate I (i.e., $\dot{p}(t)/p(t) = I$ independent of t) and the parameters δ, η, and k are also constant in time.[3] Since I is constant, we can rewrite equation (2), using (3), in the form

$$e^{It}Z(t) = e^{It}f(t) + \int_0^t C(y)[e^{I(t-y)}f(t-y)]dy. \qquad (10)$$

The quantity $e^{It}Z(t)$ has the meaning of the replacement cost at time t of the firm's physical plant at that time. Equation (10) is of the same form (in $e^{It}Z(t)$) as equation (1) is in $X(t)$, with $f(t)$ replaced by $e^{It}f(t)$. It follows (see Appendix 1) that in the steady state (long after the formation of the firm),

$$\frac{\dot{X}}{X} = \frac{\dot{Z}}{Z} + I = \gamma + I. \qquad (11)$$

Hence, from (9), we find that in the presence of a steady inflation, the franchise constraint requires

$$\rho_e(I) = \frac{1-k}{1-\eta}(\gamma + I). \qquad (12)$$

Clearly (12) does not tell the whole story of the impact of an increase in the rate of inflation on return on equity. For example, if $(\gamma+I)$ is small enough, (12) may lead to a value of ρ_e smaller than the equity investors' discount rate, and hence, as we shall see, to a market price for the firm's shares smaller than the book value per share. Equation (12) expresses the implications of the franchise constraint only.

We may use (12) to compare two situations, one with steady inflation I_1 and one with steady inflation $I_2 \equiv I_1 + \Delta$. Then, with δ, k, and η given and independent of I, the rate of return on equity necessary to support a growth rate γ is just

$$\rho_e(I_2) = \rho_e(I_1) + \frac{1-k}{1-\eta}\Delta. \qquad (13)$$

We shall see in Section 4 that because of the requirements of the equity investors we cannot in a steady state situation have $k > \eta$; hence the increase in ρ_e must always be at least as great as that in I.

To obtain the dependence of the overall return on total capital, ρ, on the rate of inflation we need the dependence of the interest rate on inflation. If we assume that inflation enters the interest rate additively (see Keran, 1971), then

$$i(I_2) = i(I_1) + \Delta. \qquad (14)$$

Using this assumption, we obtain

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[3] In reality, equity financing is usually done by means of occasional discrete stock issues, so that the function k has spikes in it. When we assume k constant, we are really thinking of some sort of smoothed behavior.

$$\rho(I_2) = \rho(I_1) + \Delta\left[\delta + \frac{(1-\delta)(1-k)}{1-\eta}\right]. \tag{15}$$

☐ **Transients.** In reality, of course, permanent steady states do not exist. The question then is how rapidly to adjust the rate of return in response to changing rates of inflation. To analyze this question completely we would have to consider general functions $I(t)$, and take account of the phenomenon of regulatory lag. We can, however, get a qualitative picture of the required response in $\rho(t)$ by considering the simple case in which there is no regulatory lag and in which inflation is steady at rate I_1 for a long time, and then jumps abruptly to a new steady level $I_2 = I_1 + \Delta$. It will be convenient to relabel the time axis so that this jump occurs at $t = 0$. Thus $p(t) = \exp(I_1 t)$ for $t < 0$ and $\exp(I_2 t)$ for $t \geq 0$.

To obtain an explicit expression for the time course of the rate of return adjustment, we must specify the form of the life table. We take

$$f(t) = e^{-\lambda t}, \tag{16}$$

where λ is the reciprocal of the mean life of average plant. We can now readily deduce from equations (1) and (2) (having relabelled the time axis as described above), the time course of $X(t)$. We find

$$\frac{\dot{X}}{X} = \begin{cases} \gamma + I_1, & t < 0 \\ \dfrac{(\gamma + I_2)(\gamma + \lambda + I_1) - \lambda\Delta e^{-(\gamma+\lambda+I_2)t}}{(\gamma + \lambda + I_1) + \Delta e^{-(\gamma+\lambda+I_2)t}}, & t \geq 0. \end{cases} \tag{17}$$

We notice that $\dot{X}/X$ is continuous across $t = 0$.

As $t \rightarrow \infty$, the negative exponential terms in (17) become negligible, and $\dot{X}/X$ approaches its new steady state value $\gamma + I_2$.

Having obtained the time trajectory of $\dot{X}/X$, we can now use (9) (which entails assuming a constant δ) to obtain explicitly, once $\eta(t)$ and $k(t)$ are specified, the amount and speed of increase in $\rho_e(t)$ required by the franchise constraint after $t = 0$.

To illustrate the time course of the increase in $\rho_e(t)$ which the regulatory body must grant (and the firm must achieve) if demand is to be met, we have carried out a numerical example, in which η and k are taken to be constant for all time. The assumed parameter values are

$$\begin{array}{ll} \gamma = 0.08 & \lambda = 0.05 \\ I_1 = 0.0 & k = 0.5 \\ I_2 = 0.03 & \eta = 0.6. \end{array}$$

The results are shown in the upper curve of Figure 1. We see that an increase in ρ_e from 10 percent to an asymptotic value of 13.75 percent is required. After the onset of inflation, $\rho_e(t)$ rises smoothly towards its asymptotic value; about 70 percent of the increase occurs in the first ten years.

The corresponding calculation can be carried out for $\rho(t)$, although further assumptions as to the debt ratio and the debt retirement schedule are required. This calculation is outlined in Appendix 2. The result is illustrated in Figure 2.

FIGURE 1

RATE OF RETURN ON EQUITY, $\rho_e(t)$, FOR AN ABRUPT CHANGE IN THE INFLATION RATE

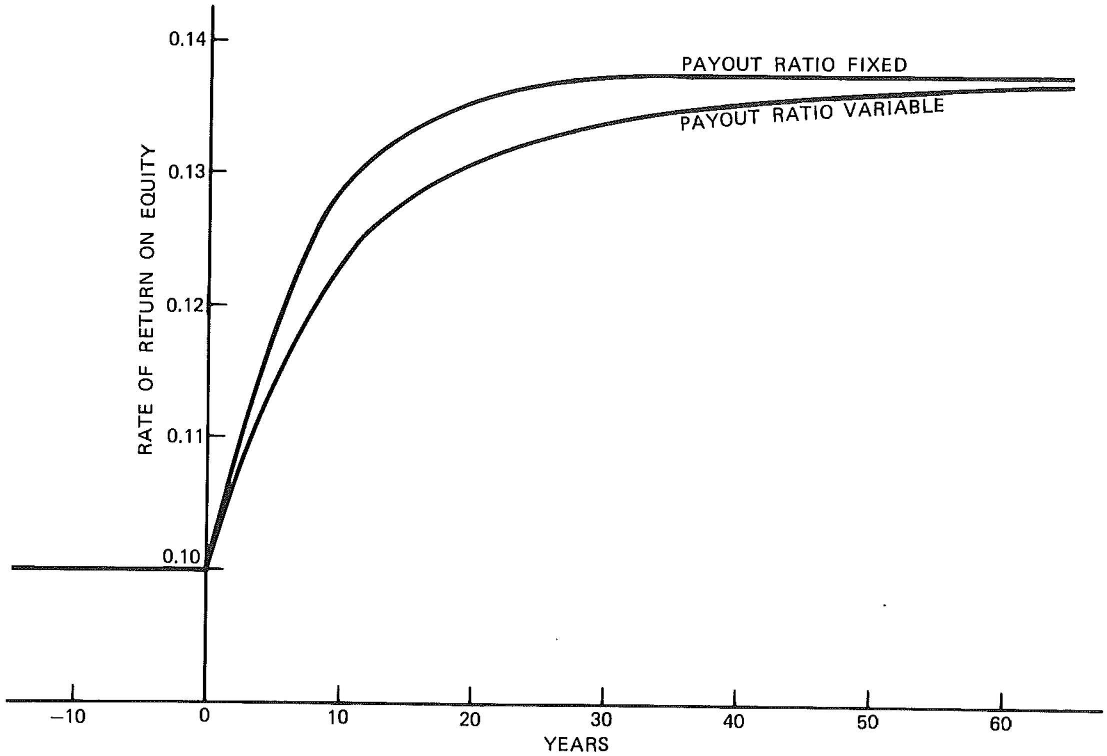
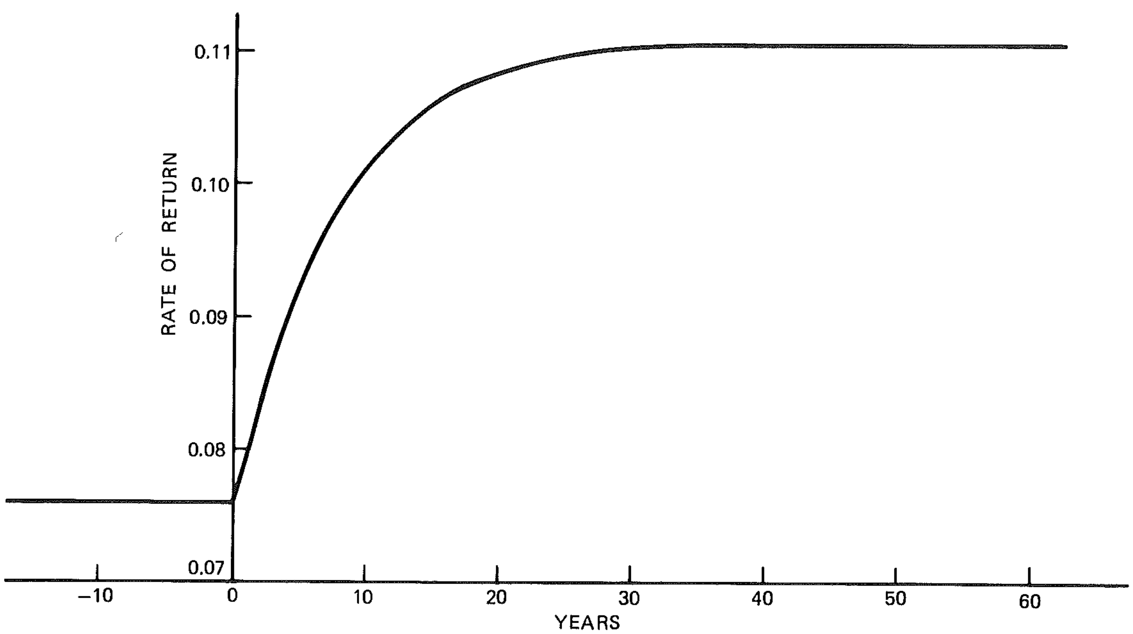


FIGURE 2

RATE OF RETURN ON TOTAL CAPITAL, $\rho(t)$, FOR AN ABRUPT CHANGE IN THE INFLATION RATE



4. Effect of inflation on equity investors —steady state

■ We have so far treated the parameters η, k, and δ as exogenously given and, for purposes of the illustrative example, as constant. These parameters, however, tie the firm to the capital markets. The firm cannot with impunity choose arbitrary values of k, η, and δ; the values it does choose will affect the experience of the investors.

We must now introduce a share valuation model. We do so very simply as follows: the "real" market price (measured in dollars of $t = 0$, for example) is taken to be the present worth of the infinite future stream of anticipated real dividends, discounted at a fixed rate σ_0, which does not vary with the rate of inflation. (σ_0 would be the discount rate for investment in a firm of this type in the absence of inflation.)[4] In symbols,

$$\frac{m(t)}{p(t)} = \int_t^\infty e^{-\sigma_0(y-t)} \frac{d(y)}{p(y)}\, dy, \tag{18}$$

where $m(t)$ is the market price and $d(\cdot)$ denotes dividends per share. If the total dividends paid are $Div(t)$, then

$$d(t) = \frac{Div(t)}{N(t)} = \frac{\eta \rho_e X_e(t)}{N(t)}, \tag{19}$$

where $N(t)$ is the number of shares outstanding at time t. If the course of inflation is known, then $X(t)$ is determined by the given demand, as previously described. If the debt ratio is specified, then $X_e(t)$ is determined also. Thus, for given η and ρ_e (for example, for $\rho_e(t)$ chosen as in Section 3 to meet the franchise constraint), equation (18) provides one relationship between two unknown functions, $m(t)$ and $N(t)$. A specification of the parameter k provides a second relationship, namely (by the definition of k)

$$k\dot{X}_e = S = m(t)\dot{N}(t). \tag{20}$$

Equation (20) implies that the proceeds price on the sale of new shares equals the market price; there is no discount.

Thus, to review, the given demand leads, through cost minimization, to a required physical plant, $z(t)$. This, plus a specification of the price level, $p(t)$, implies a construction budget $C(t)$, which solves equation (2). $C(t)$ determines $X(t)$ through (1), and hence, if the debt ratio is known, $X_e(t)$. Equation (8), plus a specification of η and k, next determines $\rho_e(t)$, and hence $Div(t)$ (see (19)). Equations (18) and (20) now determine $m(t)$ and $N(t)$. Equation (20) requires an initial condition; since the size of a share is arbitrary, we could choose $N(0) = 1$.

We shall be interested in the effects on equity investors of a changing inflation rate. Before discussing such a case, however, we wish to understand the influence of steady inflation on the market price of the firm's stock. In the steady state, with the inflation rate constant at I, equation (18) becomes

$$m(t) = \int_t^\infty e^{-\sigma(y-t)} \frac{Div(y)}{N(y)}\, dy, \tag{21}$$

where the nominal discount rate is

$$\sigma = \sigma(I) = \sigma_0 + I. \tag{22}$$

Thus, we are taking σ_0 to be a discount rate which is applied to

[4] This share valuation model is plausible if: (a) either investors hold the shares forever or there is no capital gains tax, and (b) the perceived risk of the investment does not change with the rate of inflation.

purchasing power of dividends and is independent of inflation, while $\sigma(I)$ is applied to dollar dividends and, for constant I, is given by (22).

It will be useful to put equation (21) in differential form. Define $V(t)$, the market value of the firm's equity, as

$$V(t) = N(t)m(t). \tag{23}$$

Multiplying equation (21) by $N(t)$ and differentiating, one obtains

$$\dot{V}(t) - \sigma V(t) = S(t) - Div(t). \tag{24}$$

We can now use equation (24) to find the market to book ratio. It is intuitively clear that in the steady state market value and book value per share are in constant proportion to each other: $m(t) = \alpha b(t)$, with α constant.[5] Here

$$b(t) \equiv X_e(t)/N(t) \tag{25}$$

is the book value per share. Now $Div(t) = \eta \rho_e X_e(t)$, so that by using equation (7) the right-hand side of equation (24) is $\dot{X}_e - \rho_e X_e$. Also $V = \alpha X_e$, and for α constant, $\dot{V} = \alpha \dot{X}_e$. Thus (24) becomes

$$\alpha(\dot{X}_e - \sigma X_e) = \dot{X}_e - \rho_e X_e. \tag{26}$$

But, with a constant debt ratio, $\dot{X}_e/X_e = \gamma + I$, from equation (11). Thus we obtain

$$\alpha = \frac{\rho_e - (\gamma + I)}{\sigma(I) - (\gamma + I)} = \frac{\rho_e - (\gamma + I)}{\sigma_0 - \gamma}. \tag{27}$$

We are not surprised to notice that $\alpha > 1$ if and only if $\rho_e > \sigma$. For the market price to be positive, we must have $\rho_e > \gamma + I$, that is (from equation (8)) $\eta > k$, or $Div > S$. In other words, investors are not willing to continue indefinitely putting more money into the firm than they get out of it. However, as in Miller and Modigliani (1961), η does not enter into the expression for α.

Equation (27) is illustrated in Figure 3. This diagram may be understood as follows: when $\rho_e > \sigma(I)$, rapid growth is beneficial to investors, and the market price becomes infinite when the growth rate in current dollars reaches the discount rate σ. (This behavior is a consequence of our simplistic stock price model. With a finite horizon for discounting, or with a discount rate that increases for the remote future, such behavior would be eliminated. It might also disappear in a stochastic model.) If $\rho_e < \sigma$, rapid growth is detrimental to investors; the market price goes to zero when the growth rate reaches ρ_e. Since the earnings per share are $e = p_e b$, the price-earnings multiple is given by

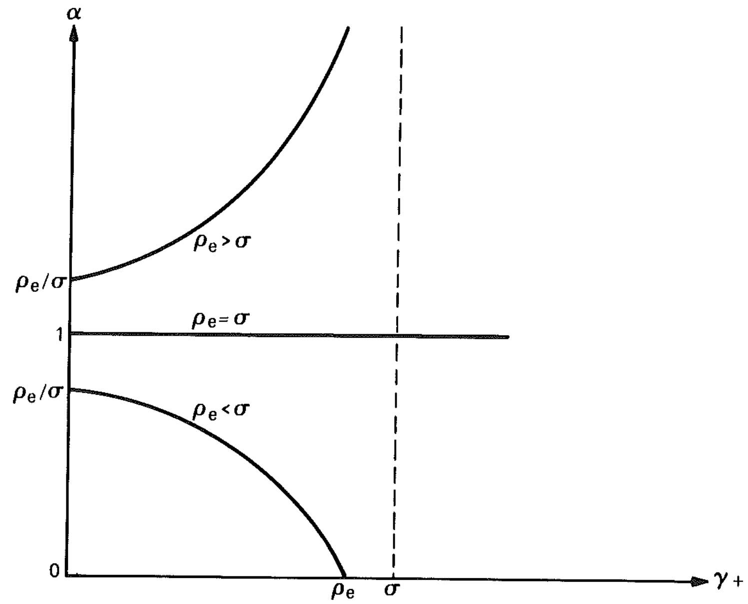
$$\frac{m}{e} = \frac{1 - (\gamma + I)/\rho_e}{\sigma(I) - (\gamma + I)}. \tag{28}$$

Nothing we have said so far forbids a steady state in which $\alpha < 1$, i.e., market is less than book. In practice such a state would not be feasible for other reasons, namely: investors would object to the continuous dilution of their stake in the firm by the sale of new shares below book, and, even more importantly, they (the owners) would be well advised to sell the firm's capital equipment, if that were possible, rather than to retain ownership.

[5] It can be shown that α = constant is in fact the only well-behaved solution of equations (20) and (24). We do not do so here.

FIGURE 3

DEPENDENCE OF MARKET–TO–BOOK RATIO ON GROWTH RATE IN CURRENT DOLLARS



5. Changing inflation and unexpected investor returns

■ We now consider the effects of changing inflation, and the adjustment of ρ_e in response to it, on the equity investors. We shall show that if the rate of return is adjusted so that the firm just meets demand, with its other financial parameters unchanged, and if the investors do not perceive a change in risk, then the "old" equity investors will experience a real return greater than their basic discount rate σ_0. We shall then show how, by variation of the financial parameters, all investors can be made to receive exactly the return σ_0. This will require a book return on equity, ρ_e, temporarily lower than that of Section 3 (although, of course, higher than the noninflationary return and increasing to the same asymptotic value), while still permitting demand to be met.

For explicitness we again consider the case in which there is a sudden jump in the rate of inflation at $t = 0$. Let us see what happens if, as before, the financial parameters (k, η, δ) are kept fixed, while the rate of return on equity, ρ_e, is allowed by the regulatory body to increase as described in Section 3.

Some assumption about investor expectations is needed at this point: we postulate that all investors believe that the current rate of inflation will continue for all time. In this the investors before $t = 0$ are incorrect, while those after $t = 0$ are correct. The investors after $t = 0$ further assume (accurately) that the regulatory body will allow the rate of return to increase, as described by equations (9) and (17), i.e., they are assured that regulation will permit, and customers will provide, the required increase in revenues. It follows that the market price of the stock will adjust itself in such a way that investors after $t = 0$ do receive their required return, $\sigma(I_2)$, i.e., σ_0 on purchasing power; however, investors before $t = 0$ need not receive their required return on purchasing power; that is, the price of the stock at $t < 0$ could have been different if inflation had been correctly anticipated.

These earlier investors may, in fact, be surprised in two ways after $t = 0$: the dollar dividends per share which they receive may increase more rapidly than anticipated (if increasing ρ_e and inflating rate base more than offset the more rapid growth in number of shares); on the other hand, the purchasing power of these dividends will be smaller than anticipated. We can see how these two competing effects offset each other by calculating the price, say $m'(t)$, which an investor gifted with perfect foresight would have paid for a share of stock at each $t < 0$, and by comparing it with the price $m(t)$ which was paid by investors who assumed that the inflationary *status quo* would continue indefinitely.

Suppose as before that the inflation rate is zero for $t < 0$ and I for $t \geq 0$, and that the life table is of the form (16). The market price paid, for $t \geq 0$, is

$$m(t) = \int_t^\infty e^{-\sigma_0(y-t)} d_0(y)dy, \tag{29}$$

where d_0 denotes the dividend per share when $I = 0$; the market price the investor gifted with perfect foresight would have been willing to pay is

$$m'(t) = \int_t^0 e^{-\sigma_0(y-t)} d_0(y)dy + \int_0^\infty e^{-\sigma_0(y-t)} e^{-Iy} d_I(y)dy, \tag{30}$$

where d_I denotes the dividend per share in the presence of inflation. Therefore

$$m'(t) - m(t) = \int_0^\infty e^{-\sigma_0(y-t)}[d_I(y)e^{-Iy} - d_0(y)]dy. \tag{31}$$

The dividend per share in the presence of inflation is

$$d_I(t) = \frac{Div(t)}{N(t)} = \eta\rho_e(t)(1 - \delta) \frac{X_I(t)}{N_I(t)}, \tag{32}$$

or, using (9),

$$d_I(t) = \frac{\eta}{1 - \eta} (1 - k)(1 - \delta) \frac{\dot{X}_I(t)}{N_I(t)}. \tag{32a}$$

Now $\dot{X}_I(t)$ is already known (from (17), with $I_1 = 0$, $I_2 = I$), so only $N_I(t)$ needs to be determined. This can be done by actually finding the time dependent solutions for $m(t)$ and $N(t)$ from equations (18) and (20), since $Div(t)$ is known. We do not give the details of the calculation. Suffice it to say that by this route we can prove that

$$d_I(t)e^{-It} > d_0(t), \tag{33}$$

for all $t > 0$. It follows from (30) that

$$m'(t) > m(t), \tag{34}$$

for all $t \leq 0$. (Since the inequality (33) applies to the integrand of (31), (34) holds whatever the value of σ_0.) That is, within the framework of our model, unanticipated inflation has been more than offset to the investor who purchased stock before $t = 0$, by subsequent dividend increases; the present worth of the purchasing power of the dividends he receives, discounted at σ_0, is greater than the price $m(t)$ he paid. Another way of putting this is to say that these early investors experience an effective real yield on their investment higher than the σ_0 they require.

The origin of this effect is, qualitatively, as follows. Depreciation based on original cost makes a smaller proportional contribution to the construction budget $C(t)$, which is determined by the franchise constraint, in the presence of inflation than in its absence. With k and δ fixed, the resulting deficiency must be made up by retained earnings. But with η fixed, dividends must increase in proportion to retained earnings, and we have assumed that this in fact occurs. Since the experience of the equity investors is described entirely in terms of dividends (not at all in terms of repayment of principal), they receive an unexpected benefit. It can be shown that as $\lambda \to 0$ (infinite plant life, no depreciation) this effect disappears.

We have worked out a numerical example, using the parameters of the example in Section 3, and in addition setting $\sigma_0 = 0.07$. We obtain, for $t < 0$,

$$ {}'m'(t) = {}'m(t) + 0.045e^{0.1t} $$

with

$$ {}'m(t) = 0.6e^{0.04t}. \tag{35} $$

Investors just before the onset of inflation, had they had foresight, would have been willing to pay 7½ percent more for a share of stock than they did in fact pay.

□ **Changing the financial parameters to give all stockholders the same real return.** We now wish to explore, within the certainty world of our model, the consequences for the "old" investors of relaxing our assumptions as to the constancy of the financial parameters k, η, and δ. In particular, we ask under what circumstances the real return to all investors would be the same, and just equal to σ.

We examine first the case in which η is allowed to vary with time after $t = 0$ (while still keeping k and δ constant) in such a way that the retained earnings still make their required contribution to the construction budget *and*

$$ {}'m'(t) = {}'m(t), \qquad \text{for all } t < 0. \tag{36} $$

It is clear that $\rho_e(t)$ must now follow a new course. In fact, the combination $\rho_e(t)[1 - \eta(t)]$ must be invariant, since (9) must be satisfied for all t and the value of $\dot{X}(t)/X(t)$ is still determined by (17).

Equation (36) can be satisfied in many ways; one way, which seems not unreasonable, is to maintain the purchasing power of the dividend, i.e.,

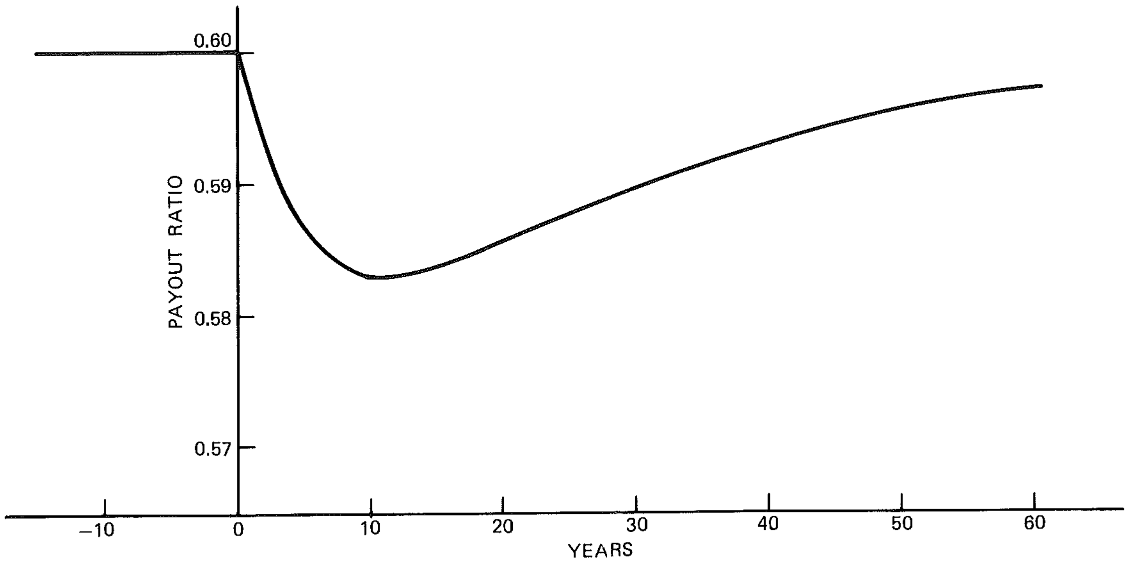
$$ d_I(t)e^{-It} = d_0(t), \tag{37} $$

for all $t \geq 0$. On this assumption, $\eta(t)$ and $\rho_e(t)$ can be determined.

Again the calculation is tedious, and we do not give the details. The numerical results for the same parameter values as before are illustrated in Figure 4, which shows $\eta(t)$, and by the lower curve of Figure 1, which shows $\rho_e(t)$. We see that a temporary and quite small dip in η (from 0.6 to 0.583) suffices to keep all investors equally (and just barely) satisfied in the sense of equation (18); this is accompanied by a much slower growth in ρ_e than before towards its asymptotic value $\rho_e(I)$. It can be shown analytically that the limiting values of $\rho_e(t)$ and $\eta(t)$ as $t \to \infty$ are the same as in the previous example.

We remind the reader at this point that the *increase* in ρ_e required

FIGURE 4
VARIATION OF PAYOUT RATIO SO THAT ALL INVESTORS ARE TREATED EQUALLY



following an increase in I is the salient effect; what is now under discussion is the time course of this increase and the associated adjustments in other parameters. Of course, if the increase in ρ_e were not adequate (as we have assumed it to be), then a slowing of this increase would make it even more inadequate.

Next, suppose that the payout ratio and debt ratio are left unchanged but that the reliance on external equity financing, as expressed by the parameter k, is allowed to vary. If demand is to be met, equation (9) must be satisfied, with $\dot{X}(t)/X(t)$ given by (17). Hence the combination $\rho_e(t)/[1 - k(t)]$ must be preserved. We again choose to satisfy (36) by maintaining the purchasing power of dividends (as per equation (37)). These relations are sufficient to determine $k(t)$. It turns out in our example that a slight increase in $k(t)$ to a maximum value of 0.5176 around the tenth year, and a gradual return thereafter to its steady state value of 0.5, accompanied by a corresponding temporary slowing of the rise of $\rho_e(t)$, reduces the real return received by the old investors to σ_0.

The calculation of the temporary change in the debt ratio required to keep $m'(t) = m(t)$ for all $t < 0$, while holding k and η constant, is a little more involved, and we have not performed it.

The firm may, of course, react to an increase in the rate of inflation in a more complicated, mixed, way. It may, for example, simultaneously change its debt ratio, decrease its payout ratio, suffer a decline in market to book ratio, and rely more heavily on external sources of equity (possibly with dilutionary consequences). The franchise constraint may also be temporarily broken, i.e., the firm may defer growth in output, or it may meet demand in a non-cost minimizing way.

We should emphasize again that we have dealt only with a model of the firm. In reality the firm may, because of regulatory lag, earn either more or less than its allowed rate of return, the stock-financing parameter may vary discontinuously, etc. These effects can be described in a more complicated model but the essential features of our analysis—in particular the required increase in ρ_e when inflation increases—would remain.

Appendix 1

■ We show here that in the case of steady inflation rate I, and with $Z(t) = e^{\gamma t}$, $\gamma > 0$, then $X(t)$ behaves asymptotically like $e^{(\gamma+I)t}$, i.e., (11) is valid. This can be verified explicitly for the case where f is a simple exponential (see (17)). To treat the general case we take the Laplace transform of (10) obtaining

$$[s - (I + \gamma)]^{-1} = \tilde{f}(s - I)[1 + \tilde{C}(s)], \tag{A1}$$

where

$$\tilde{F}(s) \equiv \int_0^\infty e^{-st}F(t)dt.$$

Taking also the Laplace transform of (1) yields

$$\tilde{X}(s) = \tilde{f}(s)[1 + \tilde{C}(s)] = \tilde{f}(s)/\{\tilde{f}(s - I)[s - (I + \gamma)]\}. \tag{A2}$$

Now the asymptotic growth of a function $F(t)$ is determined by the singularity of $\tilde{F}(s)$ with the largest real part, which will here occur at $s = I + \gamma$. Hence $C(t)$ and also $X(t)$ will grow as $e^{(I+\gamma)t}$. (It follows from the fact that $f(t)$ is nonnegative and monotone nonincreasing and that $\int_0^\infty f(t)dt < \infty$, that $\tilde{C}(s)$ in (A1) cannot have a singularity at s, such that Re $s > \gamma + I$.)

Appendix 2

■ To obtain the time dependence of return on total capital, $\rho(t)$, we must calculate the firm's interest payments. Suppose the debt ratio, δ, is constant. Since the rate of inflation, and hence the interest rate, are not constant, we must distinguish between the firm's marginal interest rate, which we denote by $i(t)$, and its embedded interest rate, $i^{em}(t)$.

Using (5) and (9), we can write

$$\rho(t) = i^{em}(t)\delta + \frac{(1 - \delta)(1 - k)}{1 - \eta}\frac{\dot{X}(t)}{X(t)}. \tag{A3}$$

To obtain, $i^{em}(t)$ we have to assume a retirement schedule for debt which will be denoted by $h(t)$, i.e., $h(t)$ is the fraction of a vintage of debt unretired at age t. We then write the total amount of debt outstanding at time t as

$$\mathscr{D}(t) = \int_{-\infty}^{t} \beta(y)h(t - y)dy, \tag{A4}$$

where $\beta(y)$ is the rate of borrowing at time y. Thus the total interest paid at time t is given by

$$Int \equiv i^{em}(t)\delta X(t) = \int_{-\infty}^{t} i(y)\beta(y)h(t - y)dy. \tag{A5}$$

The expression (A5) for total interest payment becomes particularly simple if we assume that (14) holds at all times and use an exponential debt retirement schedule, i.e., $h(t) = e^{-\lambda' t}$ where λ' is the reciprocal of the mean life of debt. Setting $\mathscr{D}(t) = \delta X(t)$, we find the rate of selling new debt to meet the construction budget, $C(t)$ to be

$$\beta(t) = \delta[C(t) + (\lambda' - \lambda)X(t)]. \tag{A6}$$

We are now in a position to solve (A5) for $i^{em}(t)$; we find

$$i^{em}(t) = \begin{cases} i_1 & t \leq 0 \\ i_1 + \Delta\left[1 - \frac{X(0)}{X(t)} e^{-\lambda' t}\right], & t \geq 0, \end{cases} \tag{A7}$$

where i_1 is the interest rate in the presence of inflation I_1.

When (A7) is combined with (17) and (A3), we find explicitly the change in the rate of return ρ necessary to maintain the growth in plant in the presence of inflation.

We have carried out a numerical example to illustrate the required change in $p(t)$, for the parameter values of Section 3, and

$$\begin{aligned} \delta &= 0.4 \\ i_1 &= 0.04 \\ \lambda' &= 0.03. \end{aligned}$$

The result is shown in Figure 2 above.

References

BAUER, J. "Updating Public Utility Regulation." *Public Administration Service,* Chapter 4. Chicago, 1966.

KAHN, A. E. *Economics of Regulation: Principles and Institutions,* Vol. 1. New York: John Wiley & Sons, Inc., 1970.

KERAN, M. W. "Expectations, Money and the Stock Market." Federal Reserve Bank of St. Louis, January 1971, pp. 16–31.

MILLER, M. AND MODIGLIANI, F. "Dividend Policy, Growth and the Valuation of Shares." *Journal of Business,* Vol. 34 (1961), pp. 411–433.