Time Symmetry in the Quantum Process of Measurement*

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We examine the assertion that the "reduction of the wave packet," implicit in the quantum theory of measurement introduces into the foundations of quantum physics a time-asymmetric element, which in turn leads to irreversibility. We argue that this time asymmetry is actually related to the manner in which statistical ensembles are constructed. If we construct an ensemble time symmetrically by using both initial and final states of the system to delimit the sample, then the resulting probability distribution turns out to be time symmetric as well. The conventional expressions for prediction as well as those for "retrodiction" may be recovered from the time-symmetric expressions formally by separating the final (or the initial) selection procedure from the measurements under consideration by sequences of "coherence destroying" manipulations. We can proceed from this situation, which resembles prediction, to true prediction (which does not involve any postselection) by adding to the time-symmetric theory a postulate which asserts that ensembles with unambiguous probability distributions may be constructed on the basis of preselection only. If, as we believe, the validity of this postulate and the falsity of its time reverse result from the macroscopic irreversibility of our universe as a whole, then the basic laws of quantum physics, including those referring to measurements, are as completely time symmetric as the laws of classical physics. As a by-product of our analysis, we also find that during the time interval between two noncommuting observations, we may assign to a system the quantum state corresponding to the observation that follows with as much justification as we assign, ordinarily, the state corresponding to the preceding measurement.

I. INTRODUCTION

ONE of the perennially challenging problems of theoretical physics is that of the "arrow of time." Everyday experience teaches us that the future is qualitatively different from the past, that our practical powers of prediction differ vastly from those of memory, and that complex physical systems tend to develop in the course of time in patterns distinct from those of their antecedents. On the other hand, all the "microscopic" laws of physics ever seriously propounded and widely accepted are entirely symmetric with respect to the direction of time; they are form-invariant with respect to time reversal.[1,2]

The *de facto* absence of time symmetry in nature enters the formal statement of the laws of nature principally in two areas. One of these is thermodynamics, particularly the second law of thermodynamics; the latter proclaims that the entropy of a thermally isolated system can only increase toward the future. The other area is that of cosmogony; our universe is expanding toward the future. Gold[1] has suggested that these two asymmetric phenomena may well be causally related to each other. A third time-asymmetric effect, the preponderance of outgoing radiation in nature over incoming radiation, may be considered to be a special aspect of the second law.

In quantum theory the dynamical laws of motion, either the Schrödinger or the Heisenberg equations, are time symmetric as are their classical counterparts, Hamilton's equations of motion. It has been suggested, though, that asymmetry in the direction of time, and even thermodynamic irreversibility, enters into quantum theory through the theory of measurement.[3,4] Any measurement performed on a quantum system changes its state discontinuously and in a manner not to be described by the Schrödinger or Heisenberg equations of the isolated system. The performance of a measurement leads to the "reduction of the wave packet." That is to say, if the result of the measurement is known, then the quantum state of the system preceding the measurement has been replaced by the eigenvector of the observable that belongs to the eigenvalue recorded. If the outcome of the measurement is not known, the original state vector must now be replaced by a density matrix diagonal with respect to the eigenvectors of the observables measured, each diagonal element equaling the absolute square of the corresponding component of the original state vector. This density matrix is inequivalent to the original state vector in that all phase relations between the components have been destroyed by the act of measurement, though their norms survive in the density matrix.

Quite aside from entropy considerations, the conventional quantum theory of measurements is concerned exclusively with the prediction of probabilities of specific outcomes of future measurements on the basis of the results of earlier observations. Indeed the reduction of the wave packet has as its operational

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[1] T. Gold, in Onzième Conseil de l'Institut International de Physique Solvay, *La Structure et l'Evolution de l'Universe* (Edition Stoops, Brussels, 1958), p. 81; Am. J. Phys. **30**, 403 (1962); Proceedings of the Conference on the Arrow of Time, Cornell University, 1963 (to be published).

[2] See, however, O. Penrose and I. C. Percival, Proc. Phys. Soc. (London) **79**, 605 (1962).

[3] J. von Neumann, *Mathematical Foundations of Quantum Mechanics*, transl. by R. T. Beyer (Princeton University Press, Princeton, 1955).

[4] D. Bohm, *Quantum Theory* (Prentice-Hall Inc., Englewood Cliffs, New Jersey, 1951), cf. in particular, p. 608.

contents nothing but this probabilistic connection between successive observations.[5]

In this paper we propose to examine the nature of the time symmetry in the quantum theory of measurement. Rather than delve into the measurement process itself, which involves a specialized interaction between the atomic system and a macroscopic device,[3–5] we shall simply accept the standard expressions for probabilities of values furnished by the conventional theory. Whereas the conventional theory deals with ensembles of quantum systems that have been "preselected" on the basis of some initial observation, we shall deduce from it probability expressions that refer to ensembles that have been selected from combinations of data favoring neither past nor future. A theory that concerns itself exclusively with such symmetrically selected ensembles (the "time-symmetric theory") will contain only time-symmetric expressions for the probabilities of observations. Logically this time-symmetric theory is contained in the conventional theory but lacks one of the latter's postulates. It will be developed in Sec. II.

In Sec. III we shall consider the case that prior to the final selection some observations are performed that completely destroy coherence of any state previously existing; we shall find that any earlier observations obey probability laws that formally resemble the conventional prediction formula. Likewise, if the initial selection ("preselection") is followed by coherence destroying measurements to be succeeded in turn by some other observations, then these latter observations obey the precise time-reflected expression of the conventional prediction formula. This reflected relationship might be called a "retrodiction" formula. Finally, in Sec. IV we shall return to the true prediction and "retrodiction" situations, i.e., to the consideration of ensembles that have been either strictly preselected or postselected. By adding to the time-symmetric theory one postulate that appears to portray accurately the conditions of our universe (and whose time-reflected proposition does not hold), we are able to recover the conventional asymmetric theory. We present an argument that this asymmetry represents the intrusion of the irreversibility of macroscopic processes into the microscopic domain, so that the totality of the basic (microscopic) laws of nature emerges completely time symmetric.

II. SEQUENCES OF OBSERVATIONS

We shall begin by considering systems which are subjected to sequences of measurements, each of which is individually "complete"; that is to say, that each observation determines a quantum state of the system. We make the conventional assumption about the selection of ensembles of such systems (and of their histories), which is to the effect that initially all systems of the ensemble have yielded a specified nondegenerate eigenvalue of an observable J; no other conditions are imposed. Under these circumstances the conventional quantum theory of measurements states that, given two successive measurements, the probability of a particular outcome of the later observation depends on the outcome of the earlier observation by being the absolute square of the scalar product of the two state-vectors belonging to the two respective eigenvalues. We shall denote the observables to be measured by symbols $A_1, A_2, \cdots, A_k, \cdots$, all of whose eigenvalues are nondegenerate; let the eigenvalues of A_k be denoted by d_k. Only when necessary will distinct eigenvalues of A_k be denoted by Greek superscripts $d_k^{(\alpha)}, d_k^{(\beta)}, \cdots$. For the sake of simplicity we shall work in a Heisenberg representation and assume further that all the A_k are constants of the motion, not necessarily explicitly time-independent. At any rate, between measurements both the quantum states of our systems and the matrix elements of our observables will be constant. If the observables A_k are to be measured in any particular sequence, which, in general, will not correspond to the order of the subscripts $\cdots, k, \cdots$, we shall indicate the sequence of measurements by Latin superscripts, thus: A_k^m.

Suppose now that we perform a sequence of observations, $A_m^{-M}, \cdots, A_i^{j-1}$, yielding the measurements $d_m, \cdots, d_i$; then the probability that the next measurement A_k^j will yield the eigenvalue d_k is

$$p(d_k/d_m,\cdots,d_i)=|\langle d_i|d_k\rangle|^2=\mathrm{Tr}(D_iD_k)\,, \quad (2.1)$$

where the symbol D_k denotes the idempotent operator

$$D_k\equiv|d_k\rangle\langle d_k|\,, \quad (2.2)$$

etc. If the measurement A_k^j is to be followed by $A_l^{j+1}, A_n^{j+2}, \cdots, A_r^{N-1}, A_s^N$, the probability that the respective outcomes will be $d_k, d_l, \cdots, d_r, d_s$ is

$$p(d_k,\cdots,d_r,d_s/d_m,\cdots,d_i)=\mathrm{Tr}(D_sD_r\cdots D_kD_iD_k\cdots D_r)\,. \quad (2.3)$$

Equations (2.1) and (2.3) hold irrespective of the outcome of the measurements $A_m^{-M}, \cdots, A_h^{j-2}$, and irrespective of the outcome of the members of the ensemble subsequent to the performance of the specified observation(s). These expressions summarize the quantitative content of the conventional theory of measurement in quantum physics.

In passing let us briefly comment on the need in quantum theory for constructing ensembles with well-defined probability characteristics. If, in classical mechanics, we had to deal with a system possessing a phase space with a finite volume Ω, then we could define an *a priori* probability density on that phase space that would be invariant with respect to canonical transformations: the constant probability density Ω^{-1}. One could then modify this density in conformity with any restrictions imposed on the physical system, so as to obtain contingency probabilities by purely deductive methods. In other words, in a finite phase space one might construct statistical mechanics employing a

[5] E. P. Wigner, Am. J. Phys. 31, 6 (1963).

standard ensemble as the point of departure. Because in every realistic physical system the phase space has an infinite volume, a transformation-invariant standard probability density does not exist, and one is led into constructing or conjecturing probability distributions to fit various conditions imposed on the ensemble.

The situation in quantum theory is analogous. If Hilbert space were finite-dimensional, then there would be one density matrix distinguished as being representation-invariant, the normalized multiple of the unit matrix, from which all other density matrices could be derived in response to various contingencies. But again, for all realistic physical systems Hilbert space is infinite-dimensional; hence, there is no "standard ensemble" existing *a priori* and independently of any information about our physical system. Thus, formally, we are forced to construct ensembles of systems having certain restrictive properties. Whether particular classes of restrictions lead to ensembles with unambiguous probability characteristics cannot be decided affirmatively by formal analysis alone, though internal inconsistencies might rule out some conjectures. It is clear that the assumptions underlying the conventional theory of quantum measurements are logically admissible.

Next we shall consider a sequence of measurements $J, A_k^{-M}, \cdots, A_j^{-1}, A_i^0, A_l^1, A_m^2, \cdots, A_n^N, F$, in that order. J and F are to be nondegenerate observables like the others, and their eigenvalues are denoted respectively by a and b. We shall now consider an ensemble of systems whose initial and final states are fixed to correspond to the particular eigenvalues a and b, respectively; we ask for the probability that the outcome of the intervening measurements are $d_j, \cdots, d_n$, respectively. This probability, on the strength of Eq. (2.3), is found to be

$$p(d_j,\cdots,d_n/a,b)=\frac{p(d_j,\cdots,d_n,b/a)}{p(b/a)}=\frac{1}{H(a,b)}\operatorname{Tr}(AD_j\cdots D_nBD_n\cdots D_j)\,, \tag{2.4}$$

where

$$H(a,b)=\sum_{j'}\cdots\sum_{n'}\operatorname{Tr}(AD_{j'}\cdots D_{n'}BD_{n'}\cdots D_{j'})\,,\quad A=|a\rangle\langle a|\,,\quad B=|b\rangle\langle b|\,. \tag{2.5}$$

This expression is manifestly time symmetric. If we change the sequence of measurements to $F, A_n^{-N}, \cdots, A_k^M, J$, Eqs. (2.4), (2.5) remain unchanged. In the exceptional case $H(a,b)=0$ the probability $p(d_j\cdots d_n/a,b)$ is not defined.

The probabilities (2.4), (2.5) refer to a sample that has been selected on the basis of required outcomes of specified initial and final observations. This procedure may appear artificial compared to the usual prescription: "Prepare a system so that the value of J (at the beginning) be a." But from a formal point of view we may legitimately specify any selection that could be performed with physical equipment, however complex.

As a matter of fact, in experimental physics selections are frequently based on combinations of initial and final characteristics. Consider a beam of particles that enters a cloud chamber or similar device controlled by a master pulse. For the device to select an event as belonging to a sample to be evaluated statistically, the particle must enter the chamber and, prior to the onset of any manipulation by magnetic fields, etc., satisfy certain requirements. But in order to be counted the particle must also activate the circuits of counters placed below the chamber; thus, we make the selection on the basis of both the initial and the final state. In some experiments even intermediate specifications may be imposed in addition to initial and final conditions. Thus, our formal treatment of initial and final states on an equivalent footing is not inconsistent with experimental procedures used in some investigations.

Equations (2.4), (2.5) may be thought of as providing the foundation for a time-symmetric theory of measurement. If we assumed the existence of ensembles with well-defined probabilities only if selected on the basis of both initial and final states, we should have a logically closed theory, though one that would never permit extrapolations to time intervals lying outside the interstice between initial and final determination. Given ensembles of any kind with well-defined probability dispersions, we can always form subensembles obeying additional restrictions and hence the time-symmetric ensembles can be obtained from those of the conventional theory by means of a deductive process. The reverse does not hold, i.e., we cannot infer the characteristics of broadly defined ensembles from those of more narrowly defined ensembles.

On the basis of Eqs. (2.4), (2.5) we may calculate probabilities involving only some of the measurements between J and F, or we may calculate contingent probabilities referring to partial samples in which the outcomes of some of these measurements are fixed. In particular we can calculate the contingent probability of the outcome d_l^1, given the outcome d_i^0. To obtain this probability we must, of course, sum over all the possible outcomes of the measurements preceding A_i^0 and over all the possible outcomes following A_l^1, keeping, as before, the outcomes of J and F fixed. The result is

$$p(d_l/d_i;a,b)=\frac{\sum_{m'}\cdots\sum_{n'}\operatorname{Tr}(D_iD_lD_{m'}\cdots D_{n'}BD_{n'}\cdots D_{m'}D_l)}{\sum_{l'}\sum_{m'}\cdots\sum_{n'}\operatorname{Tr}(D_iD_{l'}D_{m'}\cdots D_{n'}BD_{n'}\cdots D_{m'}D_{l'})}=|\langle d_i|d_l\rangle|^2\frac{\sum_{m'}\cdots\sum_{n'}\operatorname{Tr}(D_lD_{m'}\cdots D_{n'}BD_{n'}\cdots D_{m'})}{\sum_{l'}\cdots\sum_{n'}\operatorname{Tr}(D_iD_{l'}D_{m'}\cdots B\cdots D_{m'}D_{l'})}\,. \tag{2.6}$$

As expected, the history preceding the measurement A_i^0 drops out of our expression, but the coefficient of the squared matrix element of the conventional prediction (2.1) is sensitive to d_i and to d_l as well as to the subsequent history. In other words, postselection will affect the transition probability from d_i to d_l. That this is unavoidable can be understood easily by the consideration of the extreme case in which all observables $A_m, \cdots, A_n, F$ commute with each other as well as with A_l. Depending on the selection of the eigenvalue b, the transition probability in that case will be either 0 or 1.

It is obvious that the time-reflected relationship to (2.6) also holds. That is to say, if we calculate the contingent probability of d_i knowing the outcome d_l of the observation immediately following, we shall obtain an expression that is independent of the whole history subsequent to the measurement A_l^1, but which will depend on the initial selection a as well as observations scheduled prior to A_i^0.

Let us now consider incomplete measurements. The result (2.4), (2.5) can be generalized immediately if we drop the requirement that each intermediate measurement be complete. According to von Neumann,[3] an incomplete observation projects the initial state not on a particular direction but on a particular (multidimensional) linear subspace of the Hilbert space, and may be represented by an idempotent operator D_k. The form of Eqs. (2.4), (2.5) will remain unchanged under this reinterpretation of the symbols D_k. It should be noted, however, that Eq. (2.6) holds only if A_i is nondegenerate.

The replacement of the initial and final states by mixtures is a bit more involved. If we form an ensemble in which histories beginning with state $|a\rangle$ and ending with state $|b\rangle$ form a fraction c_{ab} of the whole

$$c_{ab}\geq 0\,, \quad \sum_{a'}\sum_{b'} c_{a'b'}=1\,, \tag{2.7}$$

then the probability $p(d_j,\cdots,d_n/\{c\})$ will be

$$p(d_j,\cdots,d_n/\{c\})=\sum_{a'}\sum_{b'} c_{a'b'}p(d_j,\cdots,d_n/a',b')\,. \tag{2.8}$$

There exists no simple expression that would depend on the initial and final density matrices. The probabilities (2.8) depend on the fractions of systems within the ensemble passing from specified initial to specified final states, not merely on the initial distribution $\sum_{b'} c_{ab'}$ and the final distribution $\sum_{a'} c_{a'b}$.

III. ASYMPTOTIC PROCEDURES

Whereas we have been able to obtain time-symmetric ensembles from those depending only on initial selection, the reverse procedure is impossible without an additional postulate; that is to say, given a theory of ensembles based on time-symmetric double-selection procedures, we cannot obtain probabilities for ensembles in which the selection is based only on initial (or only on final) observations by deduction alone. In this sense, the time-symmetric theory of Sec. II is more restricted than the conventional theory of measurements.

There is, however, a way to blunt the effects of either pre- or postselection. The method to be described in this section rests on the fact that in quantum theory the type of interference that we call an observation destroys the "coherence" of the state of a system, producing a new situation that is connected with the original situation only by stochastic laws. This stochastic connection, or the lack of a tighter relationship, may be expressed either in terms of the state vector, or its replacement by a density matrix, or purely in terms of probabilistic assertions. Whatever the mode of description, it is possible to sever different portions of the history of a system from each other by the interposition of certain types of measurements. By preceding the final selection in the time-symmetric theory by such "coherence destroying" manipulations, we may formally recover the prediction formula (2.1); by scheduling such procedures following the initial selection of a time-symmetric ensemble, we may obtain the time-reverse of Eq. (2.1), a "retrodiction" formula.

These possibilities are of considerable interest because they present us with a relatively large class of possible procedures all of which lead, asymptotically in most cases, to substantially similar results. Though the interpolation of coherence destroying manipulations, say before the act of final selection within the framework of the time-symmetric theory, does not relieve us of the logical necessity of performing the act of final selection, the particular choice of observable and of its numerical value used for that final selection has no effect on the probabilities of events preceding the coherence destroying acts.

We shall first indicate particular sets of measurements which destroy coherence more or less completely. Such sets of two consecutive measurements may be constructed in closed form if the Hilbert space of a system is finite dimensional, e.g., if the particles in a monochromatic and well-collimated beam can differ only in their states of polarization. Consider, in this case, two observables A_i and A_j whose eigenvectors are related to each other by a unitary matrix U and whose matrix elements all have the same absolute square $1/n$, n being the number of dimensions of the Hilbert space. One possible unitary matrix with this property is, for instance, the following:

$$U_{kl}=[1/(n)^{1/2}]e^{i\theta_{kl}}\,, \quad \theta_{kl}=(2\pi/n)kl\,. \tag{3.1}$$

Let us denote the idempotent operators to be constructed from the respective eigenvectors of the two observables by D_1 and D_2, respectively, each of these symbols representing n such different operators. Then the following expression constructed with any density matrix M whatsoever is always a multiple of the unit matrix I:

$$\textstyle\sum_{d_1'}\sum_{d_2'} D_1'D_2'MD_2'D_1'=(1/n)I\,. \tag{3.2}$$

As for the infinite-dimensional case, the situation is insofar more involved as there exists no density matrix which is precisely a multiple of the unit matrix. We shall assume that the Hilbert space admits a complete set of commuting operators, each having a continuous range of eigenvalues from $-\infty$ to ∞. We shall call these operators x_s and construct by the usual methods a set of operators p_s which satisfy standard canonical commutation relations with each other and with the x_s. In a somewhat symbolic sense the unitary operators leading from the improper joint eigenfunctions of the x_s to the improper joint eigenfunctions of the p_s, i.e., the Fourier integral operators, possess matrix elements all of the same magnitude as in the previous case. In view of the fact that idempotent operators of the type $x(x_0)$, etc., are not really defined, we introduce idempotent operators $X(x,\Delta)$, defined as integral operators whose kernel equals 1 if $x\epsilon\Delta$, and vanishes otherwise. We cover the space of numerical values of the x_s with a denumerable set of domains Δ without overlap. Similarly, we introduce idempotent operators $P(p,E)$, where the domains E cover the momentum space without overlap. The expression constructed in complete analogy to (3.1) will then not equal a multiple of the unit matrix because of the coarseness of the cell structures established in x space and in p space. However, we may establish a sensible limit if we improve the fineness of both cell structures and if we multiply the expression (3.1) on the left by a factor corresponding to the effective n, eventually becoming infinite, so that the right side can actually tend to the identity operator I (whose trace diverges).

We now return to the expression (2.4) and substitute for a certain number of factors centered on F a multiple of I, both in the numerator and the denominator. The constant of proportionality used is immaterial, as it drops out in any case, and we might use I directly. We then see, almost by inspection, that (2.4) reduces to (2.3), the pure prediction formula, and, likewise, that (2.6) reduces to (2.1). We conclude, then, that because of the asymptotic properties of expressions of type (3.2) the prediction formulas may be recovered from the time-symmetric formulas.

We may derive the corresponding "retrodiction" expression by time reversing the procedure that we have just presented. If we follow the initial selection of an ensemble in the time-symmetric theory by a set of coherence-destroying measurements, then the outcome of subsequent observations is related to the final selection as follows:

$$p(d_k,d_l,\cdots,d_s/a,b)=\mathrm{Tr}(D_kD_l\cdots D_sBD_s\cdots D_l). \tag{3.3}$$

If, in particular, we are concerned with the one observation preceding the final selection, then the probability of the outcome d is

$$p(d/b)=|\langle b|d\rangle|^2. \tag{3.4}$$

The coherence destroying properties of the procedure summarized in Eqs. (3.1), (3.2), and of the corresponding asymptotic procedure outlined for the infinite-dimensional Hilbert space may be demonstrated by straightforward computation. It would be of considerable interest if there were a broad range of procedures having the same effect. Generally, sequences of measurements will destroy coherence to a greater or lesser extent provided that they involve all directions of Hilbert space in noncommuting measurements. There are, of course, degrees of noncommutativity: The noncommutavity may involve varying numbers of directions in Hilbert space, and the eigendirections of consecutive operators may differ from each other by various angles. Formally, the extent to which coherence is destroyed by a given sequence may be evaluated in terms of the degree to which matrices of the general form (3.2) approximate a multiple of the unit matrix. That there is some approach to the unit matrix in a sequence of noncommuting measurements is assured by the results to be found in von Neumann.[3] If $D^{(\alpha)}$ is a set of idempotent operators belonging to the same measurement and with properties

$$D^{(\alpha)}D^{(\beta)}=\delta^{\alpha\beta}D^{(\alpha)},\quad \sum_\alpha D^{(\alpha)}=I, \tag{3.5}$$

and if M is an arbitrary density matrix, then

$$M'=\sum_\alpha D^{(\alpha)}MD^{(\alpha)} \tag{3.6}$$

is also a density matrix and approximates a multiple of the unit matrix I at least as well as M in the following respects: (a) If we define the entropy of M as usual by the expression

$$S=-k\,\mathrm{Tr}(M\ln M), \tag{3.7}$$

then

$$S'\geqslant S. \tag{3.8}$$

The equality holds only if the idempotent operators commute with M. (b) The range of eigenvalues of M' is not greater than the range of eigenvalues of M; that is to say, the upper limit of its eigenvalues is not larger and the lower limit not smaller. Both entropy and range of eigenvalue spectrum are yardsticks for the approach to λI.

Thus, it appears that we can destroy coherence more or less completely by a wide variety of sequences of measurements and thereby obtain the asymptotic prediction and retrodiction situations within the framework of the time-symmetric theory of measurements.

The existence of the retrodiction formula (3.3), (3.4) suggests that the customary assignment of a state vector to a system on the basis of the most recent preceding observation may be somewhat arbitrary. This assignment is based on the intuitive notion that the measurement is the "cause" and the quantum state the "effect," and that cause must precede effect in time. Also, perhaps, there is the notion that the quantum state of a system embodies the maximum of information available to us about the system at any time; ordinarily, we can know the outcome of all observations in the past but not of those yet in the future.

But, as we have seen, under suitable circumstances the usual prediction formula (2.1) may be replaced by the retrodiction formula (3.4), which bases a probabilistic statement about the outcome of one measurement on the outcome of the measurement next following in time. If the measurement of A (whose eigenvalues are being denoted by d) is preceded by coherence destroying operations as we have assumed in deriving Eqs. (3.3) and (3.4), then we know essentially nothing about the outcome of observations preceding A; that is to say, all possible outcomes of such preceding observations are approximately equally likely. Hence, our probabilistic statement about the outcome of the measurement of A is based primarily on the event immediately following, and the information on which our statement is based ought to be incorporated in an appropriate assignment of quantum state. Thus, we are led into assigning the state $|b\rangle$ to the period of time *preceding* the observation of F yielding the eigenvalue b.

From a purely operational point of view, one might eschew the assignment of quantum states to physical systems altogether and instead rely entirely on probabilistic statements referring to carefully defined ensembles. However, as long as one does assign quantum states to physical systems, it appears defensible to do so either in reliance on the (complete) observation immediately preceding (as is customary) or on the one next following, depending on circumstances. This ambiguity indicates that the quantum state of a system, though undoubtedly containing some elements of "reality" independent of any observer, also has subjective aspects.

We shall conclude this section by pointing out that, in general, the dispersion of probabilities of the outcome of one particular observation will be minimized (i.e., the "negative entropy" associated with this dispersion will be maximized) if we use all information about the system's past and future. This statement is a direct consequence of the properties of the entropy function to be found, e.g., in Khinchin.[6] Hence, if both the initial and final state of a system are known, use of the prediction formulas (2.1) or (2.3) instead of (2.4) will lead to a loss in precision of the probabilistic statement concerning the intermediate observation.

IV. DIRECT PREDICTION

By now we have established that the conventional prediction formulas can be recovered from the time-symmetric expressions (2.4) by means of a model that consists of shielding events close at hand from the terminal selection on which (2.4) is based by the interposition of a series of "coherence destroying" experiments. Each measurement constitutes an interference with the physical system which destroys in a limited and mathematically well-described manner its dynamic behavior as an isolated system.

Normally, the prediction formula (2.1) and its corollary (2.3) are not conceived of as depending on carefully managed follow-up maneuvers, but are assumed to be independent of the subsequent history of the system. That this prediction theory is indeed logically independent of the time-symmetric formula (2.4) may be deduced immediately from the circumstance that in our universe the prediction formula is considered to be universally valid, whereas the time-reflected formula, the retrodiction formula, is not.

Consider an ensemble of similar physical systems of arbitrary provenance and select a sample on the strength of a single complete measurement. The conventional theory of measurement then furnishes us, with respect to this selected subensemble, with relative frequencies of outcomes of a subsequent measurement or of a subsequent series of measurements, regardless of the events that may have preceded the initial selection procedure, as well as of those events that follow on the heels of the specified series of measurements, as long as no further selection is involved. The reverse theory would have to concern itself with the probability of the outcome of certain measurements on an ensemble of similar physical systems, the ensemble to be determined solely on the basis of a pure-state selection immediately following the specified series of measurements; the expression for the probabilities should contain no reference to any events following the terminal selection, nor to the manner in which physically similar systems were collected prior to the onset of the series of measurements. Clearly, in our universe no such "retrodiction theory" would be valid: Suppose we constructed a monochromatic and well-collimated beam of particles possessing nonzero spin, performed some observation referring to the spin distribution of the beam, and then followed up with a Stern-Gerlach experiment singling out those particles in the beam being in a very definite spin state. Suppose we ask for the percentage of particles, from among those passing the postselection test, that had specified outcomes in the antecedent experiment (which should refer to an observable not possessing the specified final state as an eigenstate); surely these probabilities would not be independent of the state of polarization of our beam prior to the performance of the first experiment.

We conclude, therefore, that in order to recover the conventional prediction statement from the time-symmetric formulas of Sec. II, we must adopt a postulate that is logically independent of the time-symmetric theory; the postulate that in our universe ensembles chosen on the basis of an initial complete measurement alone possess unambiguous and reproducible probability characteristics. Once we adopt this

[6] A. I. Khinchin, *Mathematical Foundations of Information Theory*, transl. by R. A. Silverman and M. D. Friedman (Dover Publications, Inc., New York, 1957).

postulate the conventional prediction formulas (2.1), (2.3) follow from the time-symmetric formula (2.4) and from the considerations of Sec. III. We found in that section that there are "coherence destroying" procedures that make the "prediction" expressions (2.6) independent of the particular postselection we choose to perform. But if there are methods by which we can make our probabilities independent of the manner of postselection and if, by our new postulate, there exist unambiguous probabilities even in the absence of any postselection, then these two sets of probabilities should be equal.

Logically, it is conceivable that the time reverse of our new postulate should also hold; this would mean that postselection alone results in an ensemble with well-defined and reproducible probability characteristics. Actually we know that in our universe this proposition is untrue. We are thus confronted with an indubitable asymmetry in time direction. It remains to discuss whether this asymmetry is a property of microphysics proper or whether it represents the intrusion of the macroscopic universe on the microscopic scene. Granting that this question does not lend itself to straightforward logical analysis, it appears to us that the construction of ensembles in the real physical universe is a macroscopic operation and that it depends on the realities of the universe as a whole. Let us return once more to our beam of particles endowed with spin.

If we attempt to analyze the different manner in which past and future histories affect its present characteristics, we find that no matter how we gather our beam, its constituent particles have come from one or several "sources" (e.g., a laboratory device, a distant galaxy, etc.), which determine its properties; there simply is no way of avoiding preselection completely. On the other hand, beams are not collimated toward a "sink," unless we arrange it so in our laboratory. This asymmetry is directly associated with the fact that the origins of all kinds of radiations in the universe are spatially and temporally concentrated, and their destinations are not. The nature of ensembles or beams actually occurring in nature is, in fact, macroscopic, not microscopic; it is determined by the same cause as all macroscopic irreversibility, conceivably by the expansion of the universe.[1]

As for the microscopically determined aspects of quantum measurements, we believe that they can be fairly summarized by the statement that in time-symmetrically constructed ensembles the laws of probability are also time symmetric; further, that to the extent that retrodiction situations may be said to exist, they obey the same laws as the corresponding prediction situations.

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[7] H. Margenau, Phil. Sci. **30**, 1 (1963); Ann. Phys. (N. Y.) **23**, 469 (1963). Further references are to be found in these two papers.