Quenching of Einstein coefficients in plasmas

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We investigate possible mechanisms for the recently observed selective quenching of spontaneous emission coefficients of carbon and nitrogen ions (C IV, C III, and N V) in laser-produced plasmas. Various effects of the plasma electrons on the spontaneous emission of the ions are considered. We suggest (by exclusion) that the cause of the phenomenon is the weak trapping of plasma electrons by the radiating ions (producing C III, C II, and N IV ions in highly excited states). These highly excited electrons so strongly shift and change some low-frequency lines that they are not observed. The branching ratio of line intensities from the same upper level then becomes proportional to the fraction of ions not having any trapped electrons.

I. INTRODUCTION

The motivation of the present work is a search for an explanation of the experimental observation of large selective changes in the spontaneous emission coefficients of an ionized atom in a dense plasma. The experiment reported in Ref. 1 involved the measurement of the branching ratio of two lines from a threefold ionized carbon atom C IV, in a CO_2 laser-produced plasma. The two lines correspond to transitions in the isolated ion from the same initial hydrogenlike levels $3p\ ^2P$ (denoted as level 3, which has two sublevels with a small L-S split) to the final levels $3s\ ^2S$ (5801.5 and 5812.1 Å, level 2) and $2s\ ^2S$ (312.42 and 312.46 Å, level 1). The corresponding isolated ion Einstein coefficients are $A_{32}=0.32\times10^8$ and $A_{31}=45.6\times10^8$ sec^{-1}, respectively. It was observed that, as the density of the plasma increased, the branching ratio of the intensity I_{32}/I_{31} decreased by an order of magnitude. We refer the reader to Ref. 1 for details. It was noted there, in particular, that there was (a) almost no change in the frequencies of the lines and (b) strong evidence that the change in the ratio is due almost entirely to a decrease in the emission coefficient of the lower frequency line—the higher frequency line remaining essentially unaffected by the plasma. Similar phenomena were observed in laser produced N V and C III ions[2] as well as for some spontaneous emission inside laser cavities[3] (the latter is, however, not considered in the present work).

The explanation of these observations presents a theoretical challenge. Many "obvious suspects" turn out not be involved at all or to give effects which are too small by some orders of magnitude.[2] Examples of explanations that have been tried include (a) collisions between the bound and plasma electrons, (b) long-range quasistatic effects of the plasma electrons and ions, and (c) change of index of refraction in a plasma medium. Further thought and explicit computations show, however, that (a) does not work because collisions, while giving rise to broadenings and shifts of spectral lines, do not (unless they occur with a "very high" frequency) substantially modify the total line intensity, and the experiment was carefully done to capture the whole line by fitting it with appropriate Lorentzian distributions,[2] (b) does not work because calculations show that the effects are too small. Besides, it would also predict a large line shift which was not observed in the experiment.[1,2] Finally, (c) does not work because the plasma frequency at the density $N\sim10^{19}$ cm^{-3} is much smaller than either of the two frequencies. Readers are referred to Ref. 2 for more discussions on a number of other suspects.

It should be mentioned that variations of Einstein coefficients also occur in other, very different, systems.[4] Their theoretical explanation relies on the effects of some kind of nonuniform medium (or environmental) polarizations, which effectively decouple the interactions between the radiators and the photon propagating modes. In view of this it is natural to ask whether the plasma can play a similar role: indeed, an order-of-magnitude estimate indicates that the density of the plasma electrons near an ion is greatly enhanced at low temperatures due to the Coulomb attraction. However, a detailed analysis given here does not support this conjecture.

In this work we study various effects of the plasma electrons on a partially ionized radiator. At low temperatures we find that owing to the strong, long-range Coulomb interaction between the plasma electrons and the bound electron(s) of the ion, a full quantum-mechanical description is necessary for certain types of problems. In Sec. II the general formalism for the Einstein spontaneous emission in plasmas is presented, with careful distinctions between different kinds of effects. Qualitative and quantitative computations and discussions are given in Sec. III, including comparisons with the experiments. Some algebraic work is left to the appendixes.

The major conclusions of this work can be summarized as follows. The effects of plasma polarization in the vicinity of the ion, which appear large at first sight, are suppressed when all orders are included. This is in contrast to the fact, mentioned above, that in other systems[4] nonuniform medium polarization is believed to be responsible for similar phenomena. What does seem to offer an explanation are the effects of plasma electrons weakly trapped near a partially ionized atom. A trapped plasma electron (spectator) does two things: (i) It induces screening of the zero-point electric-field fluctuations, which couple the ion with the photon field and hence cause spontaneous decays (while the photon modes and their frequency distribution remain basically unchanged since the overall density of the plasma is low); (ii) it stays close to the ion for a long time and so produces much larger shifts in the energy levels of the atom than do the collisional electrons. Our study favors (ii), although in some cases (i) might be significant too.

We believe that the phenomena observed in Refs. 1 and 2 are likely to have the following origin. The radiating ions consist of two classes: those without and those with weakly trapped plasma electrons. The latter are, of course, nothing but highly excited states of the "previous" ions. In the first class the major effect of the plasma is the usual quasistatic Stark and impact collisional broadenings of a few angstroms; these have been widely studied in the literature.[5] The line structure of the second class differs sharply from that of the first. For these ions the line shifts due to the presence of the trapped electrons are particularly large ($\sim 10^2$ Å) for the low-frequency lines. As a result, the spectral lines dissolve into a background that does not have either a Lorentzian or a Gaussian shape. The crucial point here is that the "spectator" electrons induce very large line shifts in the low-frequency lines. On the other hand, the similar (but perhaps smaller) shifts in *frequency* do not produce profound effects on the high-frequency lines since they are typically covered by instrumental broadenings (characterized in terms of wavelength). This picture is supported by our semiquantitative numerical analysis described in Sec. III. We find that there is a semiquantitative fit to the experimental results in C IV and N V by assuming a simple dependence of the trapping probability on the plasma density.

II. GENERAL FORMALISM

Consider the production of photons with wave vector $\mathbf{k}$, polarization vector $\mathbf{e}_{\mathbf{k}_\sigma}$, and frequency $\omega_{\mathbf{k}}$ by a radiator centered at the origin having (for simplicity) one bound electron with position $\mathbf{R}$ and momentum $\mathbf{P}$. The whole system can be divided into three parts: the photons in the given mode, the plasma electrons, and the bound electron. We shall denote these parts by the subscripts ph, pl, and e, respectively. The interaction between the photons and the bound electron can be written as

$$H_{\text{ph-}e}=\frac{e}{m}\left[\frac{2\pi\hbar}{\omega_{\mathbf{k}}}\right]^{1/2}(b_{\mathbf{k}_\sigma}^\dagger+b_{\mathbf{k}_\sigma})\mathbf{P}\cdot\mathbf{e}_{\mathbf{k}_\sigma}\ , \tag{2.1}$$

where $b_{\mathbf{k}_\sigma}^\dagger$ and $b_{\mathbf{k}_\sigma}$ are the usual creation and annihilation operators for the photon mode. The interactions of the photon mode with the plasma electrons are conveniently expressed in terms of $\{a_n^\dagger,a_{n'}\}$, the creation and annihilation operators of the plasma electrons with eigenstates $|n\rangle$ to be specified below. We have

$$H_{\text{ph-pl}}=\frac{e}{m}\left[\frac{2\pi\hbar}{\omega_{\mathbf{k}}}\right]^{1/2}\times\sum_{n,n'}[b_{\mathbf{k}_\sigma}^\dagger\langle n|\exp(-i\mathbf{k}\cdot\mathbf{r})\hat{\mathbf{p}}|n'\rangle\cdot\mathbf{e}_{\mathbf{k}_\sigma}\times a_n^\dagger a_{n'}+\text{H.c.}]\ , \tag{2.2}$$

with $\hat{\mathbf{p}}\equiv-i\hbar\nabla$ the momentum operator which operates on the wave functions of the plasma electron states; we shall always ignore the spin of the electrons. The interactions between the plasma electrons and the bound electron can be expanded in powers of $\mathbf{R}$ and the zero-order term included into the total static potential. The dipole and quadrupole interactions are then given by

$$H_{\text{pl-}e}=\sum_{n,n'}a_n^\dagger a_{n'}\left\langle n\left|\left[\frac{e^2\mathbf{r}}{r^3}\right]\cdot\mathbf{R}+\mathbf{R}\cdot\frac{e^2}{r^3}\left[\overleftrightarrow{\mathbf{I}}-3\frac{\mathbf{rr}}{r^2}\right]\cdot\mathbf{R}\right|n'\right\rangle\ , \tag{2.3}$$

where $\overleftrightarrow{\mathbf{I}}$ is the unit tensor. They are equally important for the line shifts, as will become evident below.

The static potential $V(r)$ felt by the plasma electrons near the radiator will, for a partially ionized atom, be mainly an attractive Coulomb potential, whereas for a neutral atom it will be of short range (either repulsion or resonance scattering). This interaction is most naturally taken care of by letting $|n\rangle$ be the eigenstates of the plasma electrons in the potential $V(r)$. The set $\{|n\rangle\}$, will, therefore, in general contain bound states as well as continuum scattering states.

A. Transition rate

In the Heisenberg picture, the equation of motion for the photon operator $b_{\mathbf{k}_\sigma}^\dagger(t)$ is

$$\frac{db_{\mathbf{k}_\sigma}^\dagger(t)}{dt}=i\omega_{\mathbf{k}}b_{\mathbf{k}_\sigma}^\dagger(t)+\frac{e}{m}\left[\frac{2\pi}{\hbar\omega_{\mathbf{k}}}\right]^{1/2}\left[\mathbf{P}(t)+\sum_{n,n'}\langle n|\exp(+i\mathbf{k}\cdot\mathbf{r})\hat{\mathbf{p}}|n'\rangle a_n^\dagger(t)a_{n'}(t)\right]\cdot\mathbf{e}_{\mathbf{k}_\sigma} \tag{2.4}$$

and its conjugate for $b_{\mathbf{k}_\sigma}(t)$. The solution of (2.4) is given by

$$b_{\mathbf{k}_\sigma}^\dagger(t)=b_{\mathbf{k}_\sigma}^\dagger(0)\exp(i\omega_{\mathbf{k}}t)+\int_0^t dt'\exp[i\omega_{\mathbf{k}}(t-t')]\frac{e}{m}\left[\frac{2\pi}{\hbar\omega_{\mathbf{k}}}\right]^{1/2}M_{\mathbf{k}}^\dagger(t')\ , \tag{2.5}$$

where

$$M_{\mathbf{k}}(t')=\left[\mathbf{P}(t')+\sum_{n,n'}\langle n|\exp(-i\mathbf{k}\cdot\mathbf{r})\hat{\mathbf{p}}|n'\rangle a_n^\dagger(t')a_{n'}(t')\right]\cdot\mathbf{e}_{\mathbf{k}_\sigma} . \tag{2.6}$$

We need to specify an initial state of the whole system. Since we are only interested in the spontaneous emission corresponding to the transition of the bound electron from an upper lever $|i\rangle$ with energy E_i (initial state) to a lower lever $|f\rangle$ with energy E_f (final state), we may assume that the photon field is initially in its ground state while the plasma electrons are described by a density matrix $\rho(\{a^\dagger,a\})$ to be specified later. The expected number of photons at time t is then

$$\langle b_{\mathbf{k}_\sigma}^\dagger(t)b_{\mathbf{k}_\sigma}(t)\rangle=\left[\frac{e}{m}\right]^2\frac{2\pi}{\hbar\omega_{\mathbf{k}}}\int_0^t dt'\int_0^t dt''\exp[-i\omega_{\mathbf{k}}(t'-t'')]\mathrm{Tr}_{\mathrm{pl}}[\rho(\{a^\dagger,a\})\langle i|M_{\mathbf{k}}^\dagger(t')|f\rangle\langle f|M_{\mathbf{k}}(t'')|i\rangle] , \tag{2.7}$$

where $\langle\ \rangle$ is the average over the ensemble density of the system at time t and $\mathrm{Tr}_{\mathrm{pl}}$ means the trace over the plasma electrons and we have inserted the final state $|f\rangle$ in the middle to project out the contribution from the particular transition we are interested in. As usual, the spontaneous emission rate per unit time is given by

$$W_{\mathbf{k}}=\lim_{t\to\infty}\frac{\langle b_{\mathbf{k}_\sigma}^\dagger(t)b_{\mathbf{k}_\sigma}(t)\rangle}{t} . \tag{2.8}$$

The total line intensity can be obtained by integration over photon modes near the frequency $\omega_0=(E_i-E_f)/\hbar$.

The formalism used above, though somewhat awkward for the conventional discussion of quasistatic or collisional line broadenings (which depends on the "fluctuations" of the plasma), is useful for discussion of the effects of polarization as well as trapping of plasma electrons. In any case, the use of second quantization is merely symbolic since in reality one is always far away from having a degenerate Fermi gas.

B. Polarization of the plasma electrons

Note that the second term in (2.6) does not explicitly involve $\mathbf{R}(t)$. The dependence on $\mathbf{R}(t)$ will come from the coupling of the plasma electrons with the bound electron. Physically it represents the effect of medium polarization—the bound electron can be indirectly coupled to the photon field through the medium, i.e., the plasma electrons. The interactions (2.2) induce perturbations of the plasma electron states. We shall need $a_n^\dagger(t)$ and $a_n(t)$ to first order in $\mathbf{R}(t)$. This is sufficient for the dipole transition since the linear polarization will dominate the contribution [for other cases, e.g., forbidden transitions, inclusion of higher orders of $\mathbf{R}(t)$ might be necessary]. The equation of motion for $a_n^\dagger(t)$ is

$$\dot{a}_n^\dagger(t)=\frac{i}{\hbar}\left[\varepsilon_n a_n^\dagger(t)+\sum_{n'}a_{n'}^\dagger(t)\left\langle n'\left|\left[\frac{e^2\mathbf{r}}{r^3}\right]\right|n\right\rangle\cdot\mathbf{R}(t)\right] , \tag{2.9}$$

and its conjugate for $a_n(t)$, where ε_n and $\varepsilon_{n'}$ are the eigenenergies. For the calculation of the matrix element $\langle f|M_{\mathbf{k}}|i\rangle$ the time dependence of $\mathbf{R}(t)$ may be taken proportional to $\exp(-i\omega_0 t)$. Writing

$$a_n^\dagger(t)=a_n^{\dagger(0)}(t)+\delta a_n^\dagger(t) , \tag{2.10}$$

and similarly for $a_n(t)$, then, to first order in the perturbation, (2.9) yields

$$\delta_n(t)=\sum_{n'}\frac{\langle n|(e^2\mathbf{r}/r^3)|n'\rangle\cdot\mathbf{R}(t)}{(\varepsilon_{n'}-\varepsilon_n)+\hbar\omega_0}a_{n'}^{(0)}(t) , \tag{2.11a}$$

$$\delta a_n^\dagger(t)=\sum_{n'}a_{n'}^{\dagger(0)}(t)\frac{\langle n'|(e^2\mathbf{r}/r^3)|n\rangle\cdot\mathbf{R}(t)}{(\varepsilon_{n'}-\varepsilon_n)-\hbar\omega_0} , \tag{2.11b}$$

where $\{a_n^{\dagger(0)}(t),a_n^{(0)}(t)\}$ are the zeroth-order Heisenberg operators and some time-independent terms have been ignored.

Using (2.6), (2.10), (2.11), and the relation

$$\langle f|\mathbf{R}(t)|i\rangle=i\frac{\langle f|\mathbf{P}(t)|i\rangle}{m\omega_0} , \tag{2.12}$$

we obtain for the transition matrix element (ignoring the effects of photons on the operators of the plasma and bound electrons)

$$\langle f|M_{\mathbf{k}}(t)|i\rangle=\langle f|\mathbf{P}(t)|i\rangle\cdot\left[\overleftrightarrow{\mathbf{I}}+\sum_{n,n'}\overleftrightarrow{\mathbf{T}}_{n,n'}(t)\right]\cdot\mathbf{e}_{\mathbf{k}_\sigma} , \tag{2.13}$$

where

$$\overleftrightarrow{\mathbf{T}}_{n,n'}(t)=-i\frac{a_n^{\dagger(0)}(t)a_{n'}^{(0)}(t)}{m\omega_0}\sum_{n''}\left[\frac{\langle n|(e^2\mathbf{r}/r^3)|n''\rangle\langle n''|e^{-i\mathbf{k}\cdot\mathbf{r}}\hat{\mathbf{p}}|n'\rangle}{\hbar\omega_0+(\varepsilon_{n''}-\varepsilon_n)}-\frac{\langle n''|(e^2\mathbf{r}/r^3)|n'\rangle\langle n|e^{-i\mathbf{k}\cdot\mathbf{r}}\hat{\mathbf{p}}|n''\rangle}{\hbar\omega_0-(\varepsilon_{n''}-\varepsilon_{n'})}\right] . \tag{2.14}$$

Going back to (2.7), we need to carry out the average over the initial distribution of the plasma electrons. We must distinguish the discrete line ω_0 from the continuous background. There are two types of diagrams in (2.7): (a) The operators $a_n^{(0)}$ and $a_{n'}^{\dagger(0)}$ in $\sum_{n,n'}\overleftrightarrow{\mathbf{T}}_{n,n'}$ are connected to those in $\sum_{n,n'}\overleftrightarrow{\mathbf{T}}_{n,n'}^\dagger$. This corresponds to transitions of the plasma electrons into different final states and results in a continuous large spreading of the spectrum over the frequencies near ω_0. (b) The operators $a_n^{(0)}$ and $a_{n'}^{\dagger(0)}$ are connected within each $\sum_{n,n'}\overleftrightarrow{\mathbf{T}}_{n,n'}$ or $\sum_{n,n'}\overleftrightarrow{\mathbf{T}}_{n,n'}^\dagger$ itself, which gives rise to modifications of the discrete line at ω_0. Evidently only the latter is relevant to our consideration and we can bring the average inside $\sum_{n,n'}\overleftrightarrow{\mathbf{T}}_{n,n'}$ and $\sum_{n,n'}\overleftrightarrow{\mathbf{T}}_{n,n'}^\dagger$. This yields an effective matrix element

$$\langle f|M_{\mathbf{k}}(t)|i\rangle_{\text{eff}}=\langle f|\mathbf{P}(t)|i\rangle\cdot\mathbf{e}_{\mathbf{k}_\sigma}(\overleftrightarrow{\mathrm{I}}+\langle\overleftrightarrow{\mathrm{T}}\rangle_{\text{av}}) , \tag{2.15}$$

with

$$\langle\overleftrightarrow{\mathrm{T}}\rangle_{\text{av}}=\sum_{n,n'}\exp\left[\frac{i}{\hbar}t(\varepsilon_n-\varepsilon_{n'})\right]\frac{\langle a_n^{\dagger(0)}a_{n'}^{(0)}\rangle_{\text{av}}}{im\omega_0}$$
$$\times\sum_{n''}\left[\frac{\langle n|(e^2\mathbf{r}/r^3)|n''\rangle\langle n''|e^{ik\cdot r}\hat{\mathbf{p}}|n'\rangle}{\hbar\omega_0+(\varepsilon_{n''}-\varepsilon_n)}-\frac{\langle n''|(e^2\mathbf{r}/r^3)|n'\rangle\langle n|e^{-ik\cdot r}\hat{\mathbf{p}}|n''\rangle}{\hbar\omega_0-(\varepsilon_{n''}-\varepsilon_{n'})}\right] , \tag{2.16}$$

where $\langle\ \rangle_{\text{av}}$ refers to the average over the distribution of the plasma electrons.

Initial distribution of the plasma electrons

The determination of the initial state of the plasma electrons presents some problems. A simple choice would be the usual grand canonical ensemble which leads to the Fermi (or in our case, Maxwell) distribution function $f(\varepsilon_n)$. Some care needs to be taken, however, since the plasma electrons are not just centered around this particular radiator and there are bound states. We show in Appendix A that as far as the continuum states are concerned this choice is fairly convincing and, furthermore $\{|n\rangle\}$ can be most naturally identified as the set of scattering states $\{|\psi_{\mathbf{p}}\rangle\}$, a plane wave plus outgoing spherical scattering waves.

On the other hand, there are some problems with the trapping of plasma electrons near the radiator. These evidently depend on the details of the inelastic processes which determine how the whole system evolves (e.g., C IV→C III→ $\cdots$). The simple canonical ensemble description therefore may fail for the bound states and the actual trapping probability becomes uncertain. Since a trapped electron stays for a long time close to the ion and may cause large effects on its spectral properties, we divide ions into two classes according to whether they do or do not have trapped electrons. The effects of the plasma on the two classes are quite different and we discuss them separately. We then investigate the statistics and dynamics of the trapping and escape of the plasma electrons.

C. Line shifts and broadenings

The collisional line shifts and broadenings do not appear in (2.16). In fact, they arise from the perturbations of $\mathbf{P}(t)$ in the matrix element $\langle f|M_{\mathbf{k}}(t)|i\rangle$ and we have to study the perturbations of all the eigenstates of the bound electron. Denoting the creation and annihilation operators of these states by $\{A_\alpha^\dagger, A_\alpha\}$, we have

$$\mathbf{P}(t)=\sum_{\alpha,\alpha'}\langle\alpha|\mathbf{P}|\alpha'\rangle A_\alpha^\dagger(t)A_{\alpha'}(t) , \tag{2.17}$$

while

$$\dot{A}_\alpha^\dagger(t)=i\frac{E_\alpha}{\hbar}A_\alpha^\dagger(t)+\sum_{\alpha',n,n'}A_{\alpha'}^\dagger(t)a_n^\dagger(t)a_{n'}(t)\left\{\langle\alpha'|\left\langle n\left|e^2\left[\frac{\mathbf{r}}{r^3}\cdot\mathbf{R}+2\mathbf{R}\cdot\left[\frac{\overleftrightarrow{\mathrm{I}}}{r^3}-3\frac{\mathbf{rr}}{r^5}\right]\cdot\mathbf{R}\right]\right|n'\right\rangle|\alpha\rangle\right\} , \tag{2.18}$$

where the quadrupole term has now been included. Note that it is customary to ignore the perturbations of $\{a_n^\dagger(t),a_n(t)\}$ in this context.[5] As one can see, the nonvanishing diagonal matrix element of the quadrupole term already gives energy shifts to the unperturbed states, while the shifts due to the dipole term usually have to enter via the mixing of different states—their magnitude depends on the spacing between the levels. The former is, in fact, a dominant source of the so-called impact broadenings and the latter causes the well-known quasistatic Stark broadenings; both of them play important roles in practice.[5] In this work we turn our attention to a less extensively discussed problem, the effects of trapping of plasma electrons near the radiator. These "spectator electrons" are, we believe, most relevant to the understanding of the observations reported in Refs. 1 and 2.

III. IONIZED ATOM IN A PLASMA

To study the radiation of a partially ionized atom in a dense plasma we have to consider the wave functions of the plasma electrons inside the static long-range Coulomb potential. Let the net charge of the ion be Ze, then the attractive potential is

$$V(r)=-\frac{Ze^2}{r}\quad\text{or}\quad V_{\mathbf{p},\mathbf{p}'}=-\frac{4\pi Ze^2}{(\mathbf{p}-\mathbf{p}')^2} \tag{3.1}$$

in the momentum representation. Note that the two matrix elements in (2.16) can be transformed into each other via

$$\left\langle n\left|\left[\frac{e^2\mathbf{r}}{r^3}\right]\right|n'\right\rangle=\frac{i}{Z\hbar}(\varepsilon_{n'}-\varepsilon_n)\langle n|\hat{\mathbf{p}}|n'\rangle , \tag{3.2}$$

where we have ignored the factor $\exp(i\mathbf{k}\cdot\mathbf{r})$ in (2.16) since $\mathbf{k}\cdot\mathbf{r}\ll 1$. Even if one includes various screenings, i.e., when (3.1) is no longer valid, (3.2) still holds since we also replace

$$\left[\frac{e^2\mathbf{r}}{r^3}\right]\to\frac{1}{Z}\nabla V(r)=\frac{i}{Z\hbar}[\hat{\mathbf{p}},H] . \tag{3.3}$$

A. Plasma electrons in the continuum states

Collisions of the incident plasma electrons with the ion usually induce spectral line shifts and broadenings, which has been a subject of extensive studies in the literature. In addition, there might also be effects of plasma polarization. For $\varepsilon_n > 0$, we have $\{|n\rangle = \{|\psi_{\mathbf{p}}\rangle\}$ (where $|\psi_p\rangle$ is the scattering wave function mentioned in Sec. II B). Therefore

$$\langle \overleftrightarrow{\mathrm{T}} \rangle_{\mathrm{av}} = \sum_{\mathbf{p},\mathbf{p}'} \frac{[f(\varepsilon_{\mathbf{p}})-f(\varepsilon_{\mathbf{p}'})]}{mZ} \times \frac{(\varepsilon_{\mathbf{p}'}-\varepsilon_{\mathbf{p}})\langle \psi_{\mathbf{p}}|\hat{\mathbf{p}}|\psi_{\mathbf{p}'}\rangle\langle \psi_{\mathbf{p}'}|\hat{\mathbf{p}}|\psi_{\mathbf{p}}\rangle}{(\hbar\omega_0)-(\varepsilon_{\mathbf{p}'}-\varepsilon_{\mathbf{p}})^2} . \quad (3.4)$$

To proceed further, it is convenient to use the so-called Coulomb unit where all lengths are scaled by the unit length

$$r_0 \equiv \frac{\hbar^2}{me^2Z} = \frac{0.529\ \text{Å}}{Z} . \quad (3.5)$$

In terms of this, the dimensionless momentum p,p' are usually very small (in our case, $\sim 0.2-0.1$). It is well known that in a Coulomb potential the wave function $\langle \mathbf{r}|\psi_p\rangle$ has the simple expression[6]

$$\langle \mathbf{r}|\psi_{\mathbf{p}}\rangle = e^{\pi/2p}\Gamma(1-i/p)\exp(i\mathbf{p}\cdot\mathbf{r}) \times F(i/p,1,ipr-i\mathbf{p}\cdot\mathbf{r}) , \quad (3.6)$$

where $F(\alpha,\gamma,z)$ is the confluent hypergeometric function and $\Gamma(z)$ is the Γ function. The calculation of the matrix element in (3.4) is rather lengthy and is left to Appendix B. The result is

$$\langle \psi_{\mathbf{p}}|\hat{\mathbf{p}}|\psi_{\mathbf{p}'}\rangle = \frac{-8\hbar\pi\psi_{\mathbf{p}}^*(0)\psi_{\mathbf{p}'}(0)\exp(-\pi/p')}{(\mathbf{p}-\mathbf{p}')^2(p'^2-p^2+i0^+)} \times A(\mathbf{p},\mathbf{p}')[\mathbf{p}B(\mathbf{p},\mathbf{p}')-\mathbf{p}'B^*(\mathbf{p}',p)] , \quad (3.7a)$$

where

$$A(\mathbf{p},\mathbf{p}') = \left[\frac{p'^2-p^2+i0^+}{(\mathbf{p}-\mathbf{p}')^2}\right]^{i/p-i/p'} \quad (3.7b)$$

and

$$B(\mathbf{p},\mathbf{p}') = \left[1+\frac{i}{p}\right]F\left[-\frac{i}{p},1+\frac{i}{p'},2,1-\frac{(\mathbf{p}-\mathbf{p}')^2}{(p+p')^2}\right] , \quad (3.7c)$$

with $F(\alpha,\beta,\gamma,z)$ the hypergeometric function. For $z=0$ it is 1, whereas for $z=1$ we have

$$\left[1+\frac{i}{p}\right]F\left[-\frac{i}{p},1+\frac{i}{p'},2,1\right] = \frac{\Gamma(1+i/p-i/p')}{\Gamma(1+i/p)\Gamma(1-i/p')} . \quad (3.8)$$

Furthermore, at the origin

$$|\psi_{\mathbf{p}}(0)|^2 = \frac{2\pi}{p[1-\exp(-2\pi/p)]} . \quad (3.9)$$

Combining these together and substituting them into (3.4) we then obtain the influence of the incident plasma polarization on the spontaneous decay rate.

In the limit $p,p' \gg 1$ the result agrees with the first-order perturbation result (with the conventional units restored)

$$\langle \overleftrightarrow{\mathrm{T}}^{(1)} \rangle_{\mathrm{av}} = \sum_{\mathbf{p},\mathbf{p}'} \frac{(4\pi)^2Z\hbar^2e^4[(\mathbf{p}-\mathbf{p}')(\mathbf{p}-\mathbf{p}')][f(\varepsilon_{\mathbf{p}})-f(\varepsilon_{\mathbf{p}'})]}{m[(\mathbf{p}-\mathbf{p}')^2]^2[(\hbar\omega_0)^2-(\varepsilon_{\mathbf{p}'}-\varepsilon_{\mathbf{p}})^2](\varepsilon_{\mathbf{p}'}-\varepsilon_{\mathbf{p}})} . \quad (3.10)$$

In the opposite limit $p,p' \ll 1$ one might expect, at first sight, that (3.9) would increase this by some orders of magnitude. However, $\langle \overleftrightarrow{\mathrm{T}} \rangle_{\mathrm{av}}$ is further suppressed by including all the higher-order corrections (which is precisely our case). This is because the matrix element (3.7) is always suppressed (at least partially) by the exponential factor $\exp(-\pi/p)$ or $\exp(-\pi/p')$ except for $(\mathbf{p}-\mathbf{p}')^2 \ll (p+p')^2$ where the first-order result (3.10) persists. Furthermore, inclusion of additional scattering waves from short-range non-Coulomb interactions[6] or virtual transitions between the continuum states and bound states does not change this feature. Finally, $\langle \overleftrightarrow{\mathrm{T}} \rangle_{\mathrm{av}}$ always tends to increase the spontaneous emission rate, as can be easily understood from the well-known effect that the refraction index in the plasma is usually smaller than 1 for frequencies higher than the plasma oscillation frequency.

One can easily estimate the order of magnitude of the plasma density required for the effect of polarization to be significant. Equation (3.10) may be further simplified

$$\|\langle \overleftrightarrow{\mathrm{T}}^{(1)} \rangle_{\mathrm{av}}\| = N\left[\frac{2\pi\hbar^2}{mkT}\right]^{3/2}\left[\frac{8Zme^4}{3\pi^2kT\hbar^2}\right]Y\left[\frac{\hbar\omega_0}{kT}\right] , \quad (3.11)$$

where

$$Y(x) = \int_0^\infty \int_0^\infty dp\,dp' \frac{pp'[\exp(-p^2)-\exp(-p'^2)]}{[x^2-(p^2-p'^2)^2](p^2-p'^2)} \times \ln\left|\frac{p+p'}{p-p'}\right| . \quad (3.12)$$

Taking, for example, the data given in Ref. 1 for the lower-frequency lines of the C IV ion, $Z=3$, $kT \cong 5$ eV, $\omega_0\hbar \cong 2$ eV, $\langle p \rangle \cong (2mkT/\hbar^2)^{1/2} \cong 10^8/\text{cm}$, we get $\|\langle \overleftrightarrow{\mathrm{T}}^{(1)} \rangle_{\mathrm{av}}\| \cong (2\times10^{-22})N$, which is obviously too small for the density $N \sim 10^{19}\ \text{cm}^{-3}$ considered here.

B. Weak trapping of plasma electrons

Screening of trapped electrons

The existence of many bound states for the long-range Coulomb potential raises new questions and possibilities. Note that we are basically interested in the weak trapping case where the trapped plasma electrons are still far away from the center. We first look at the screening of a trapped electron. Let us assume that the electron is located in a given energy eigenstate, say $|n_0\rangle$. Equation (2.16) can be readily generalized to this case. In fact, we should now add an additional contribution from the trapped electron to (3.4). This is given by

$$\Delta\langle\overleftrightarrow{\mathrm{T}}\rangle_{\mathrm{av}}=\sum_n \frac{2(\varepsilon_n-\varepsilon_{n_0})\langle n_0\rangle\langle n_0|\hat{\mathbf{p}}|n\rangle\langle n|\hat{\mathbf{p}}|n_0\rangle}{Zm\,[(\hbar\omega_0)^2-(\varepsilon_{n_0}-\varepsilon_n)^2]} \;. \tag{3.13}$$

Using

$$\langle n|\hat{\mathbf{p}}|n'\rangle=\frac{i}{\hbar}m\,(\varepsilon_n-\varepsilon_{n'})\langle n|\mathbf{r}|n'\rangle \;, \tag{3.14}$$

then in the limit $\hbar\omega_0\to 0$ (3.13) yields

$$\Delta\langle\overleftrightarrow{\mathrm{T}}\rangle_{\mathrm{av}}|_{\hbar\omega_0\to 0}=-\frac{1}{Z}\overleftrightarrow{\mathrm{I}}, \quad \text{for } \hbar\omega_0\to 0 \tag{3.15}$$

due to the completeness of the eigenfunctions. This is obviously a "perfect" screening of the photon electron field by the trapped electron. Equation (3.15) still holds when the trapped electron spreads over several eigenstates, except that now the off-diagonal correlation might induce some satellite lines [cf. (2.16) and its derivation]. A crucial question is then when does (3.15) become invalid as $\hbar\omega_0$ is increased. Clearly, from (3.13) a sufficient criterion for the validity of (3.15) would be $\hbar\omega_0 \ll$ the energy spacing around the state $|n_0\rangle$. This is too strict, however, and, in fact, in our case $\hbar\omega_0$ is roughly of the order of the energy spacing between different eigenstates. This causes further complications, but since the matrix elements in (3.13) are well known,[7] numerical evaluations can and should be obtained.

Energy shifts and line broadenings

Another and, we believe, the most important consequence of the trapping of the plasma electrons is to give rise to large energy shifts in the electronic levels of the ion (and therefore broadenings). This is due to the fact that the trapped plasma electrons remain close to the ion for a long time. We first roughly estimate the order of magnitude of the induced energy shifts. For the dipole interaction, the energy shifts are about

$$(\delta E)_d \sim \frac{|\tilde{\mathbf{R}}|^2(eE)^2}{\hbar\omega_0} \sim \frac{|\tilde{\mathbf{R}}|^2 e^4}{\hbar\omega_0 r^4} \;, \tag{3.16a}$$

where $\hbar\omega_0$ and $\tilde{\mathbf{R}}$ stand for the relevant level spacing and matrix element between eigenstates, whereas for the quadrupole,

$$(\delta E)_q \sim \frac{|\tilde{\mathbf{R}}|^2 e^2}{r^3} \;. \tag{3.16b}$$

Note that from (2.18) the average over the quadrupole induced shifts is zero since the average distribution of the trapped electrons should be spherically symmetric so that the quadrupole vanishes. In contrast, the average over the dipole induced shifts is, in general, nonzero and therefore require more attention.

Considering now as an illustration the C IV ion, $\hbar\omega_0=2.1$ eV and a good estimate of $|\tilde{\mathbf{R}}|$ can actually be made from the knowledge of the Einstein A coefficient. Using the value presented in the Introduction and the well-known formula $A=4e^2\omega^3|\tilde{\mathbf{R}}|^2/(3\hbar c^3)$, we obtain $|\tilde{\mathbf{R}}|\cong 0.9$ Å. Thus

$$(\delta E)_d \sim (8.3\times 10^4)\left[\frac{r_0}{r}\right]^4 \;, \tag{3.17a}$$

$$(\delta E)_q \sim (2\times 10^3)\left[\frac{r_0}{r}\right]^3 \;, \tag{3.17b}$$

in eV. For a trapped plasma electron in a hydrogenlike state $|\psi_{nlm}\rangle$ we have[7]

$$\left[\frac{r_0}{r}\right]^4=\frac{3n^2-l(l+1)}{2n^5(l+3/2)(l+1)(l+1/2)l(l-1/2)} \;, \tag{3.18a}$$

$$\left[\frac{r_0}{r}\right]^3=\frac{1}{n^3(l+1)(l+1/2)l} \;. \tag{3.18b}$$

Taking, for example (cf. the discussion below), $n=10$ and $l=8$, we have $(\delta E)_d \sim 6.5\times 10^{-3}$ eV and $(\delta E)_q \sim 3.9\times 10^{-3}$ eV; whereas for $n=7$ and $l=5$, we get $(\delta E)_d \sim 0.2$ eV and $(\delta E)_q \sim 0.03$ eV.

It is possible to study more quantitatively the dipole induced energy shifts. Going back to (2.18) we need the matrix elements between two hydrogenlike eigenstates $|\psi_{nlm}\rangle$ and $|\psi_{n'l'm'}\rangle$ for the trapped electron. From (3.2) and (3.14), we obtain

$$\left\langle \psi_{nlm}\left|\left[\frac{e^2\mathbf{r}}{r^3}\right]\right|\psi_{n'l'm'}\right\rangle =\frac{e^2}{4r_0^2}\left[\frac{1}{n^2}-\frac{1}{n'^2}\right]^2\left\langle \psi_{nlm}\left|\frac{\mathbf{r}}{r_0}\right|\psi_{n'l'm'}\right\rangle \;. \tag{3.19}$$

Taking, for example, the $3\,^2S$ state of C IV and assuming that there is a trapped electron with quantum number nl, then the perturbing level $n'(l-1)$ will induce an energy shift (within a factor of 2)

$$(\delta E)_{nl,n'(l-1)} \cong 2500\frac{|\langle r/r_0\rangle_{nl,n'(l-1)}|^2}{2.13-(\varepsilon_n-\varepsilon_{n'})}\left[\frac{1}{n^2}-\frac{1}{n'^2}\right]^4 \;, \tag{3.20}$$

where $\langle r/r_0\rangle_{nl,n'(l-1)}$ stands for the usual radial integrals of the hydrogen wave functions (see, e.g., pp. 131–133 of Ref. 7) and all energies are measured in units of eV. Equation (3.20) can be used to evaluate quite precisely the energy shifts. Note that for large n it is possible to find an n' such that the denominator $[2.13-(\varepsilon_n-\varepsilon_{n'})]$ becomes very small and therefore induces a large shift. For instance we take $l=n'=6$ then $n=9$ $\delta E=0.057$; $n=10$, $\delta E=-0.041$; $n=11$, $\delta E=-0.005$; and $n=12$, $\delta E=-0.0022$. These shifts will all lie well outside the line profile of $3\,^2P\to 3\,^2S$ (except, perhaps, the last one). Nevertheless, since the maximum angular momentum is given by n', for a given energy level the number of states inducing large energy shifts remain nearly the same. The crucial point here is that one either gets a very large shift (much larger than the L-S coupling), with an "effective" trapped electron, or a very small shift without it.

We now summarize the expected behavior of the radiation in the presence of a trapped electron. We believe that there are three important consequences for C IV (similarly for N V, but there are some differences for C III, see below): (a) The additional interaction with the trapped electron may overcome the L-S interaction so that the L-S splitting of the upper $3P$ states will be changed. It therefore follows that we no longer observe the simple structure of two lines with the statistical weight 2:1 and the distance 11 Å between them. Consequently, fitting of plasma broadenings based on this assumption does not work appropriately. (b) The energy shifts are distributed over a large scale. For a given ion the shifts may have discrete values, but these will be further heavily broadened by the collisional electrons since the effective size of the ion becomes huge. (c) The upper frequency line is affected much less than the lower frequency one due to either smaller energy shifts or the "visual" effect: A shift of 0.05 eV that will induce for the former a change of ~ 150 Å in wavelength and thus totally destroy it within the range of observation, gives the former only a small shift of 0.4 Å which still stays inside the line profile. We therefore conclude that for ions with effective trapped electrons the lower (but not the upper) frequency line will be so drastically altered that it will appear missing.

C. Modeling the trapping of the plasma electron

The above discussion suggests a possible explanation of the "quenching" of the Einstein coefficients. It is simply that the observed intensities of the lower frequency lines come entirely from those ions which do not have any trapped plasma electrons. They therefore preserve the usual L-S splitting, cf. (a). The fraction of such ions can become very small as the density of the plasma electrons increases. The low-frequency spectrum of the other ions dissolves into the background. The branching ratio should therefore be multiplied, in this simple picture, by the ratio R of "normal" radiating ions N_n, i.e., those with no trapped electrons, to the total density of ions,

$$R=\frac{N_n}{N_n+N_t}=\frac{1}{1+N_t/N_n}\ , \tag{3.21}$$

with N_t the radiating ions with trapped electrons. The ratio N_t/N_n will certainly be a function of the plasma density N, the temperature, and the internal structure of the ions.[8,9] It may also depend, if the laser produced plasma observed in Refs. 1 and 2 is not in local thermodynamic equilibrium on the specific kinetics of the different competing processes, involving ionization, weak trapping, and recombination. It appears that while the ion recombination rate, e.g., C IV$\rightarrow$C III is two orders of magnitude smaller than the relevant spontaneous emission rate,[2,8,9] collisional transition rates (with energy transfer of the order of the lower-frequency lines) are much larger than the latter.[1,2] Therefore during the course of a spontaneous emission, electrons can be captured into, and escape from, the highly excited states many times. This would suggest that a local equilibrium treatment may be reasonable.

Saha-Boltzmann approximation

It is argued[9] that the populations of these highly excited states (cf. the criteria given in Ref. 9) should be in Saha-Boltzmann equilibrium. It then follows that the ratio N_t/N_n is determined by the Saha formula

$$\frac{N_t}{N_n}=N\sum_n\frac{g_n\exp(|\varepsilon_n|/kT)}{2(2\pi mkT/\hbar^2)^{3/2}}=a(T)N\ , \tag{3.22}$$

with g_n the statistical weight of the nth level and N the density of the plasma electrons. Taking $kT\cong 5$ eV, $N\cong 10^{19}$ cm^{-3}, and $n_{\max}\cong 12$, we have $N_t/N_n\cong 10^{-1}$. This appears slightly small for the effect of trapping to be important. However, since there are some uncertainties in the temperature and density (perhaps a factor of 2 for the former and 3 for the latter[2]), the two densities may become comparable.

Nonequilibrium distribution of the trapped electrons

We tried to fit the experimentally observed R in Refs. 1 and 2 with the form (3.22), assuming T to be constant over the region of observation and using a as a fitting parameter. This, however, did not work well. What does work is the ansatz that N_t/N_n is proportional to the square of the plasma electron density

$$R\cong\frac{1}{1+(N/N_0)^2}\ , \tag{3.23}$$

where N_0, is the density at which R is reduced by a half, is a fitting parameter. Since the C IV and N V ions have similar internal electronic structures, we expect that they should have roughly the same N_0. We find indeed that (3.23) is consistent with the results of the experiments with the same N_0 for C IV and N V. This is presented in Fig. 1. The data are taken from Refs. 1 and 2 and we converted the density N into the experimentally measured "distance from the target" according to Fig. 3 of Ref. 1. The only fitting parameter for the two curves is $N_0=1.2\times 10^{18}$ cm^{-3}. The agreement between (3.22) and experiment for the N V ion is exceptionally good. There are some fluctuations in the C IV data; at large distances it is probably due to the self-absorption discussed in Ref. 1 and at small distances the intensities of the lower line might have been underestimated due to strong plasma broadenings and background radiations (data for N V were taken later and more carefully treated[2]). The experiments on C IV and N V were carried out with the same CO_2 laser and the relation between electron density and distance was the same in both.[10]

We do not have, at the present time, any convincing kinetic model which would yield the required N^2 dependence for the ratio N_t/N_n. Assuming, as is usual, that the capture of an electron is a three-body process and therefore proportional to N^2 while escape is by collisions

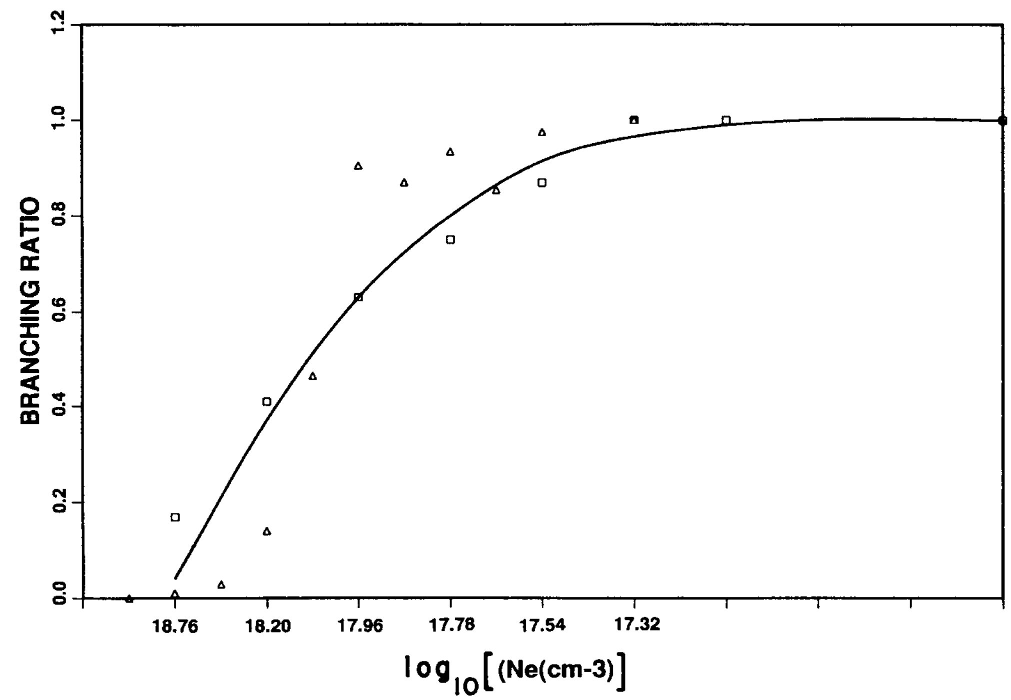

FIG. 1. Comparison of the theoretical prediction using (3.23) with the experimentally observed normalized branching ratios: □ for N V data and △ for C IV data; the solid line is the corresponding theoretical curve (distance from the target is measured in centimeters). The common fitting parameter for both cases is $N_0=1.2\times10^{18}$ cm^{-3}.

leads to the linear dependence (3.22). The N^2 dependence might be caused by some deficiency in the escape mechanism causing an accumulation of ions with trapped electrons or by the existence of some other detrapping mechanism which is effectively independent of the plasma density in the region considered. It is important to note that the C III ion does not fall into the same groups as N V and C IV. The branching ratio for C III starts decreasing at rather lower densities[2] compared to those of C IV and N V. This evidently indicates that any explanation ignoring the detailed internal electronic structure of the ions would face serious difficulties. The difference is, we think, related to the fact that the spacings between the relevant energy levels are small so that more trapping states are important. See, for example, pp. 213 and 214 of Ref. 5 and references therein, and note that the Stark effect for C III is much larger than that for the other ions.

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APPENDIX A: PLASMA DISTRIBUTION

We now discuss the initial distribution problem. Assume that at time $t=-t_0$ the density matrix of the plasma electrons is given by the grand canonical ensemble of noninteracting particles

$$\rho(-t_0)=\exp\left[-\beta\sum_{\mathbf{p}}(\varepsilon_{\mathbf{p}}-\mu)a_{\mathbf{p}}^{\dagger(0)}a_{\mathbf{p}}^{(0)}\right]\Big/\mathrm{Tr}_{\mathrm{pl}}\left[\exp\left[-\beta\sum_{\mathbf{p}}(\varepsilon_{\mathbf{p}}-\mu)a_{\mathbf{p}}^{\dagger(0)}a_{\mathbf{p}}^{(0)}\right]\right], \tag{A1}$$

where μ is the chemical potential and $\{a_{\mathbf{p}}^{\dagger(0)},a_{\mathbf{p}}^{(0)}\}$ are the creation and annihilation operators of the momentum states $\langle\mathbf{r}|\mathbf{p}\rangle=\exp(i\mathbf{p}\cdot\mathbf{r})$. We now consider two ways of imposing the static potential: (a) suddenly switch on $V(r)$ at $t=-t_0$ and (b) adiabatically switch on $V(r)$.

1. Sudden switch on of the static potential

In case (a), the density matrix at time $t=0$ becomes

$$\rho(0)=\exp\left[-\frac{i}{\hbar}H_{\mathrm{pl}}t_0\right]\rho(-t_0)\exp\left[\frac{i}{\hbar}H_{\mathrm{pl}}t_0\right], \tag{A2}$$

where H_{pl} is the true Hamiltonian (i.e., including the static potential) of the plasma electrons. Applying (A2) to (2.16), we have

$$\langle a_n^{\dagger(0)} a_{n'}^{(0)} \rangle_{\mathrm{av}} = \exp[-it_0(\varepsilon_n - \varepsilon_{n'})] \sum_{\mathbf{p}} \langle \mathbf{p}|n\rangle \langle n'|\mathbf{p}\rangle f(\varepsilon_{\mathbf{p}}) , \tag{A3}$$

with $f(\varepsilon)$ the Fermi distribution function. This is a highly oscillating function unless $\varepsilon_n = \varepsilon_{n'}$. For $\varepsilon_n > 0$ the energy spectrum is continuous. Summing over all possible n and n' in (2.16) then leads to the result that only the δ functions at $\varepsilon_n = \varepsilon_{\mathbf{p}}$ and $\varepsilon_{n'} = \varepsilon_{\mathbf{p}}$ contribute. Summing (or integrating) over all directions of $\mathbf{p}$ then yields

$$\langle a_n^{\dagger(0)} a_{n'}^{(0)} \rangle_{\mathrm{av}} = f(\varepsilon_n)\delta_{n,n'} , \tag{A4}$$

which coincides with the usual Fermi distribution. This is easily seen from the fact that in the region far away from the radiator (which contributes the δ function discussed above), $\{|n\rangle\}$ approaches to a different (or same) set of eigenstates for a free electron, therefore $\langle n|p\rangle \to 0$ for $\varepsilon_{\mathbf{p}} \neq \varepsilon_n$ and $\sum_{\mathbf{p}} |\mathbf{p}\rangle\langle\mathbf{p}|$ can be simply taken out. Note that (A4) is independent of possible choices of different sets of $\{|n\rangle\}$.

For $\varepsilon_n < 0$, there is a finite probability of having the plasma electrons trapped near the radiator even though we started with all plasma electrons having positive energies. This is presumably due to the sudden switch on of the interaction which involves some inelastic parts. This cannot, however, be used for quantitative results since it depends on how we switch on the potential, cf. below.

2. Adiabatic switch-on of the static potential

In case (b) we let $t_0 \to \infty$ and switch on the interaction $V(r)$ adiabatically. Note that we can equivalently describe the problem by slowly switching on $V(r)$ starting at $t=0$ and observing the system at $t = t_0 \to \infty$, i.e.,

$$\langle a_n^{\dagger(0)} a_{n'}^{(0)} \rangle_{\mathrm{av}} = \mathrm{Tr}_{\mathrm{pl}}[\rho(0) a_n^{\dagger(0)}(t) a_{n'}^{(0)}(t)]|_{t=t_0} , \tag{A5}$$

where $\rho(0)$ is given by (A1) and $\{a_n^{\dagger(0)}(t), a_n^{(0)}(t)\}$ are the time-dependent Heisenberg operators evolving under the adiabatic switch on of $V(r)$ and the initial condition $a_n^{\dagger(0)}(0) \equiv a_n^{\dagger(0)}$, $a_n^{(0)}(0) \equiv a_n^{(0)}$. They can be further expressed as linear combinations of $\{a_{\mathbf{p}}^{\dagger(0)}(t), a_{\mathbf{p}}^{(0)}(t)\}$,

$$a_n^{\dagger(0)}(t) = \sum_{\mathbf{p}} a_{\mathbf{p}}^{\dagger(0)}(t) \langle \mathbf{p}|n\rangle , \tag{A6}$$

and its conjugate for $a_n^{(0)}(t)$. The Heisenberg operator $a_{\mathbf{p}}^{(0)}(t)$ can be solved perturbatively as $t \to \infty$. This yields

$$a_{\mathbf{p}}^{\dagger(0)}(t) = \sum_{\mathbf{p}'} a_{\mathbf{p}'}^{\dagger(0)} \exp\left[\frac{i}{\hbar}\varepsilon_{\mathbf{p}'} t\right] \langle \psi_{\mathbf{p}'}|\mathbf{p}\rangle \quad \text{for } t \to \infty \tag{A7a}$$

where

$$\langle \psi_{\mathbf{p}'}|p\rangle = \delta_{\mathbf{p}',\mathbf{p}} + \sum_{n=1}^{\infty} \sum_{\{\mathbf{p}_1,\ldots,\mathbf{p}_n\}} \delta_{\mathbf{p}',\mathbf{p}_n} \frac{\tilde{V}(\mathbf{p}-\mathbf{p}_1)\cdots\tilde{V}(\mathbf{p}_{n-1}-\mathbf{p}_n)}{(\varepsilon_{\mathbf{p}_n} - i0^+ - \varepsilon_{\mathbf{p}})\cdots(\varepsilon_{\mathbf{p}_n} - i0^+ - \varepsilon_{\mathbf{p}_{n-1}})} . \tag{A7b}$$

Here $\tilde{V}(\mathbf{p})$ is the Fourier transform of $V(r)$ and the states $\{|\psi_{\mathbf{p}}\rangle\}$ can be identified as the usual outgoing-wave scattering states.[6] Combining these we have, as $t_0 \to \infty$,

$$\langle a_n^{\dagger(0)} a_{n'}^{(0)} \rangle_{\mathrm{av}} = \sum_{\mathbf{p}} f(\varepsilon_{\mathbf{p}}) \langle \psi_{\mathbf{p}}|n\rangle \langle n'|\psi_{\mathbf{p}}\rangle . \tag{A8}$$

Choosing for $\varepsilon_n > 0\{|n\} \equiv \{|\psi_{\mathbf{p}}\rangle\}$, we then recover the result (A4). But the right-hand side of (A8) vanishes for ε_n or $\varepsilon_{n'} < 0$ because the eigenstates are orthogonal. This shows that a smooth switching on of the static potential tends to reduce the trapping probability. Therefore the trapping of the plasma electrons need separate considerations.

APPENDIX B: MATRIX ELEMENT FOR THE COULOMB POTENTIAL

Equation (3.7) can be obtained by applying the method described in the appendixes of Ref. 6 (this must exist somewhere in the literature, but we failed to find a good reference). To start with, we use the integral representation of the confluent hypergeometric function[6]

$$\langle \mathbf{r}|\psi_{\mathbf{p}}\rangle = \psi_{\mathbf{p}}(0) \oint_C \frac{dt}{-2\pi i} (-t)^{i/p-1} (1-t)^{-i/p} \exp[i(1-t)\mathbf{p}\cdot\mathbf{r} + i(pt + i0^+)r] , \tag{B1}$$

where the contour C passes round both the points $t=0$ and 1, see Fig. 2. Using this, we have

$$\langle \psi_{\mathbf{p}}|\hat{\mathbf{p}}|\psi_{\mathbf{p}'}\rangle = A \oint_C \frac{dt dt'}{(-2\pi i)^2} f(t,t')[p't'\mathbf{r}/r + (1-t')\mathbf{p}'] \exp[-i(1-t)\mathbf{p}\cdot\mathbf{r} - iptr + i(1-t')\mathbf{p}'\cdot\mathbf{r} + ip't'r - 0^+ r] , \tag{B2}$$

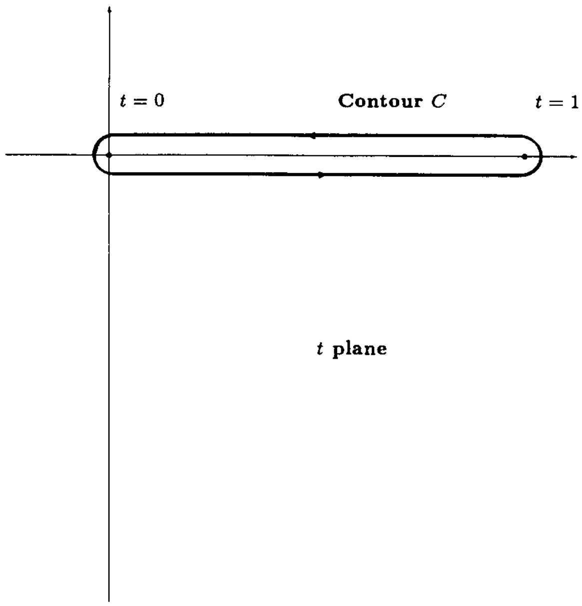

FIG. 2. Contour of integration C for the integral expression for the wave function $\psi_{\mathbf{p}}(\mathbf{r})$.

where

$$A = \hbar\psi_{\mathbf{p}}^*(0)\psi_{\mathbf{p}'}(0) \tag{B3a}$$

and

$$f(t,t') = (-t)^{-i/p-1}(1-t)^{i/p} \times(-t')^{i/p'-1}(1-t')^{-i/p'} . \tag{B3b}$$

Integration over $\mathbf{r}$ can now be done and we obtain

$$\langle \psi_{\mathbf{p}}|\hat{\mathbf{p}}|\psi_{\mathbf{p}'}\rangle = -8\pi A \oint_C \frac{dt\,dt'}{(-2\pi i)^2} f(t,t') \times \frac{i[p't'\mathbf{B}+a(1-t')\mathbf{p}']}{(a^2-\mathbf{B}^2)^2} , \tag{B4}$$

with

$$a = p't' - pt + i0^+ \tag{B5a}$$

and

$$\mathbf{B} = (1-t')\mathbf{p}' - (1-t)\mathbf{p} . \tag{B5b}$$

Both the numerator and denominator in (B4) can be further written as

$$i[(1-t')\mathbf{p}'a - p'\mathbf{B}] = i\mathbf{p}p't'(1-t) - i\mathbf{p}'pt(1-t') \tag{B6a}$$

and

$$a^2 - \mathbf{B}\cdot\mathbf{B} = t'S_1(t) - S_2(t) , \tag{B6b}$$

where

$$S_1(t) = 2(\mathbf{p}\cdot\mathbf{p}' - pp')t + 2(\mathbf{p}' - p)\cdot\mathbf{p}' - 2i0^+p' \tag{B6c}$$

and

$$S_2(t) = 2[(\mathbf{p}'-\mathbf{p})\cdot\mathbf{p} + 2i0^+p]t + (\mathbf{p}-\mathbf{p}')^2 . \tag{B6d}$$

Equations (B6b)–(B6d) are written for convenience of integration over t' and the second term in the right of (B6a) can be transformed into the first term. We therefore write

$$\langle \psi_{\mathbf{p}}|\hat{\mathbf{p}}|\psi_{\mathbf{p}'}\rangle = 8\pi A\{C(\mathbf{p},\mathbf{p}') + [\mathbf{C}(\mathbf{p}',\mathbf{p})]^*\} , \tag{B7}$$

where

$$\mathbf{C}(\mathbf{p},\mathbf{p}') = \mathbf{p}\oint_C \frac{dt\,dt'}{(-2\pi i)^2} f(t,t') \frac{-ip't'(1-t)}{[t'S_1(t)-S_2(t)]^2} = \mathbf{p}\oint_C \frac{dt}{(-2\pi i)} \left[\frac{1-t}{-t}\right]^{i/p+1} [-S_2(t)]^{i/p'-1} \times [S_1(t)-S_2(t)]^{-i/p'-1} . \tag{B8}$$

It can be further simplified by the following transformation

$$t \to \frac{(\mathbf{p}-\mathbf{p}')^2 t}{[(\mathbf{p}-\mathbf{p}')^2 - (p'^2-p^2) - 2i0^+p]t + (p'^2-p^2) + 2i0^+p} . \tag{B9}$$

This then leads to

$$\mathbf{C}(\mathbf{p},\mathbf{p}') = \frac{-\mathbf{p}}{(\mathbf{p}-\mathbf{p}')^2(p'^2-p^2+2i0^+p')} \left[\frac{p'^2-p^2+2i0^+p}{(\mathbf{p}-\mathbf{p}')^2}\right]^{i/p} \left[\frac{-(\mathbf{p}-\mathbf{p}')^2}{p'^2-p^2+2i0^+p'}\right]^{i/p'} \times \oint_C \frac{dt}{(-2\pi i)} \left[\frac{1-t}{-t}\right]^{i/p+1} \left[1 - \frac{(\mathbf{p}-\mathbf{p}')^2}{(p+p')^2}t\right]^{-i/p'-1} . \tag{B10}$$

Now using the formula (see, e.g., the Appendix E of Ref. 6)

$$\oint_C \frac{dt}{(-2\pi i)} \left[\frac{1-t}{-t}\right]^{i/p+1} (1-zt)^{-i/p'-1} = \left[1+\frac{i}{p}\right] F\left[-\frac{i}{p}, 1+\frac{i}{p'}, 2, z\right] , \tag{B11}$$

and combining (B6)–(B9), yields Eqs. (3.7).

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