Ginzburg-Landau minimizers with prescribed degrees. Capacity of the domain and emergence of vortices

Let Ω be a 2D domain with holes $\omega_0, \omega_1, \ldots, \omega_j, j = 1...k$. In domain $A = \Omega \setminus (\cup_{j=0}^{k} \omega_j)$ consider class $\mathcal{J}$ of complex valued maps having degrees 1 and -1 on $\partial\Omega$, $\partial\omega_0$ respectively and degree 0 on $\partial\omega_j, j = 1...k$.

We show that if $\mathrm{cap}(A) \geq \pi$, minimizers of the Ginzburg-Landau energy E_κ exist for each κ. They are vortexless and converge in $H^1(A)$ to a minimizing S^1-valued harmonic map as the coherency length κ^{-1} tends to 0. When $\mathrm{cap}(A) < \pi$, we establish existence of quasi-minimizers, which exhibit a different qualitative behavior: they have exactly two zeroes (vortices) rapidly converging to ∂A.