Eddie Cohen, long time tails, and FM quantum phase transitions
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Review of LTT
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- $J_{xy}(t) = \sum v_{ix}(t) v_{iy}(t) + \ldots$
- $C(t) = \langle J_{xy}(t) J_{xy}(0) \rangle$
- $\eta = \int_0^\infty C(t)$
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FIG. 3 Normalized velocity autocorrelation function \( \rho_D(t) = C_D(t)/(\langle \nu^2(0) \rangle) \) as a function of the dimensionless time \( t^* = t/t_0 \), where \( t_0 \) is the mean-free time. The crosses indicate computer results obtained by Wood and Erpenbeck (1975) for a system of 4000 hard spheres at a reduced density corresponding to \( V/V_0 = 3 \), where \( V \) is the actual volume and \( V_0 \) is the close-packing volume. The dashed curve represents the theoretical curve \( \rho_D(t) = \alpha_D (t^*)^{-3/2} \). The solid curve represents a more complete evaluation of the mode-coupling formula with contributions from all possible hydrodynamic modes and with finite-size corrections included (Dorfman, 1981). From Dorfman et al. (1994).
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\[ J_{xy}(t) = 0 J_{xy}(t) + \int dk \, \delta u_x(k,t) \, \delta u_y(-k,t) \leftarrow \text{product of soft modes} \]

\[ \rightarrow \delta C_{\eta \eta}(t) \sim \int dk \, < |u_x(k,t)| > < |u_y(k,t)| > \sim \int dk \, \exp(-2vk^2 t) \sim 1/t^{d/2} \quad \text{LTT!!!} \]
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\]

Universal effects in liquids due to soft or slow modes coupling to........stuff.
• Another example: Heisenberg magnet in FM phase-0rdered in z-direction

• \( S=(\pi_x,\pi_y,\sigma) \)

• \( \sigma=m+\delta\sigma \)
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• $<|\pi(k)|^2| \sim 1/k^2 \rightarrow \text{GM due to BS and LRO}$

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• \( \delta \sigma(k) = \delta \sigma^{(0)}(k) + \sum \pi(k-q)\pi(q) + \cdots \)
• Another example: Heisenberg magnet in FM phase-ordered in z-direction

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• $<|\delta\sigma(k)|^2> \sim \text{const} + \Sigma <|\pi(k-q)|^2><|\pi(q)|^2> \sim 1/k^{4-d} \rightarrow$ singular in all $d<4$
• What does this have to do with QPT??
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Focus on FM transition in clean metallic systems where the PM<->FM transition happens at low T
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Experiment: Without fail, at low enough T (with a non-thermal control variable, say pressure) the PT changes from 2nd order to first order!!!
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  ![UGe$_2$ graph](image)

(Pfleiderer & Huxley 2002)
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TABLE I: Systems with low-\(T\) ferromagnetic transitions and their properties. \(T_c\) = Curie temperature, \(T_{ic}\) = tricritical temperature. \(\rho_0\) = residual resistivity. FM = ferromagnet, SC = superconductor. N/A = not applicable; n.a. = not available.

<table>
<thead>
<tr>
<th>System</th>
<th>Order of Transition</th>
<th>(T_c/\text{K})</th>
<th>magnetic moment/(\mu\text{B})</th>
<th>tuning parameter</th>
<th>(T_{ic}/\text{K})</th>
<th>wings observed ((\rho_0/\mu\Omega\text{cm}))</th>
<th>Disorder</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{MnSi}^{27})</td>
<td>1st</td>
<td>29.5 (^{28})</td>
<td>0.4 (^{28})</td>
<td>hydrostatic pressure (^{18})</td>
<td>(\approx 10) (^{18})</td>
<td>yes (^{25}) (0.33) (^{25})</td>
<td>weak helimagnet, exotic phases (^{25,26})</td>
<td></td>
</tr>
<tr>
<td>(\text{ZrZn}_2^{27})</td>
<td>1st</td>
<td>28.5 (^{29})</td>
<td>0.17 (^{29})</td>
<td>hydrostatic pressure (^{29})</td>
<td>(\approx 5) (^{29})</td>
<td>yes (^{29}) (\geq 0.31) (^{30})</td>
<td>confusing history, see Ref. (^{27})</td>
<td></td>
</tr>
<tr>
<td>(\text{Sr}_3\text{Ru}_2\text{O}_7)</td>
<td>1st (^f)</td>
<td>0 (^g)</td>
<td>0 (^g)</td>
<td>pressure (^g)</td>
<td>n.a.</td>
<td>yes (^{31}) (&lt; 0.5) (^{31})</td>
<td>foliated wing tips, nematic phase (^{31})</td>
<td></td>
</tr>
<tr>
<td>(\text{UGe}_2^{33})</td>
<td>1st (^{34})</td>
<td>52 (^{35})</td>
<td>1.5 (^{35})</td>
<td>hydrostatic pressure (^{22,35})</td>
<td>24 (^{36})</td>
<td>yes (^{35,36}) (0.2) (^{22})</td>
<td>easy-axis FM coexisting FM+SC (^{22})</td>
<td></td>
</tr>
<tr>
<td>(\text{URhGe}^{33})</td>
<td>1st (^{37})</td>
<td>9.5 (^{23})</td>
<td>0.42 (^{23})</td>
<td>transverse B-field (^{37,39})</td>
<td>(\approx 1) (^{37})</td>
<td>yes (^{37}) (8) (^{38})</td>
<td>easy-plane FM coexisting FM+SC (^{23})</td>
<td></td>
</tr>
<tr>
<td>(\text{UCoGe}^{33})</td>
<td>1st (^{40})</td>
<td>2.5 (^{40})</td>
<td>0.03 (^{24})</td>
<td>none (^{h})</td>
<td>(&gt; 2.5?) (^{h})</td>
<td>no (^{24}) (12) (^{24})</td>
<td>coexisting FM+SC (^{24})</td>
<td></td>
</tr>
<tr>
<td>(\text{CoS}_2)</td>
<td>1st (^{41})</td>
<td>122 (^{41})</td>
<td>0.84 (^{41})</td>
<td>hydrostatic pressure (^{41})</td>
<td>(\approx 120) (^{41})</td>
<td>no (^{41}) (0.7) (^{41})</td>
<td>rather high (T_c)</td>
<td></td>
</tr>
<tr>
<td>(\text{La}_{1-x}\text{Ce}_x\text{In}_2)</td>
<td>1st (^{42})</td>
<td>22 - 19.5 (^{42}) (^i)</td>
<td>n.a.</td>
<td>composition (^{42}) (&gt; 22?) (^{j})</td>
<td>no</td>
<td>n.a.</td>
<td>third phase between FM and PM? (^{42})</td>
<td></td>
</tr>
<tr>
<td>(\text{Ni}_3\text{Al}^{27})</td>
<td>(1st) (^k)</td>
<td>41 - 15 (^{1})</td>
<td>0.075 (^{35})</td>
<td>hydrostatic pressure (^{43})</td>
<td>n.a.</td>
<td>no (^{44}) (0.84) (^{44})</td>
<td>order of transition uncertain</td>
<td></td>
</tr>
<tr>
<td>(\text{YbIr}_2\text{Si}_2^{n})</td>
<td>1st (^{45})</td>
<td>1.3 - 2.3 (^{9})</td>
<td>n.a.</td>
<td>hydrostatic pressure (^{45})</td>
<td>n.a.</td>
<td>no (^{45}) (\approx 22) (^{7})</td>
<td>FM nature of ordered (\approx 22) phase suspected (^{45})</td>
<td></td>
</tr>
<tr>
<td>(\text{YbCu}_2\text{Si}_2^{n})</td>
<td>n.a.</td>
<td>4 - 6 (^{46})</td>
<td>n.a.</td>
<td>hydrostatic pressure (^{46})</td>
<td>n.a.</td>
<td>no</td>
<td>n.a.</td>
<td>nature of magnetic order unclear</td>
</tr>
<tr>
<td>(\text{URu}_{2-x}\text{Re}_x\text{Si}_2)</td>
<td>2nd (^{47,48})</td>
<td>25 - 2 (^{7})</td>
<td>0.4 - 0.03 (^{48})</td>
<td>composition (^{47})</td>
<td>N/A</td>
<td>N/A (\approx 100) (^{7})</td>
<td>strongly disordered</td>
<td></td>
</tr>
<tr>
<td>(\text{Ni}<em>x\text{Pd}</em>{1-x})</td>
<td>2nd (^{50})</td>
<td>600 - 7 (^{7})</td>
<td>n.a.</td>
<td>composition (^{50})</td>
<td>N/A</td>
<td>N/A</td>
<td>disordered, lowest (T_c) rather high</td>
<td></td>
</tr>
<tr>
<td>(\text{YbNi}_4\text{P}_2)</td>
<td>2nd (^{51})</td>
<td>0.17 (^{51})</td>
<td>(\approx 0.05) (^{51})</td>
<td>none</td>
<td>N/A</td>
<td>N/A (2.6) (^{51})</td>
<td>quasi-1d, disordered</td>
<td></td>
</tr>
</tbody>
</table>
II. Quantum Ferromagnetic Transitions: Theory

1. Conventional (= mean-field) theory

- **Hertz 1976**: Mean-field theory correctly describes T=0 transition for d>1 in clean systems, and for d>0 in disordered ones.
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- **Conclusion**: Conventional theory not viable
Universal mechanism for tri-critical point in low T metallic FM

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Soft modes in FL?
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    - \( \sigma=\sigma^{(0)}+\sum\pi\pi+\cdots \leftarrow \text{mmc term} \)
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• Integrate out soft electronic modes to obtain a generalized MFT,
2. Renormalized mean-field theory.

- In general, conventional theory misses effects of fermion soft modes:
  
  - Contribution to $f_0$: 
    \[
    \int d\mathbf{k} \int d\Omega \ln((k + \Omega)^2 + m^2)
    \]
  
  - Contribution to eq. of state:

  - Renormalized mean-field equation of state:
    \[
    \frac{d}{dm} \int d\mathbf{k} \int d\Omega \ln[(k + \Omega)^2 + m^2] \sim m \left\{ \begin{array}{ll}
    \text{const.} - m^{d-1} & (1 < d < 3) \\
    \text{const.} + m^2 \ln m & (d = 3)
    \end{array} \right.
    \]

  $\Rightarrow$ $h = tm + vm^3 \ln m + um^3$ (clean, $d=3$, $T=0$)

  - $v>0$

  Transition is generically 1st order! (TRK, T Vojta, DB 1999)

Physics? - Free energy gain by making soft fluctuations massive.

$\Rightarrow$ Coleman-Weinberg mechanism
II. Quantum Ferromagnetic Transitions: Theory

2. Renormalized mean-field theory

- External field $h$ produces tricritical wings: (DB, TRK, J. Rollbühler 2005)
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**Remarks:**
- Landau theory with a TCP also produces tricritical wings (Griffiths 1970)
- So far no OP fluctuations have been considered
- More generally, Hertz theory works if field conjugate the OP does not change the soft-mode spectrum (DB, TRK, T Vojta 2002)
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• Sufficiently strong non-magnetic disorder drives transition 2nd order. Also understood.
TABLE I: Systems with low-$T$ ferromagnetic transitions and their properties. $T_c$ = Curie temperature, $T_{ic}$ = tricritical temperature, $\rho_0$ = residual resistivity. FM = ferromagnet, SC = superconductor. N/A = not applicable; n.a. = not available.

<table>
<thead>
<tr>
<th>System $^a$</th>
<th>Order of Transition $^i$</th>
<th>$T_c$/K $^b$</th>
<th>magnetic moment/µB $^d$</th>
<th>tuning parameter</th>
<th>$T_{ic}$/K</th>
<th>wings observed ($\rho_0$/µ1cm)$^g$</th>
<th>Disorder</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>MnSi $^{27}$</td>
<td>1st $^{18}$</td>
<td>29.5 $^{28}$</td>
<td>0.4 $^{28}$</td>
<td>hydrostatic pressure $^{18}$</td>
<td>$\approx 10$ $^{18}$</td>
<td>yes $^{25}$</td>
<td>0.33 $^{25}$</td>
<td>weak helimagnet, exotic phases $^{17}$</td>
</tr>
<tr>
<td>ZrZn$_2$ $^{27}$</td>
<td>1st $^{29}$</td>
<td>28.5 $^{29}$</td>
<td>0.17 $^{29}$</td>
<td>hydrostatic pressure $^{29}$</td>
<td>$\approx 5$ $^{29}$</td>
<td>yes $^{29}$</td>
<td>$\geq 0.31$ $^{30}$</td>
<td>confusing history, see Ref. 27</td>
</tr>
<tr>
<td>Sr$_3$Ru$_2$O$_7$</td>
<td>1st $^f$</td>
<td>0 $^g$</td>
<td>0 $^g$</td>
<td>pressure $^g$</td>
<td>n.a.</td>
<td>yes $^{31}$</td>
<td>$&lt; 0.5$ $^{31}$</td>
<td>foliated wing tips, nematic phase $^{31}$</td>
</tr>
<tr>
<td>UGe$_2$ $^{33}$</td>
<td>1st $^{34}$</td>
<td>52 $^{35}$</td>
<td>1.5 $^{35}$</td>
<td>hydrostatic pressure $^{22,35}$</td>
<td>24 $^{36}$</td>
<td>yes $^{35,36}$</td>
<td>0.2 $^{22}$</td>
<td>easy-axis FM coexisting FM+SC $^{22}$</td>
</tr>
<tr>
<td>URhGe $^{33}$</td>
<td>1st $^{37}$</td>
<td>9.5 $^{23}$</td>
<td>0.42 $^{23}$</td>
<td>transverse $B$-field $^{37,39}$</td>
<td>$\approx 1$ $^{37}$</td>
<td>yes $^{37}$</td>
<td>8 $^{38}$</td>
<td>easy-plane FM coexisting FM+SC $^{23}$</td>
</tr>
<tr>
<td>UCoGe $^{33}$</td>
<td>1st $^{40}$</td>
<td>2.5 $^{40}$</td>
<td>0.03 $^{24}$</td>
<td>none</td>
<td>$&gt; 2.5$ $^h$</td>
<td>no</td>
<td>12 $^{24}$</td>
<td>coexisting FM+SC $^{24}$</td>
</tr>
<tr>
<td>CoS$_2$</td>
<td>1st $^{41}$</td>
<td>122 $^{41}$</td>
<td>0.84 $^{41}$</td>
<td>hydrostatic pressure $^{41}$</td>
<td>$\approx 120$ $^{41}$</td>
<td>no</td>
<td>0.7 $^{41}$</td>
<td>rather high $T_c$</td>
</tr>
<tr>
<td>La$_{1-x}$Ce$_x$In$_2$</td>
<td>1st $^{42}$</td>
<td>22 - 19.5 $^{42}$</td>
<td>n.a.</td>
<td>composition $^{42}$</td>
<td>$&gt; 22$ $^j$</td>
<td>no</td>
<td>n.a.</td>
<td>third phase between FM and PM? $^{42}$</td>
</tr>
<tr>
<td>Ni$_3$Al $^{27}$</td>
<td>(1st) $^k$</td>
<td>41 - 15 $^l$</td>
<td>0.075 $^{m}$</td>
<td>hydrostatic pressure $^{43}$</td>
<td>n.a.</td>
<td>no</td>
<td>0.84 $^{44}$</td>
<td>order of transition uncertain</td>
</tr>
<tr>
<td>YbIr$_2$Si$_2$ $^n$</td>
<td>1st $^{45}$</td>
<td>1.3 - 2.3 $^o$</td>
<td>n.a.</td>
<td>hydrostatic pressure $^{45}$</td>
<td>n.a.</td>
<td>no</td>
<td>$\approx 22$ $^p$</td>
<td>FM nature of ordered phase suspected $^{45}$</td>
</tr>
<tr>
<td>YbCu$_2$Si$_2$ $^n$</td>
<td>n.a.</td>
<td>4 - 6 $^{46}$</td>
<td>n.a.</td>
<td>hydrostatic pressure $^{46}$</td>
<td>n.a.</td>
<td>no</td>
<td>n.a.</td>
<td>nature of magnetic order unclear</td>
</tr>
<tr>
<td>URu$_{2-x}$Re$_x$Si$_2$</td>
<td>2nd $^{47,48}$</td>
<td>25 - 2 $^r$</td>
<td>0.4 - 0.03 $^{48}$</td>
<td>composition $^{47}$</td>
<td>N/A</td>
<td>N/A</td>
<td>$\approx 100$ $^t$</td>
<td>strongly disordered</td>
</tr>
<tr>
<td>Ni$<em>x$Pd$</em>{1-x}$</td>
<td>2nd $^{50}$</td>
<td>600 - 7 $^t$</td>
<td>n.a.</td>
<td>composition $^{50}$</td>
<td>N/A</td>
<td>N/A</td>
<td>n.a.</td>
<td>disordered, lowest $T_c$ rather high</td>
</tr>
<tr>
<td>YbNi$_4$P$_2$</td>
<td>2nd $^{51}$</td>
<td>0.17 $^{51}$</td>
<td>$\approx 0.05$ $^{51}$</td>
<td>none</td>
<td>N/A</td>
<td>N/A</td>
<td>2.6 $^{51}$</td>
<td>quasi-1d, disordered</td>
</tr>
</tbody>
</table>
Many More BDs Eddie!!!

Helena and Ted