# Single-Electron Tunneling and the Fluctuation Theorem


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|---|---|
| Yasuhiro Utsumi | University of Tokyo |
| Dmitry Golubev | Karlsruhe Inst. of Technology |
| Matthias Hettler | " |
| Michael Marthaler | University of Karlsruhe |
| **Gerd Schön** | **University of Karlsruhe** |

*Experiment:*

| | |
|---|---|
| Toshimasa Fujisawa | Tokyo Inst. of Technology & NTT BRL |


Karlsruhe Institute of Technology

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# Fluctuation Theorem

Evans, Cohen, Morriss 1993; Gallavotti, Cohen 1995
Lebowitz, Spohn 1999

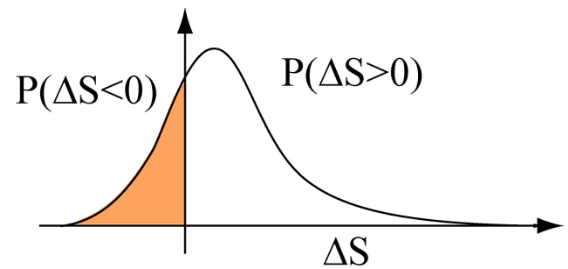
$$\frac{P_\tau(+\Delta S)}{P_\tau(-\Delta S)} = \exp(\Delta S)$$

$\Delta S$ entropy production during measurement $\tau$

- $\Rightarrow$ Fluctuation-dissipation & Onsager's theorems
- valid also in non-equilibrium, violation of 2nd law of thermodynamics for short $\tau$,

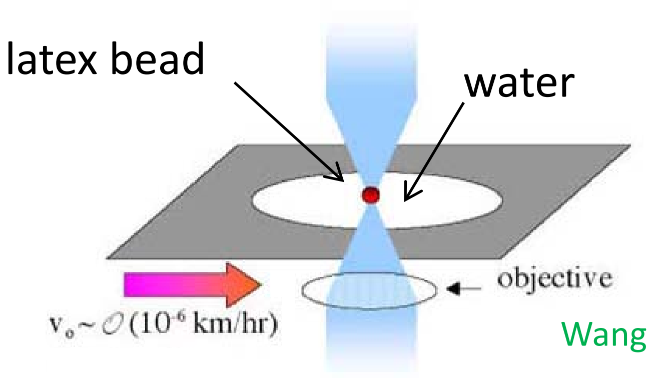
## Integrated version

$$\frac{P_\tau(\Delta S < 0)}{P_\tau(\Delta S > 0)} = \langle \exp(-\Delta S) \rangle_{\Delta S > 0}$$

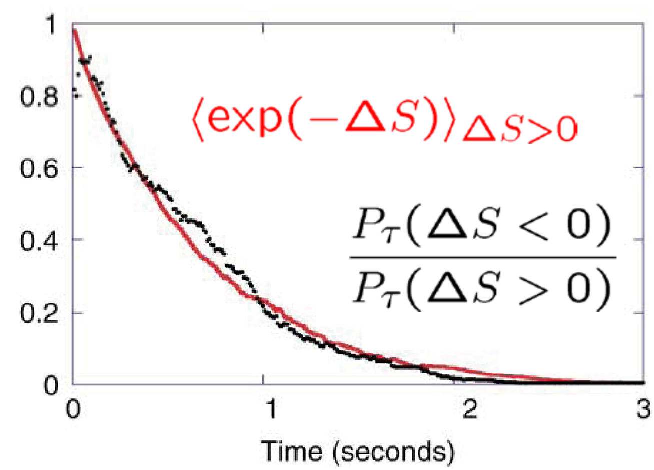



## Example:

Entropy production of colloidal particles

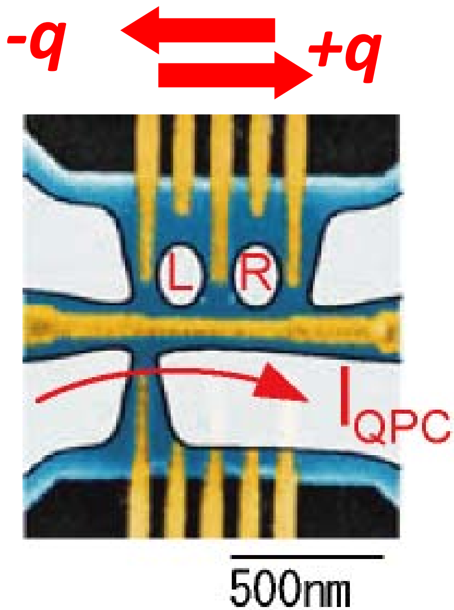


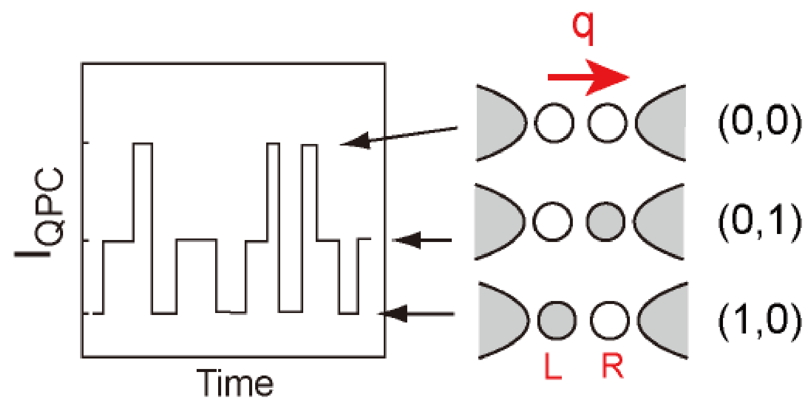

Wang et al. 2002



# Bidirectional Single-Electron Counting

T. Fujisawa et al., Science **312**, 1634 (2006)



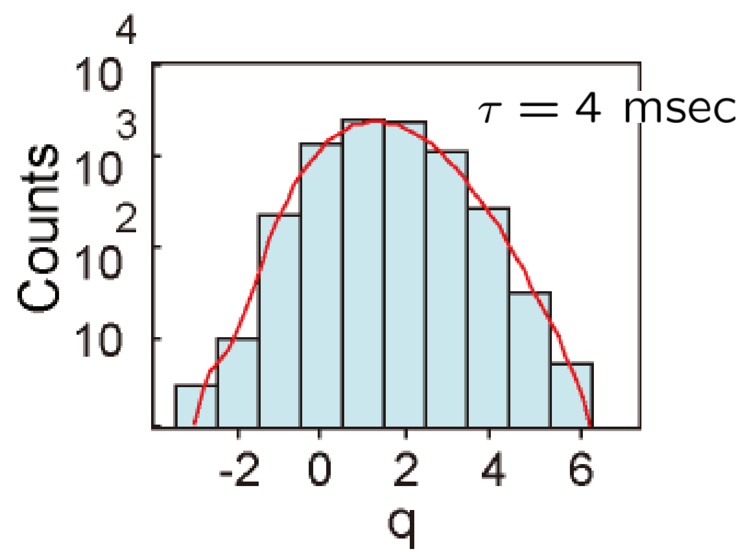




distinguish { (1,0) → (0,1) forward tunneling
(0,1) → (1,0) backward tunneling

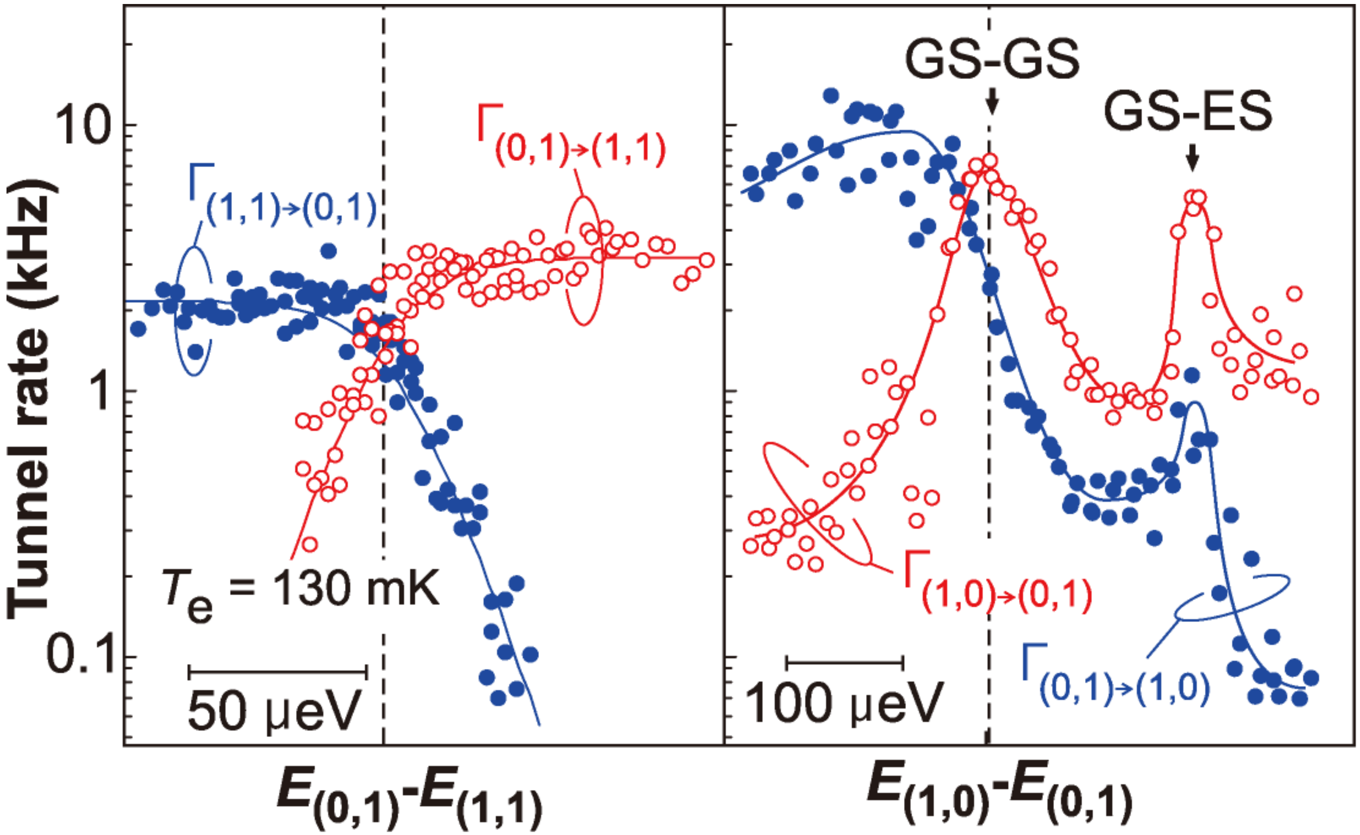
## Full-Counting Statistics

$P_\tau(q)$ transmitted charge $q$ during time $\tau$

Levitov, Lesovik, Lee 93, 96



# Tunnel Rate




Lead – dot tunneling:
broadening ≈ bath temperature

dot- dot tunneling:
broadening » bath temperature

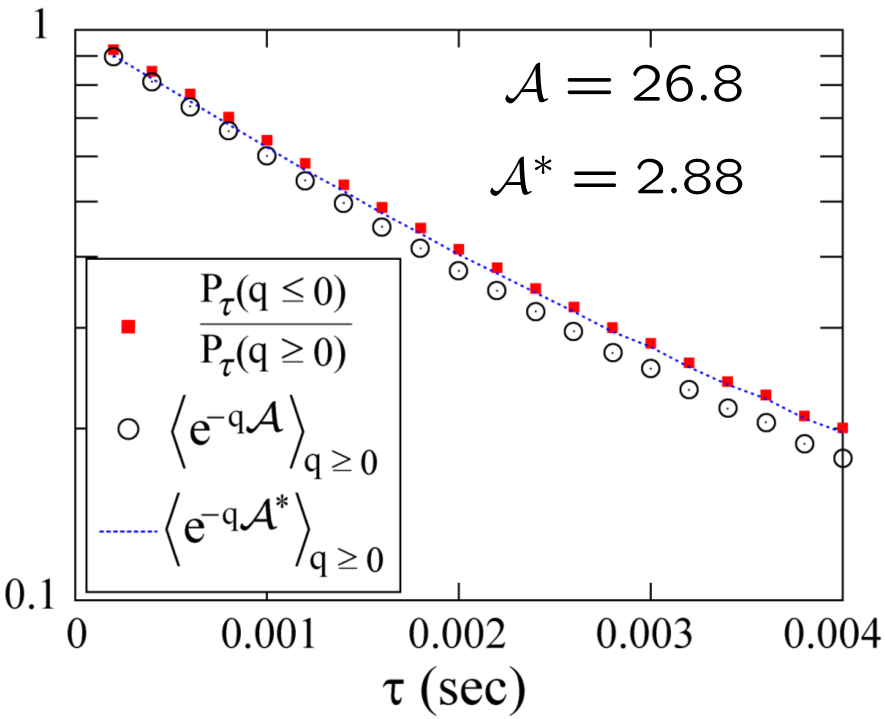
Fluctuation Theorem
for transmitted charge

$$\frac{P_\tau(-q)}{P_\tau(q)} = \exp(-qeV/T) \rightarrow \exp(-q\mathcal{A})$$

$$\frac{P_\tau(q \le 0)}{P_\tau(q \ge 0)} = \langle \exp(-q\mathcal{A}) \rangle_{q \ge 0}$$

## Comparison with experiment

Fujisawa et al. 2006



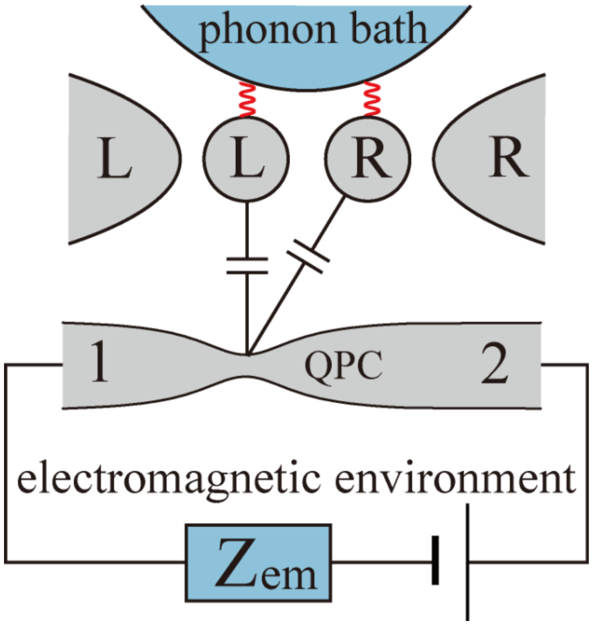

fit to experiment with $\mathcal{A}^* \ll eV/T$

⇔ substantially enhanced “temperature”

**Deviations due to**

- **nonequilibrium phonons?**
- **back-action of measurement?**
- **environment induced transitions?**
- **dephasing?**

# Model Hamiltonian




$$H_0 = H_{\text{Res}} + H_{\text{QD}} + H_{\text{QPC}} + H_{\text{ph}} + H_{\text{em}}$$

$$H_{\text{Res}} = \sum_{r=L,R,1,2} \sum_{k,\sigma} \epsilon_{r,k}\, a^{\dagger}_{r,k\sigma}\, a_{r,k\sigma}$$

$$H_{\text{QD}} = \sum_{r=L,R} \left( \sum_{\sigma} \varepsilon_r\, d^{\dagger}_{r\sigma} d_{r\sigma} + U_r\, n_{r\uparrow} n_{r\downarrow} \right) + U_{LR}\, n_L\, n_R$$
$$+ H^{\text{tunn}}_{\text{dot-dot}} + H^{\text{tunn}}_{\text{res-dot}}$$

$$H_{\text{QPC}} = \left( t_0 - \sum_{r=L,R} t_r\, n_r \right) \sum_{qq'\sigma} a^{\dagger}_{2,q\sigma} a_{1,q'\sigma} + h.c.$$

Acoustic phonons – dot

$$H_{\text{ph}} = \sum_q v_s |q|\, b^{\dagger}_q b_q + \sum_{r=R,L} \sum_q g_{rq} (b^{\dagger}_q + b_q) n_r$$

Electromagnetic environment

$$H_{\text{em}} = \sum_{i=1,2} \delta V_i(t)\, n_i \quad \Rightarrow \quad \sum_{r=L,R} \delta V_r(t)\, n_r$$

linear circuit analysis

Aguado, Kouwenhoven 2000

$$S_V(\omega) = |Z_t(\omega)|^2 S_I(\omega)$$

QPC current noise in read-out circuit

# Dressing of tunneling matrix elements

- Polaron & gauge transformations $\Rightarrow$ $d_{L/R\,\sigma} \to d_{L/R\,\sigma}\mathrm{e}^{i\varphi_{L/R}}$

$$\varphi_{\mathrm{ph}\,L/R} = \mathrm{Re}\sum_q \frac{i\, g^*_{L/R\,q} b_q}{v_s|q|} \qquad \varphi_{\mathrm{em}\,L/R} = \int^t dt' \delta V_{L/R}(t')$$

- tunneling matrix element $\tilde{X} = T_{RL}\, d_L^\dagger d_R\, \mathrm{e}^{-i(\varphi_L - \varphi_R)}$

$$\Pi_{RL}(t) = \left\langle \tilde{X}(t)\tilde{X}(0)^\dagger \right\rangle$$

$$= |T_{RL}|^2 \left\langle \mathrm{e}^{iH^{(L)}_{\mathrm{QPC}}\,t}\, \mathrm{e}^{-iH^{(R)}_{\mathrm{QPC}}\,t} \right\rangle \left\langle \mathrm{e}^{i\varphi_{\mathrm{ph}}(t)} \mathrm{e}^{-i\varphi_{\mathrm{ph}}(0)} \right\rangle \left\langle \mathrm{e}^{i\varphi_{\mathrm{em}}(t)} \mathrm{e}^{-i\varphi_{\mathrm{em}}(0)} \right\rangle \mathrm{e}^{i(\varepsilon_L - \varepsilon_R)t}$$

1. Measurement back action
2. Electron-phonon coupling
3. Electromagnetic environment

# 1. Measurement back-action

$$\left\langle e^{iH_{\mathrm{QPC}}^{(L)}t} e^{-iH_{\mathrm{QPC}}^{(R)}t} \right\rangle_{t\to\infty} \approx e^{-\Gamma_{\mathrm{QPC}}t/2}$$

- perturbation expansion in tunneling: Gurvitz 97; Shnirmann, G.S. 98

- stronger tunneling, FCS approach: Averin, Sukhorukov 05

$$\Gamma_{\mathrm{QPC}} = \frac{-1}{\pi}\int d\omega \ln\Big(1-\Big[\sqrt{1-\mathcal{T}_L}\sqrt{1-\mathcal{T}_R}+\sqrt{\mathcal{T}_L}\sqrt{\mathcal{T}_R}-1\Big] \times [f_1(\omega)(1-f_2(\omega))+f_2(\omega)(1-f_1(\omega))]\Big)$$

$$\mathcal{T}_L \approx \mathcal{T}_R \approx 0.38 \quad \Longrightarrow \quad \Gamma_{\mathrm{QPC}} \approx 3.4 \text{ neV}$$

$\Rightarrow$ **weak effect, although the transmission probabilities are not small!**

# 2. Electron-phonon coupling

Bruus, Flensberg, Smith 93
Fujisawa et al. 98; Brandes, Kramer 98

Piezoelectric phonons

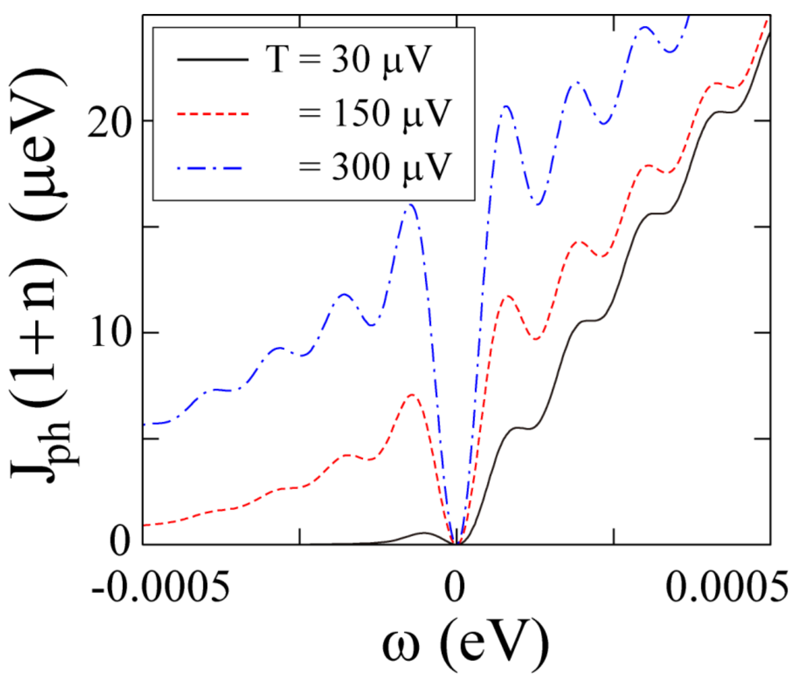
$$g_{rq} = \sqrt{\frac{P_{\mathrm{GaAs}}}{2M_{\mathrm{GaAs}}\, v_s |q|}}\, \mathrm{e}^{-i\vec{q}\cdot\vec{R}_r - (q\, r_0/2)^2}$$

$\vec{R}_r$ : position of QD $r$
$r_0$ : size of QDs

$$J_{\mathrm{ph}}(\omega) = \sum_{q,r,r'} g^*_{rq}\, g_{r'q}\, \delta(\omega - \omega_q)$$

$$\ln \left\langle \mathrm{e}^{i\varphi_{\mathrm{ph}}(t)} \mathrm{e}^{i\varphi_{\mathrm{ph}}(0)} \right\rangle = \int d\omega\, \frac{J_{\mathrm{ph}}(\omega)}{\omega^2} [1 + n(\omega)]\, (\mathrm{e}^{-i\omega t} - 1)$$

Bose distribution




- Oscillation with the period

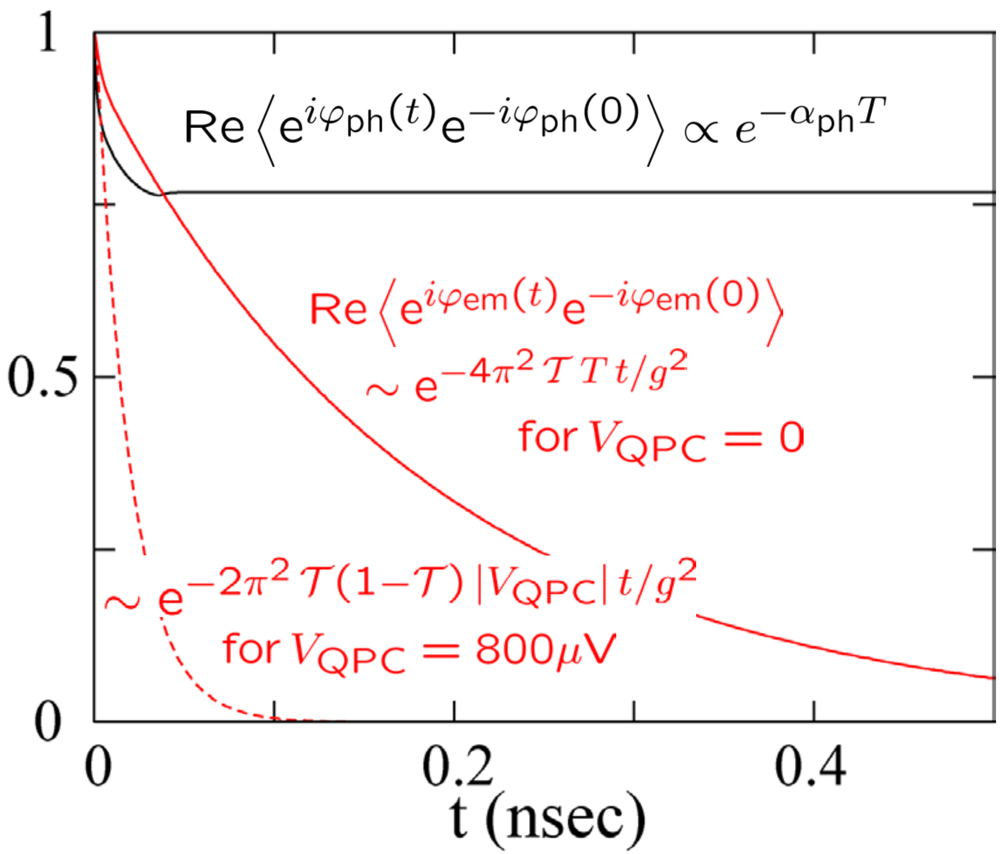
$$|\vec{R}_L - \vec{R}_R| / v_s$$

- Super-Ohmic for small $\omega$

$$J_{\mathrm{ph}} \propto |\omega|^3$$

**Coherence persists even for finite $T$**

# Dephasing by the environments




$$\alpha_{\mathrm{ph}} = \frac{P_{\mathrm{GaAs}} |\vec{R}_L - \vec{R}_R|^2}{3\pi^{3/2} r_0 v_s^4 \rho_{\mathrm{GaAs}}}$$

$$T = 30\mu\mathrm{eV}$$

$$\mathcal{T} = 0.3$$

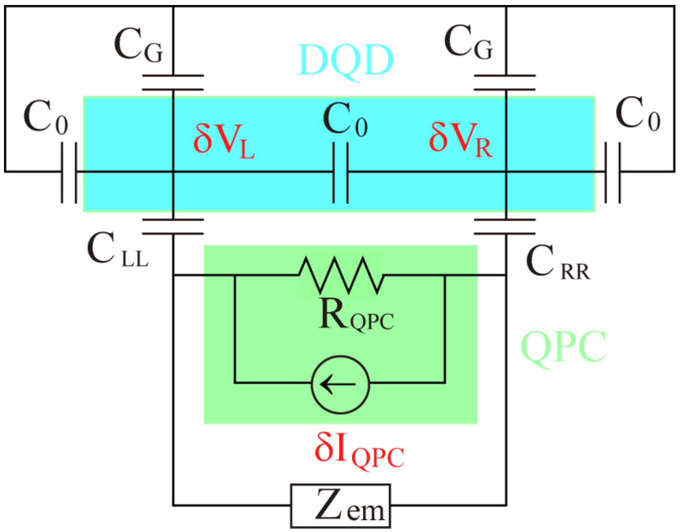
$$R \sim 500\Omega$$

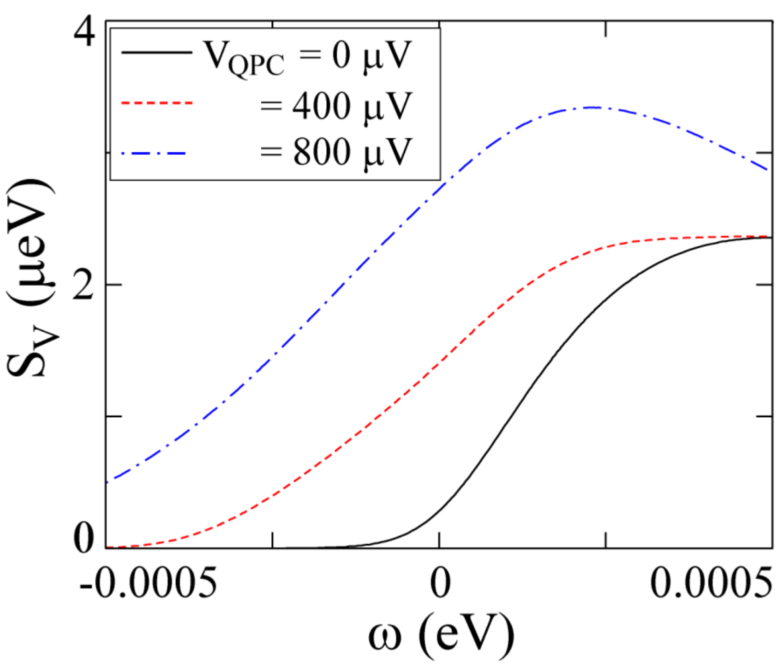
$$C \sim 0.1 \text{ fF}$$

$$g = \frac{R_{\mathrm{K}}}{R}$$

- **Phonons have only weak dephasing effect**

# 3. Electromagnetic environment




QPC shot noise    Khlus 87

$$S_I(\omega) = \left\langle \delta I_{\mathrm{QPC}}(t)\, \delta I_{\mathrm{QPC}}(0) \right\rangle_\omega$$

$$= \frac{e}{R_{\mathrm{K}}} \Big\{ \mathcal{T}(1-\mathcal{T}) \Big[ (\omega + V_{\mathrm{QPC}}) \left[1 + n(\omega + V_{\mathrm{QPC}})\right] + (\omega - V_{\mathrm{QPC}}) \left[1 + n(\omega - V_{\mathrm{QPC}})\right] \Big] + 2\mathcal{T}^2 \omega \left[1 + n(\omega)\right] \Big\}$$

linear circuit analysis    Aguado, Kowenhoven 2000

$$S_V = |Z_t|^2\, S_I \qquad Z_t = \frac{-1}{i\omega C - 1/R}$$

$$\ln \left\langle \mathrm{e}^{i\varphi_{\mathrm{em}}(t)} \mathrm{e}^{i\varphi_{\mathrm{em}}(0)} \right\rangle = 4 \int d\omega \ \ \frac{S_V(\omega)}{\omega^2} (\mathrm{e}^{-i\omega t} - 1)$$

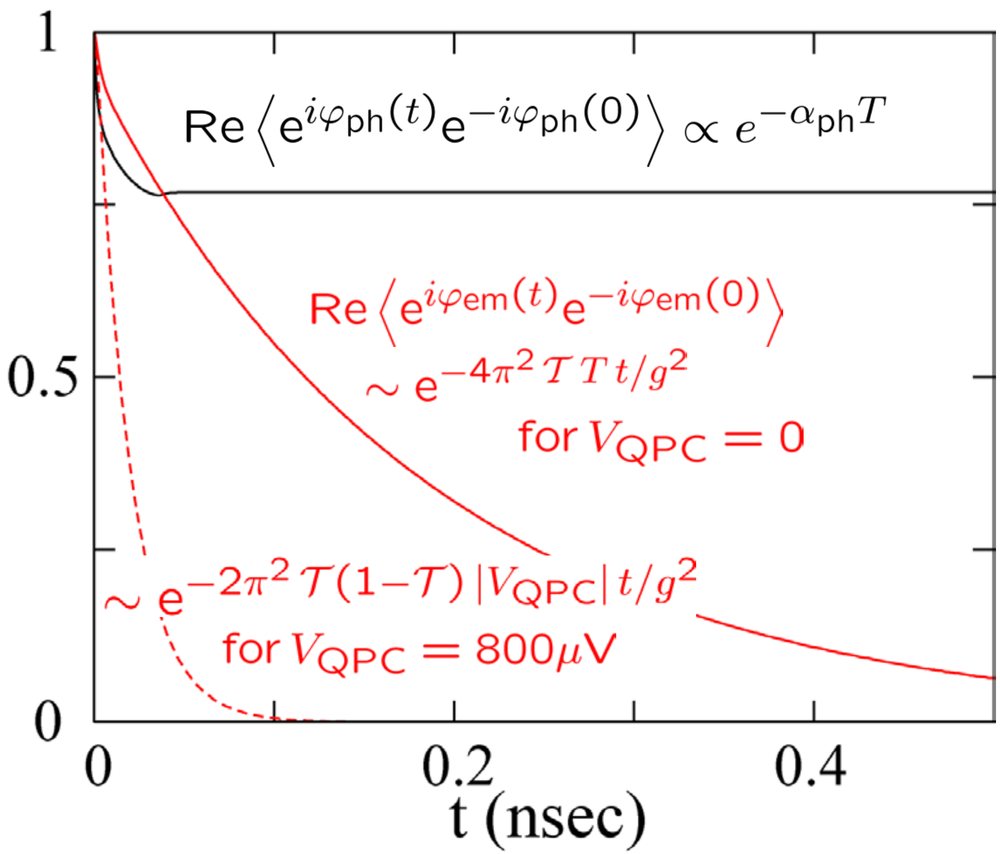
- In equilibrium    $V_{\mathrm{QPC}} = 0$

  $\Rightarrow$ Ohmic dissipation  $S_V(\omega) \propto \omega$

**- Non-equilibr. electromagnetic environment**

$V_{\mathrm{QPC}} \neq 0$

# Dephasing by the environments




$$\alpha_{\mathrm{ph}} = \frac{P_{\mathrm{GaAs}} |\vec{R}_L - \vec{R}_R|^2}{3\pi^{3/2} r_0 v_s^4 \rho_{\mathrm{GaAs}}}$$

$$T = 30\mu\mathrm{eV}$$

$$\mathcal{T} = 0.3$$

$$R \sim 500\Omega$$

$$C \sim 0.1\ \mathrm{fF}$$

$$g = \frac{R_{\mathrm{K}}}{R}$$

- **Electromagnetic environment may cause large dephasing effect**

# Full Counting Statistics

Characteristic Function $\mathcal{Z}_\tau(\lambda) = \sum_q P_\tau(q) e^{iq\lambda} = \mathrm{Tr}_{\mathrm{QD}}\left(\rho_\tau(\lambda)\right)$

counting field

"Reduced Density Matrix"

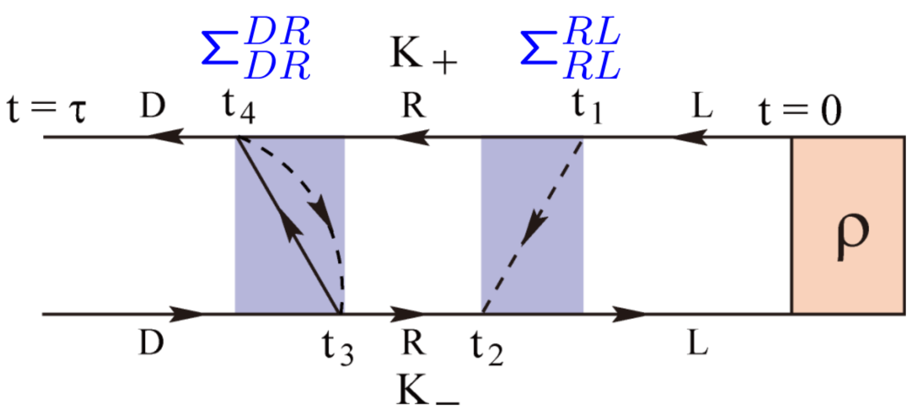
$$\rho_\tau(\lambda) = \mathrm{Tr}_{\mathrm{Res}} \mathrm{Tr}_{\mathrm{ph}} \mathrm{Tr}_{\mathrm{el}} \left( e^{-iUHU^\dagger \tau}\, \hat{\rho}_0\, e^{iU^\dagger HU\tau} \right)$$

$$\hat{U} = e^{-i \sum_{r=L,R} \lambda_r (\hat{N}_r + \hat{n}_r)}$$

$$\lambda = \lambda_L - \lambda_R$$

- Coulomb interaction ⇒ 3 charge states $|L\rangle\,,\ |R\rangle\,,\ |D\rangle$

- Real-time expansion in tunneling Schoeller, G.S. 94




time evolution of density matrix
due to tunneling processes
and a phonon emission

$$\frac{d\rho_\tau(\lambda)}{d\tau} = -i[H_0, \rho_\tau(\lambda)] + \int_0^\tau d\tau' \, \Sigma(\tau, \tau'; \lambda)\rho_{\tau'}(\lambda)$$

Bloch-Master equation

Lowest order expansion

$$\begin{cases} \Sigma_{RL}^{RL}(t) \leftrightarrow \Pi_{LR}(t)\mathrm{e}^{i\lambda} \\ \Sigma_{DR}^{DR}(t) \leftrightarrow \Pi_{DR}(t) \end{cases}$$

Rates are strongly influenced by dephasing

Markov approximation $M(\lambda) \approx \int_0^\infty dt \, \Sigma(t; \lambda)$

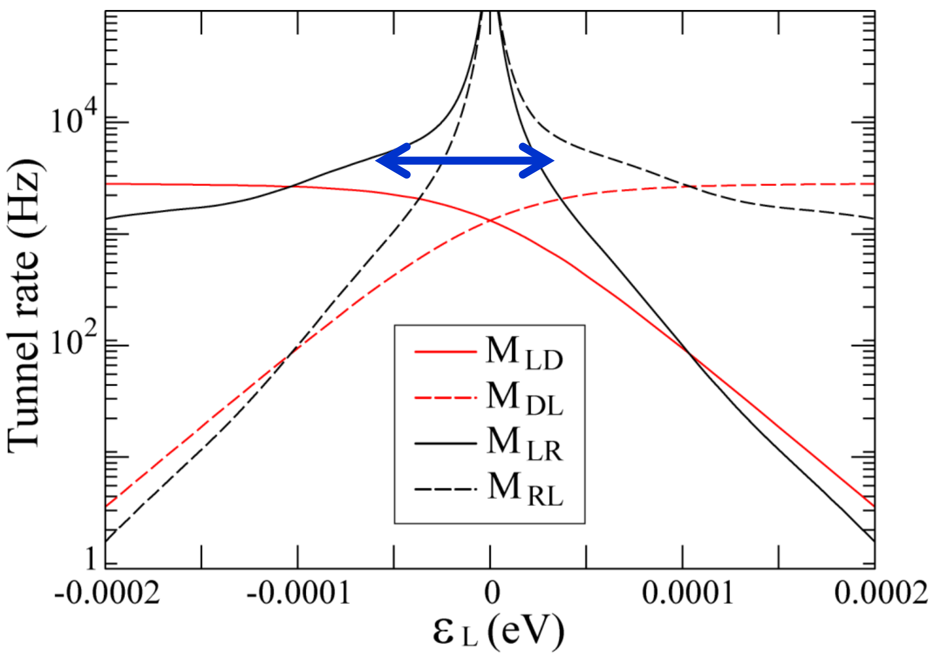
$$\mathcal{Z}_\tau(\lambda) = \mathbf{e}^T \exp(\tau \mathrm{M}(\lambda)) \, \boldsymbol{\rho}_0$$

$$\begin{cases} \mathbf{e}^T = (1, 1, 1) \\ \boldsymbol{\rho}_0^T = (\rho_{\mathrm{st}\,L}, \rho_{\mathrm{st}\,D}, \rho_{\mathrm{st}\,R}) \end{cases}$$

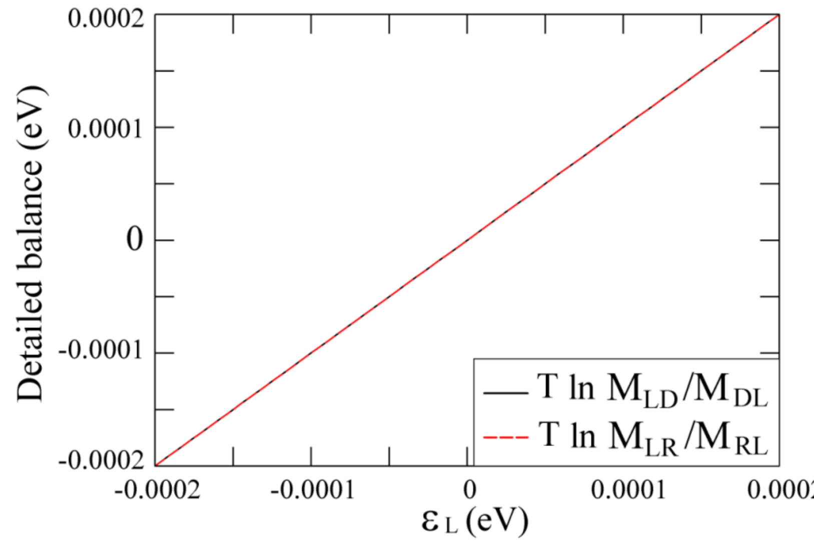
Bagrets, Nazarov 2003

## QPC electrometer off $V_{\mathrm{QPC}} = 0$

### Tunnel rates




### Detailed balance




$$\varepsilon_R = \varepsilon_D = \mu_L = \mu_R \quad (\varepsilon_D \equiv \varepsilon_L + \varepsilon_R + U_{LR})$$

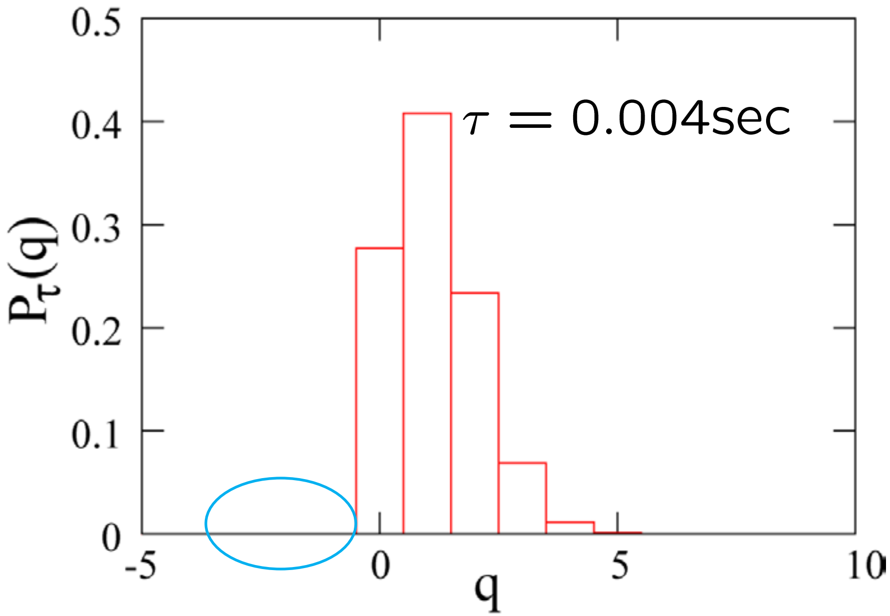
- **Transitions broadened by environments** ("Dynamical Coulomb blockade")

- Oscillations due to phonon interferences

**Detailed balance is satisfied**

## QPC electrometer off $V_{\mathrm{QPC}} = 0$

### Full Counting Statistics

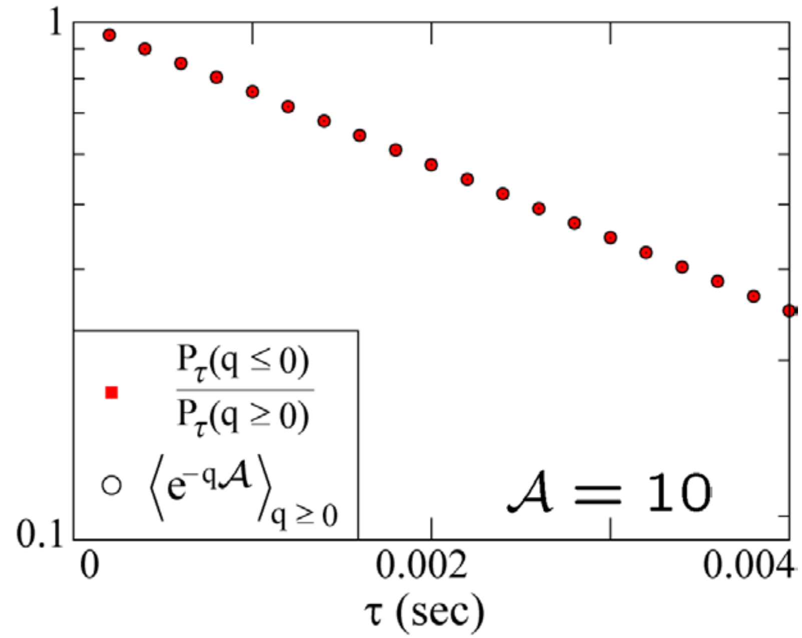



Backward tunneling processes (thermal fluctuations) are suppressed

$$eV \gg k_{\mathrm{B}}T$$

$$V = 300\mu\mathrm{V} \quad T \approx 300\mathrm{mK}$$

### Fluctuation Theorem

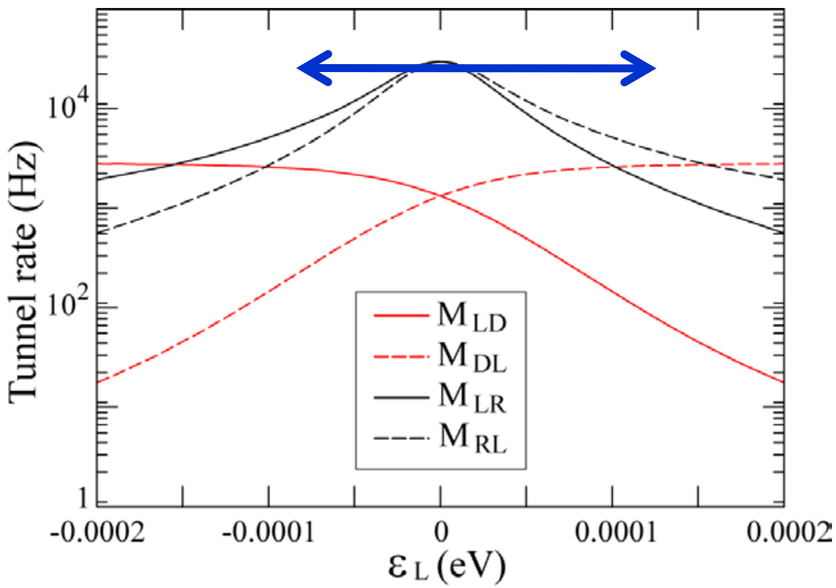


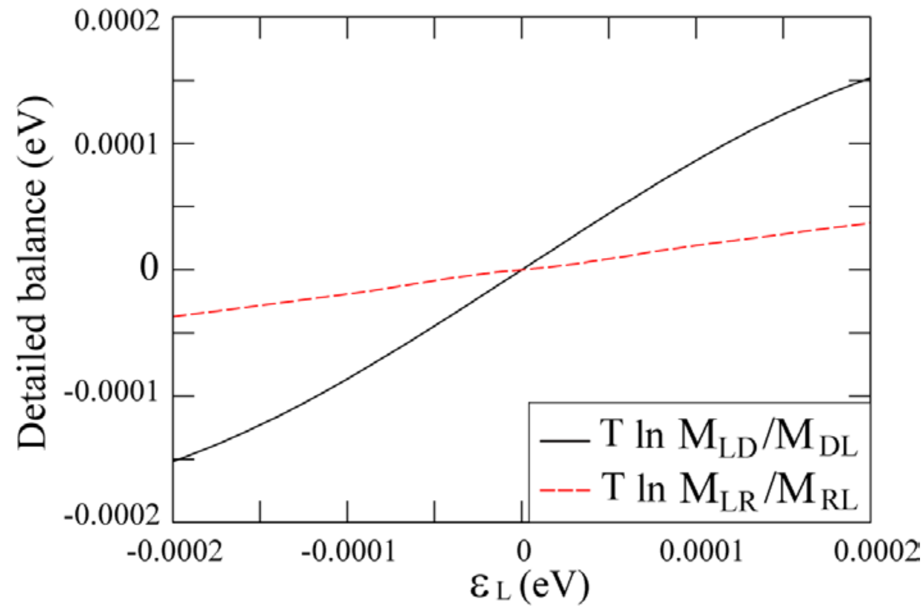

**Fluctuation Theorem is satisfied**

## QPC electrometer on $V_{\mathrm{QPC}} = 800\mu\mathrm{eV}$

**Non-equilibrium electromagnetic environment**

## Tunnel rates



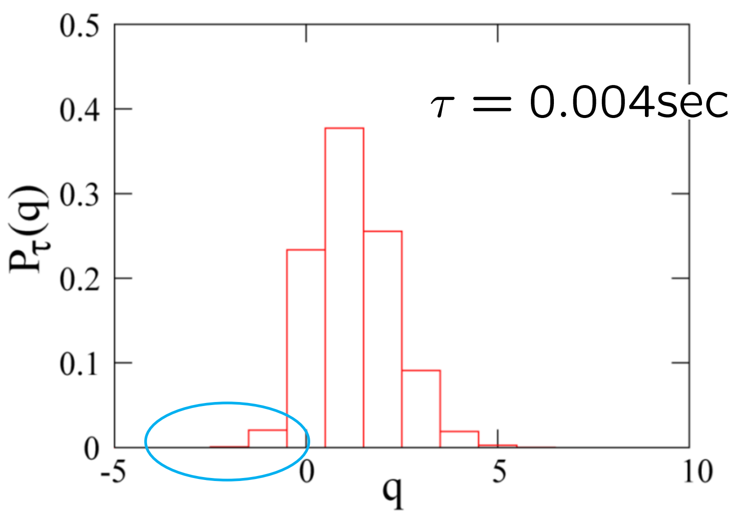

**Transition rates broadened by non-equilibrium QPC current noise**

**- Detailed balance is violated**

## QPC electrometer on $V_{\mathrm{QPC}} = 800\mu\mathrm{eV}$

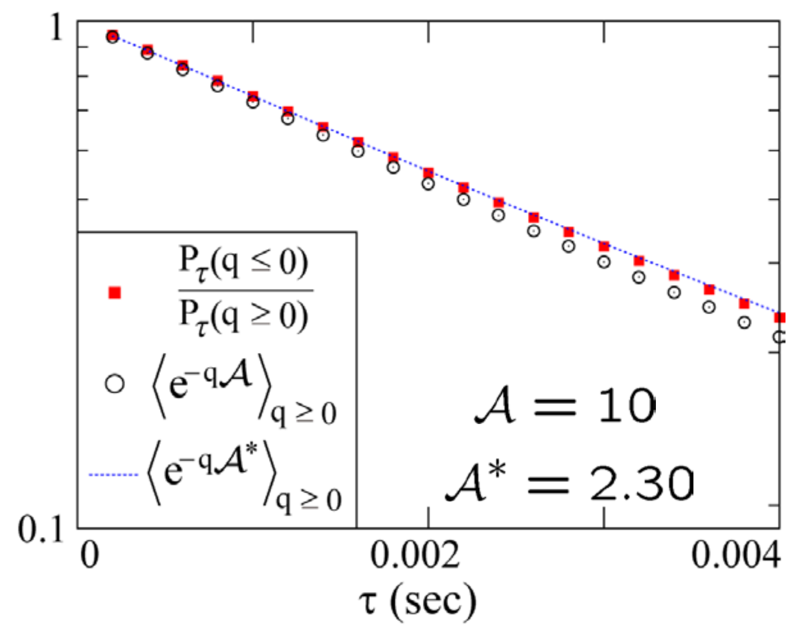
**Non-equilibrium electromagnetic environment**

### Full Counting Statistics




Backward tunneling induced

### Fluctuation Theorem




**- Fluctuation Theorem is violated**

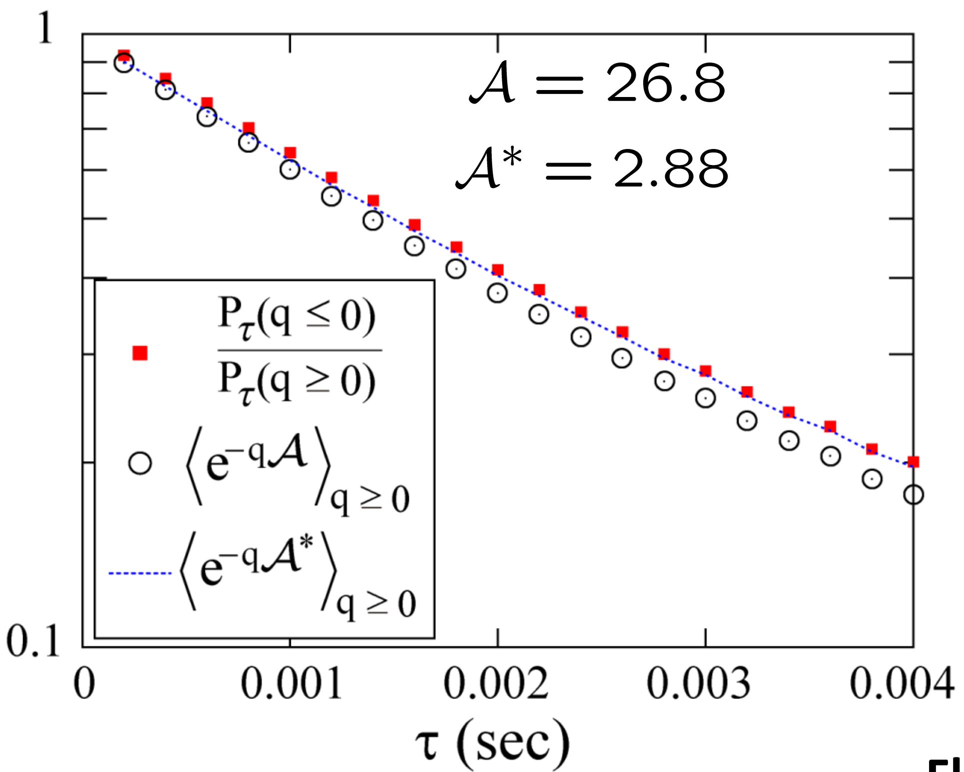
FT reproduced with effective temperature

for 3 state system:

$$T^* = eV/\ln\frac{\Gamma_{LD}\Gamma_{DR}\Gamma_{RL}}{\Gamma_{LR}\Gamma_{RD}\Gamma_{DL}}$$

# Comparison with Experiment

Fujisawa, Hayashi, Tomita, Hirayama, Science 2006




**- Fluctuation Theorem is violated**

but reproduced with effective temperature

# Summary

1. We compare predictions of Fluctuation Theorem (FT)
   with FCS of electron tunneling through a double QD coupled to
   phonons,
   measurement device,
   and electromagnetic environment.

2. Strong coupling to phonons and **equilibrium** electromagnetic environment strongly modify the tunneling rates, but detailed balance and FT remain valid

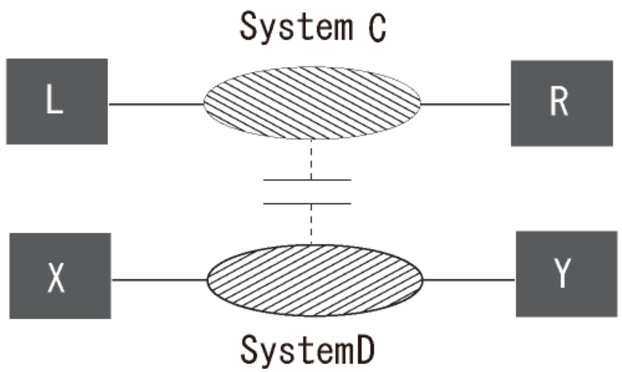
3. **Non-equilibrium** QPC current noise causes deviation from detailed balance and FT

4. FT is recovered with **effective heating**.

# Related works

- Non-equilibrium environment → Onsager relations break down

Sánchez & Kang, PRL (2008)




Onsager symmetry for "system" D breaks down when the "environment" C is out of equilibrium

Here: Extension to the nonlinear regime

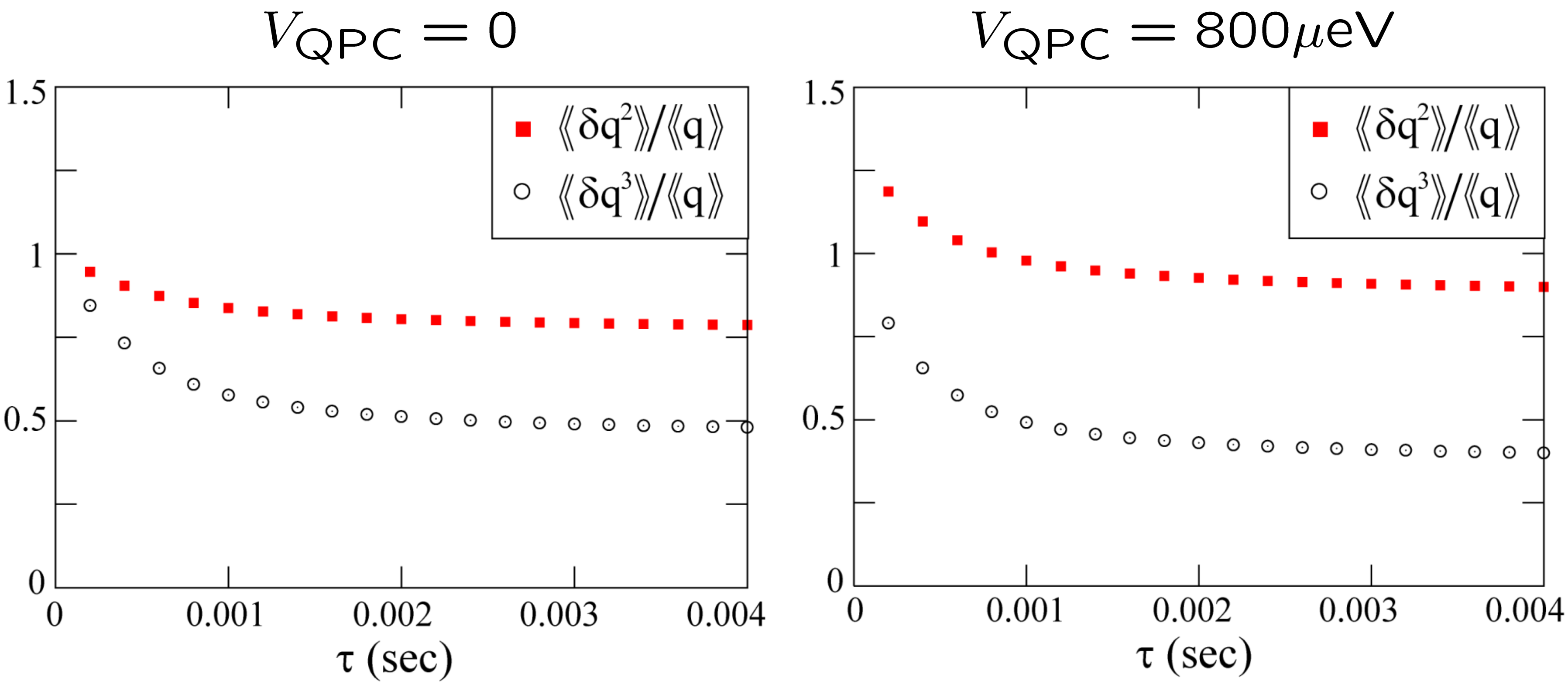
- Temperature is `enhanced' by non-equilibrium current noise

Hashisaka, Yamauchi, Nakamura, Kasai, Ono, Kobayashi, PRB (2009)

# Parameters

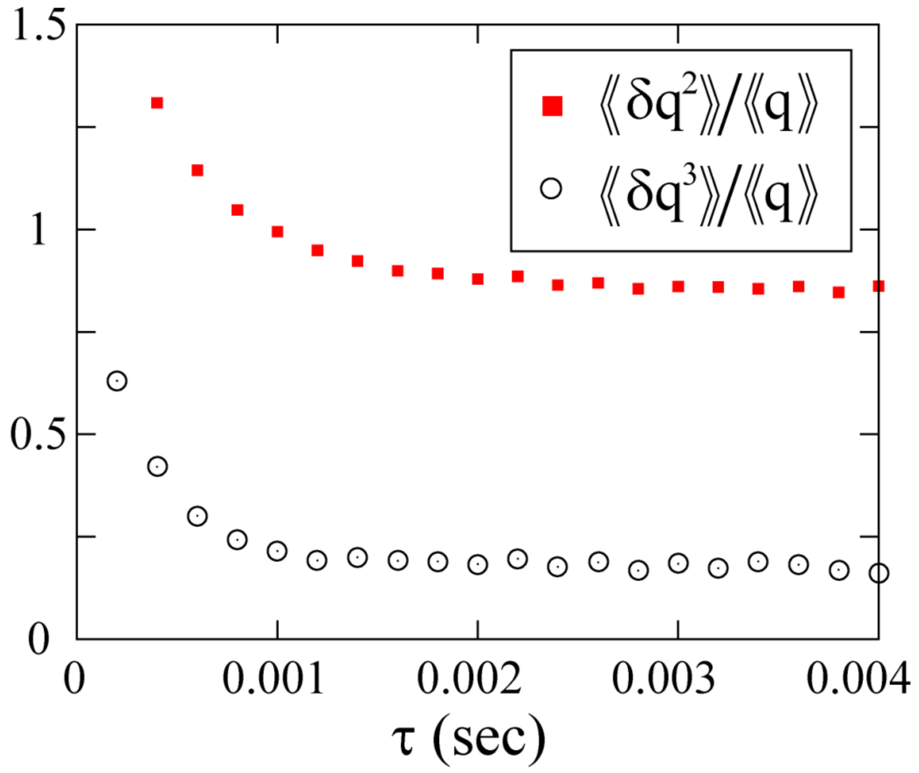
| | |
|---|---|
| QPC transmission probability<br>QPC bias voltage<br>$RC$-time of the external circuit<br>resistance of external circuit | $\mathcal{T} = 0.3$<br>$V_{\mathsf{PC}} = 800\mu\mathsf{eV}$<br>$1/(RC) = 0.5\mathsf{meV}$<br>$R_K/R = 0.05$ |
| phonon coupling constant | $\frac{P_{\mathsf{GaAs}}}{2\pi^2 \rho_{\mathsf{GaAs}} v_s^3} = 0.005$<br>$2v_s/r_0 = 1$ meV<br>$v_s/\|\vec{R}_L - \vec{R}_R\| = 16\mu\mathsf{eV}$ |
| dot bias voltage<br>dot level<br><br><br>coupling strength | $\mu_L = -\mu_R = 150\mu$ eV<br>$\varepsilon_L = 140\mu$ eV<br>$\varepsilon_R = -110\mu$ eV<br>$\varepsilon_D \equiv \varepsilon_L + \varepsilon_R + U_{LR} = 0$ eV<br>$\Gamma_L = 3$ kHz<br>$\Gamma_R = 3$ kHz<br>$T_{LR} = 30$ MHz |
| temperature | $T = 30\mu$ eV |

# Generalized Fano factor



# Comparison with Experiment

Fujisawa, Hayashi, Tomita, Hirayama, Science 2006



# Full Counting Statistics

$$P_\tau(q) \;=\; \sum_{if} \left|\langle f|\mathrm{e}^{-iH\tau}|i\rangle\right|^2 \langle i|\hat{\rho}_0|i\rangle\, \delta_{q\,,N_L^{(f)}+n_L^{(f)}-N_L^{(i)}-n_L^{(i)}}$$

$|i\rangle,\ |f\rangle$ : eigen vectors of $H_0$ $\qquad \hat{N}_L|m\rangle = N_L^{(m)}|m\rangle \quad \hat{n}_L|m\rangle = n_L^{(m)}|m\rangle$

Characteristic Function $\quad \mathcal{Z}_\tau(\lambda) = \sum_q P_\tau(q)\mathrm{e}^{iq\lambda} = \mathrm{Tr}_{\mathrm{QD}}\left(\rho_\tau(\lambda)\right)$ $\quad$ counting field

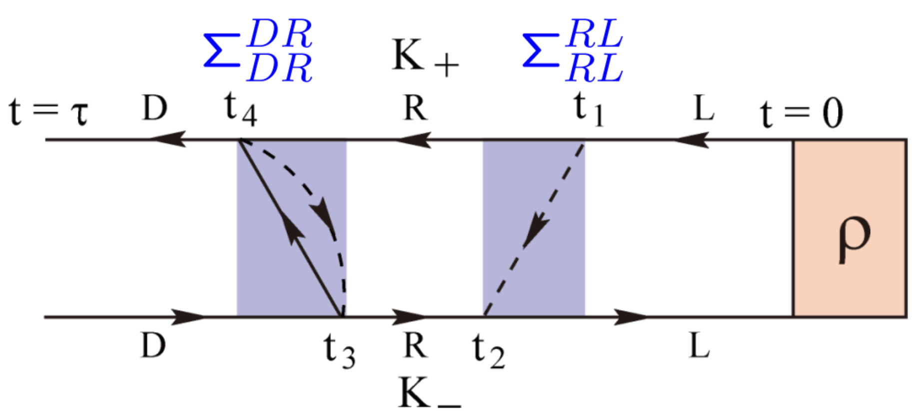
"Reduced Density Matrix"

$$\rho_\tau(\lambda) = \mathrm{Tr}_{\mathrm{Res}}\mathrm{Tr}_{\mathrm{ph}}\mathrm{Tr}_{\mathrm{el}}\left(\mathrm{e}^{-i\hat{U}\hat{H}\hat{U}^\dagger\tau}\,\hat{\rho}_0\,\mathrm{e}^{i\hat{U}^\dagger\hat{H}\hat{U}\tau}\right)$$

$$\hat{U} = \mathrm{e}^{-i\sum_{r=L,R}\lambda_r(\hat{N}_r+\hat{n}_r)}$$

$$\lambda = \lambda_L - \lambda_R$$

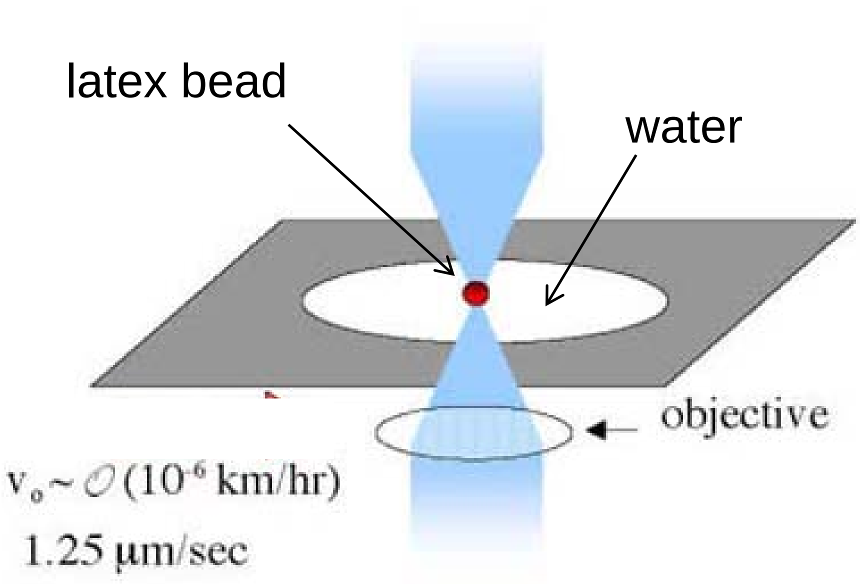
Real-time expansion in tunneling on Keldysh contuour $\quad$ Schoeller , G.S. 94

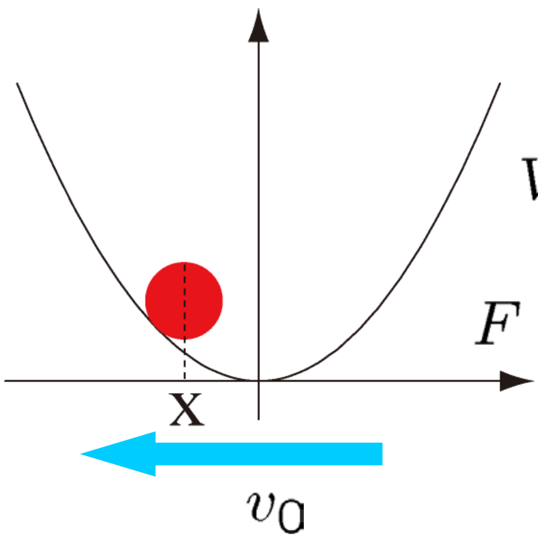


# Experiment

Wang, Sevick, Mittag, Searles, Evans, PRL (2002); Carberry, Reid, Wang, Sevick, Searles, Evans, PRL (2002)

**Colloidal Particle captured in an Optical Trap**




$$V = \frac{1}{2}kx^2$$

$$F = -k\,x(t)$$

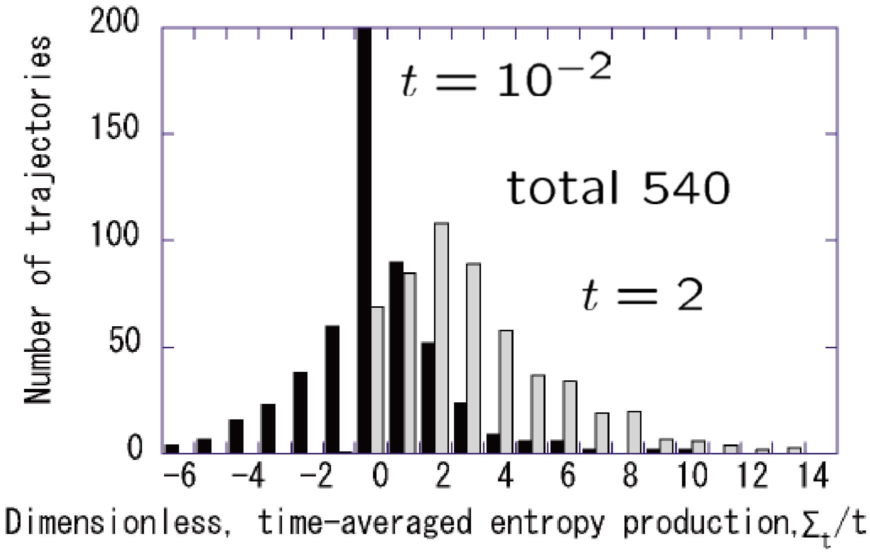
$T \approx 300$K

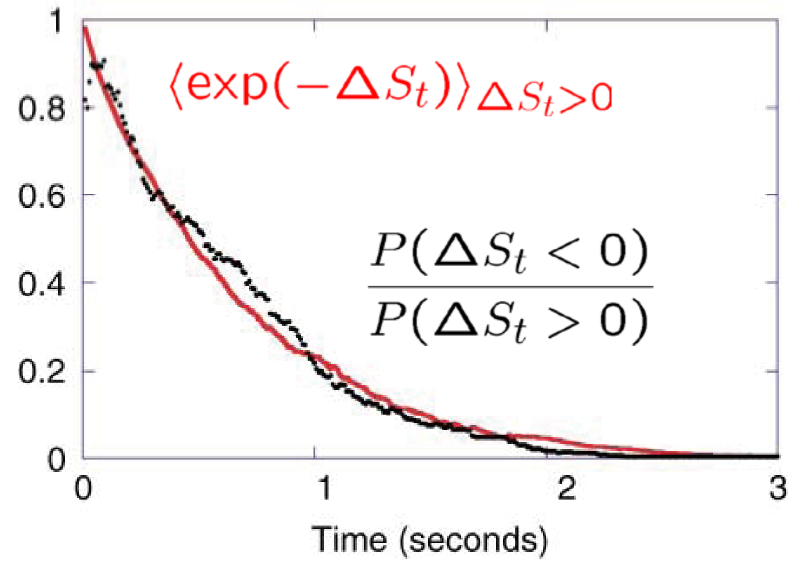
Entropy Production

$$\Delta S = \int_{-\tau/2}^{\tau/2} dt\; v_o\, F(t)/(k_{\mathrm{B}}T)$$

# Experiment

Wang, Sevick, Mittag, Searles, Evans,
PRL (2002)

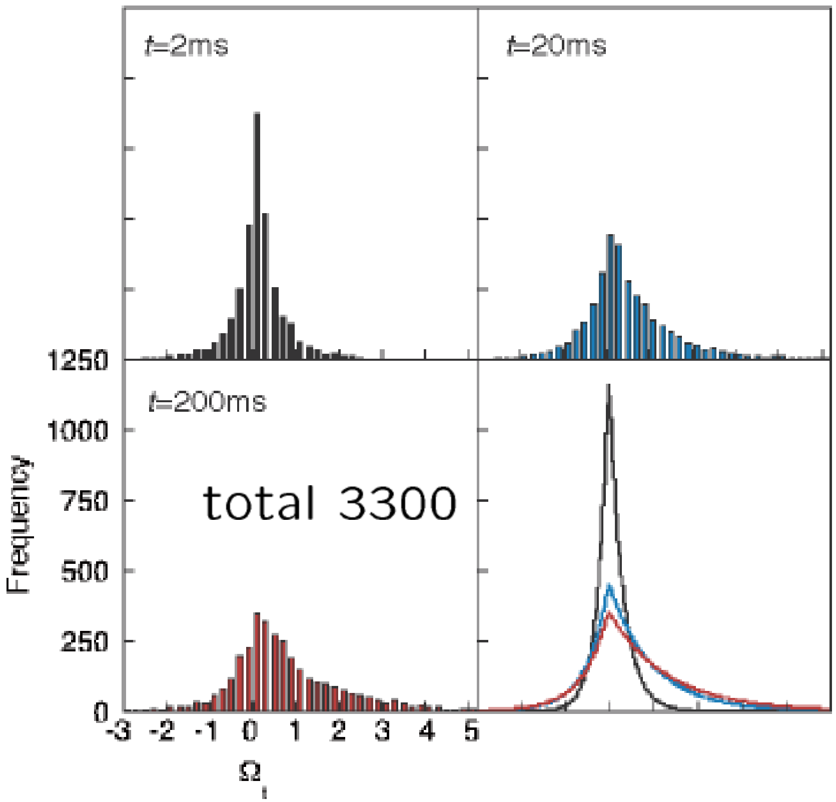


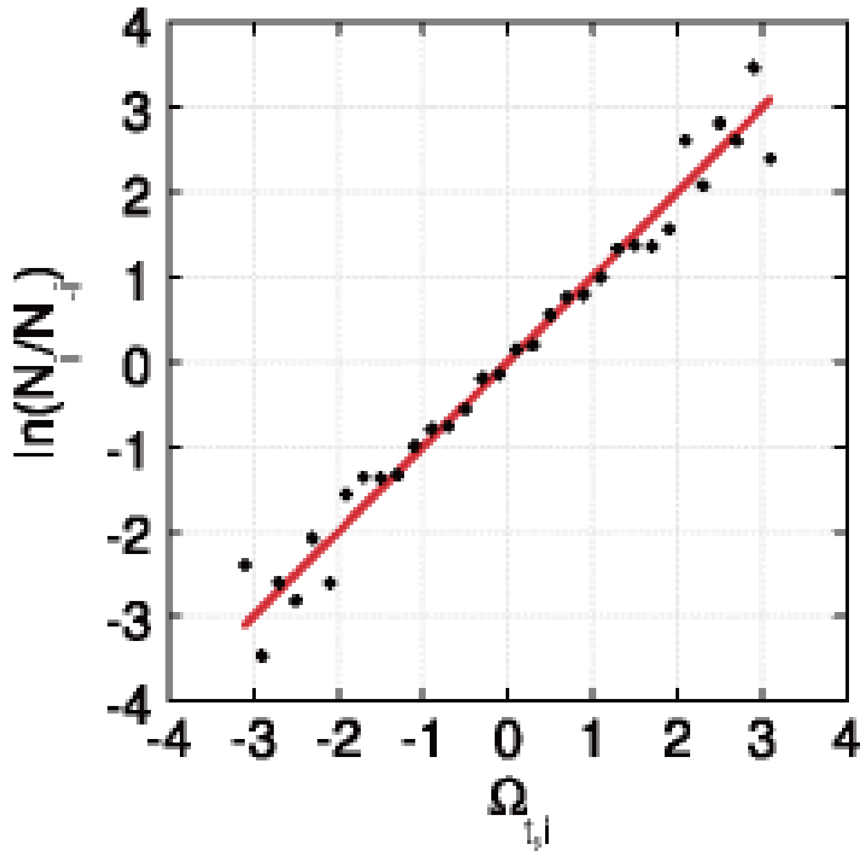




$$\frac{P(\Delta S < 0)}{P(\Delta S > 0)} = \langle \exp(-\Delta S) \rangle_{\Delta S > 0}$$

# Experiment

Carberry, Reid, Wang, Sevick, Searles, Evans,
PRL (2002)



# Bidirectional Single Electron Counting

Fujisawa, Hayashi, Tomita, Hirayama, Science 2006

Band width $\Delta f \approx 10\text{kHz}$