Nonequilibrium physics & heat transport in a quenched Luttinger model

Edwin Langmann

Theoretical Physics KTH, Stockholm, Sweden

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work with J Lebowitz, V Mastropietro, P Moosavi (arXiv:1701.06620)[1]

[1] Phys. Rev. B (accepted)

Plan

Present result on spacetime evolution of heat in a quantum wire

- Energy profile corresponding to temperature profile $T(x)$:

$$\boxed{E_0(x) = \frac{\pi}{6v} T(x)^2 - \frac{v}{12\pi} \left(\frac{T''(x)}{T(x)} - \frac{3}{2} \left(\frac{T'(x)}{T(x)} \right)^2 \right)}$$

- Sketch of derivation
- Discussion

Definitions ("Protocol")

- Model defined by Hamiltonian

$$H = \int dx\, \mathcal{H}(x)$$

- $t = 0$: Non-equilibrium state with temperature profile $T(x)$

$$\rho_{init} = \frac{1}{Z} \exp\left(- \int \beta(x)\mathcal{H}(x)dx \right), \quad \beta(x) = 1/T(x)$$

- Observables at time $t > 0$

$$\langle \mathcal{O}(t) \rangle = \mathrm{Tr}(\rho_{init} e^{iHt} \mathcal{O} e^{-iHt})$$

- Energy density, heat current density

$$E(x,t) = \langle \mathcal{H}(x,t) \rangle, \quad J(x,t) = \langle \mathcal{J}(x,t) \rangle$$

with $\mathcal{J}$ s.t. $\partial_t \mathcal{H} + \partial_x \mathcal{J} = 0$

Result

- (local) Luttinger model (a.k.a. massless Thirring model):[2]

$$\mathcal{H}(x) = v_F \sum_{r=\pm} \psi_r^\dagger(x) r(-i \overset{\leftrightarrow}{\partial}_x)\psi_r(x) + g\psi_+^\dagger(x)\psi_+(x)\psi_-^\dagger(x)\psi_-(x)$$

- Exact computation gives:

$$E(x,t) = \frac{1}{2}\left(E_0(x-vt) + E_0(x+vt)\right)$$

$$J(x,t) = \frac{v}{2}\left(E_0(x-vt) - E_0(x+vt)\right)$$

$$\boxed{E_0(x) = \frac{\pi}{6v}T(x)^2 - \frac{v}{12\pi}\left(\frac{T''(x)}{T(x)} - \frac{3}{2}\left(\frac{T'(x)}{T(x)}\right)^2\right)}$$

[2] $\psi^\dagger(-i \overset{\leftrightarrow}{\partial}_x)\psi_r = (1/2)(\psi^\dagger(-i\partial_x)\psi_r + h.c.)$

Implication

$$J(x,t) = \frac{v}{2}(E_0(x - vt) - E_0(x + vt))$$

$$E_0(x) = \frac{\pi}{6v}T(x)^2 - \frac{v}{12\pi}\left(\frac{T''(x)}{T(x)} - \frac{3}{2}\left(\frac{T'(x)}{T(x)}\right)^2\right)$$

- $\Rightarrow$ Universal heat current

$$\lim_{t\to\infty} J(x,t) = \frac{v}{2}(E_0(-\infty) - E_0(+\infty)) = \frac{\pi}{12}(T_L^2 - T_R^2)$$

Derivation I

- Write $\beta(x) = \beta(1 + \epsilon W(x))$ and expand in ϵ
- Use general result ($\nu \in (2\pi/\beta)\mathbb{Z}$)

$$
\begin{aligned}
&\frac{\mathrm{Tr}(e^{-\beta d\hat{\Gamma}(K+\epsilon W)} d\hat{\Gamma}(O))}{\mathrm{Tr}(e^{-\beta d\hat{\Gamma}(K+\epsilon W)})} - \frac{\mathrm{Tr}(e^{-\beta d\hat{\Gamma}(K)} d\hat{\Gamma}(O))}{\mathrm{Tr}(e^{-\beta d\hat{\Gamma}(K)})} \\
&\qquad = \mathrm{tr}\left(\left\{\left[e^{2\beta(K+\epsilon W)} - 1\right]^{-1} - \left[e^{2\beta K} - 1\right]^{-1}\right\} O\right) \\
&\qquad = \frac{1}{\beta}\sum_{\nu} \mathrm{tr}\left(\left\{[i\nu - 2(K+\epsilon W)]^{-1} - [i\nu - 2K]^{-1}\right\} O\right) \\
&\qquad = \sum_{n=1}^{\infty} \epsilon^n \frac{1}{\beta}\sum_{\nu} \mathrm{tr}\left([i\nu - 2K]^{-1}(2W[i\nu - 2K]^{-1})^n O\right),
\end{aligned}
$$

Derivation II

- $(\cdots)$

$$E_0(x) = \frac{\pi}{6v\beta^2} + \sum_{n=1}^{\infty} \epsilon^n \frac{v}{4\pi} \int_{\mathbb{R}^n} \frac{dq_1 \ldots dq_n}{(2\pi)^n} I_n(\mathbf{q}) \left(\prod_{j=1}^{n} \hat{W}(q_j) e^{iq_j x} \right)$$

$$I_n(\mathbf{q}) = \frac{2}{v} \int_{\mathbb{R}} dp \, \frac{1}{\beta} \sum_{\nu} \left(\prod_{j=0}^{n} \frac{v(p+Q_j)}{i\nu - v(p+Q_j)} \right), \quad Q_j = \sum_{\ell=j}^{n} q_\ell$$

- $(\cdots)$

$$\boxed{I_n(\mathbf{q}) \simeq \frac{(-1)^n}{6} \left\{ (n+1) \left(\frac{2\pi}{\beta v} \right)^2 + 2q_1^2 + (n-1) q_1 q_2 \right\}}$$

$q_j q_k \simeq q_1^2$ if $j = k$ and $q_1 q_2$ otherwise

Derivation III

- Recall $\beta(1+\epsilon W(x)) = \beta(x)$

$$E_0(x) = \frac{\pi}{6v\beta^2} + \sum_{n=1}^{\infty} \epsilon^n (-1)^n \Bigg(\frac{(n+1)\pi}{6v\beta^2} W(x)^n$$
$$- \frac{v}{12\pi} \left[W''(x) W(x)^{n-1} + \frac{n-1}{2} W'(x)^2 W(x)^{n-2} \right] \Bigg)$$

$$= \frac{\pi}{6v} \frac{1}{\beta(x)^2} + \frac{v}{12\pi} \left(\frac{\beta''(x)}{\beta(x)} - \frac{1}{2} \left(\frac{\beta'(x)}{\beta(x)} \right)^2 \right)$$

$$= \frac{\pi}{6v} T(x)^2 - \frac{v}{12\pi} \left(\frac{T''(x)}{T(x)} - \frac{3}{2} \left(\frac{T'(x)}{T(x)} \right)^2 \right)$$

CFT interpretation

$$\boxed{E_0(x) = \frac{\pi}{6v}T(x)^2 - \frac{v}{12\pi}\left(\frac{T''(x)}{T(x)} - \frac{3}{2}\left(\frac{T'(x)}{T(x)}\right)^2\right)}$$

- $E_0(x)$ can be written as (S= Schwarzian derivative)

$$E_0(x) = \frac{\pi}{6v}T^2 g'(x)^2 - \frac{v}{12\pi}(Sg)(x), \quad g(x) = \int_0^x \frac{T(x')}{T}dx'$$

- Compare with CFT transformation of energy-momentum tensor

$$\mathcal{T}(z) = \left(\frac{dw}{dz}\right)^2 \mathcal{T}(w) + \frac{cv}{12}(Sw)(z)$$

$\Rightarrow$ Non-equilibrium results for E and J can be obtained from equilibrium ones by conformal transformation given by $g(x)$

- True also for other observables? for other CFT's?

Other results in our paper

- Explicit description of NESS even for non-local Luttinger model
- Universal heat current even for the non-local Luttinger model
- Explicit formulas for correlation functions
- $E(x, t)$ and $J(x, t)$ for non-local Luttinger model in first order in ϵ
- Corresponding result for charge transport (LLMM, CMP 2016)