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RANDOM FIELDS AND TOPOLOGY

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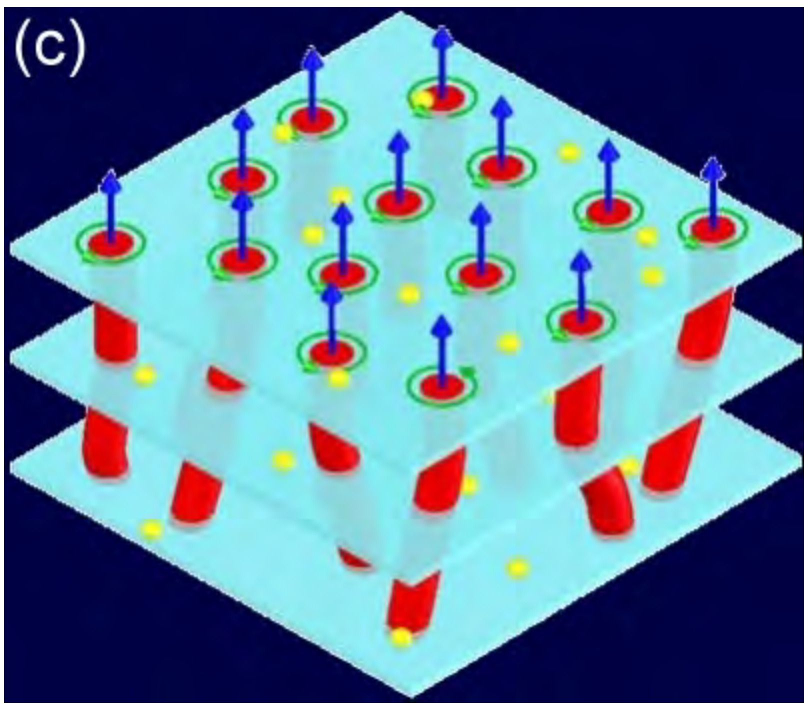
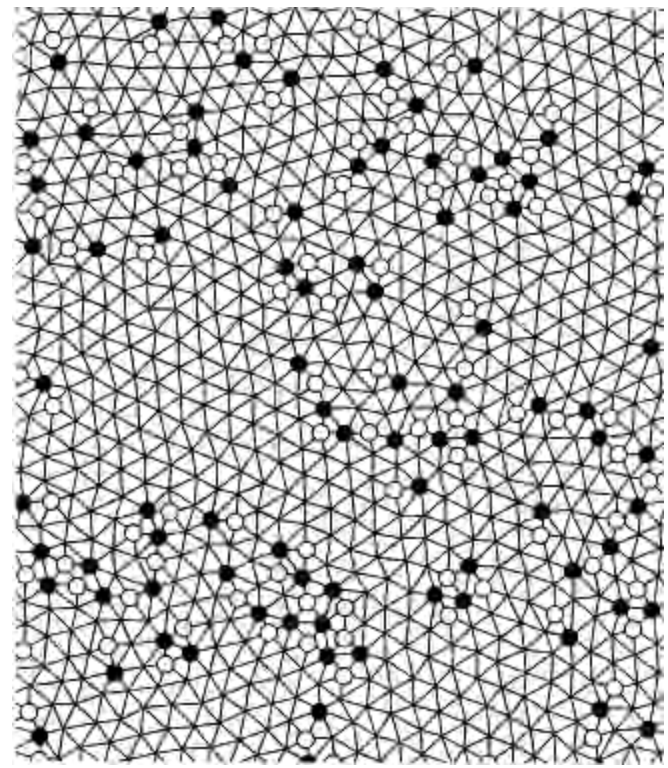
Random Pinning of Flux Lattices (Larkin 1970s)

$$H = \frac{1}{2}\int d^3r\left[\left(C_{11} - C_{66}\right)\left(\partial_\alpha u_\alpha\right)^2 + C_{66}\left(\partial_\alpha u_\beta\right)^2 + C_{44}\left(\partial_z u_\alpha\right)^2\right] - \int d^3r\, u_\alpha f_\alpha$$





Imry-Ma Argument (1975)

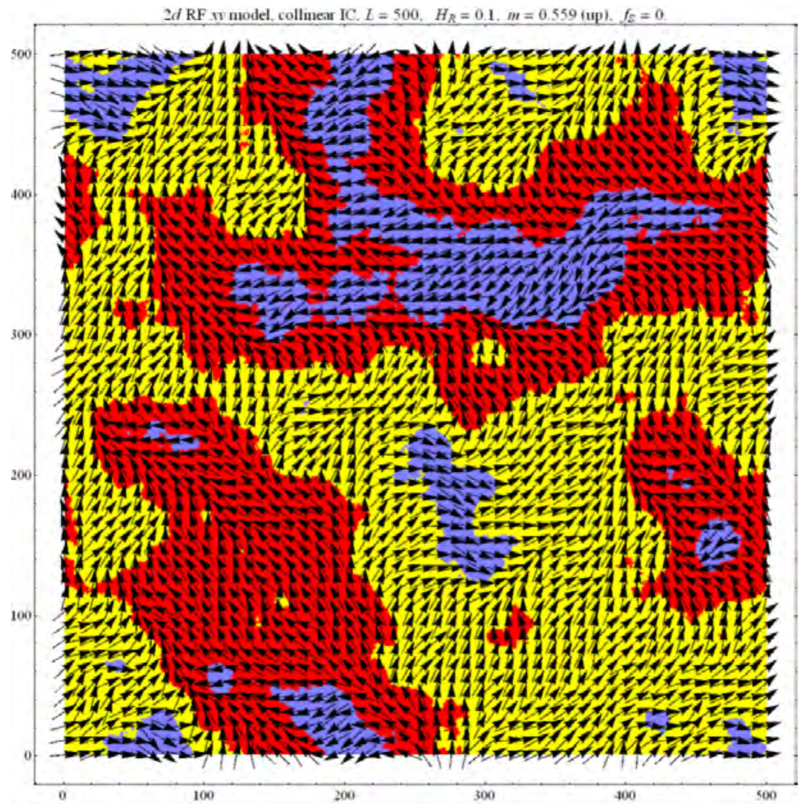
$$\mathrm{H} = \int d^d r \left[\frac{1}{2} \alpha \left(\frac{d\vec{S}}{dr_i} \right)^2 - \vec{h} \cdot \vec{S} \right]$$

If the order exists in volumes of size R, the average exchange energy per spin scales as α / R^2.

The average Zeeman energy per spin scales as $-h / R^{d/2}$.

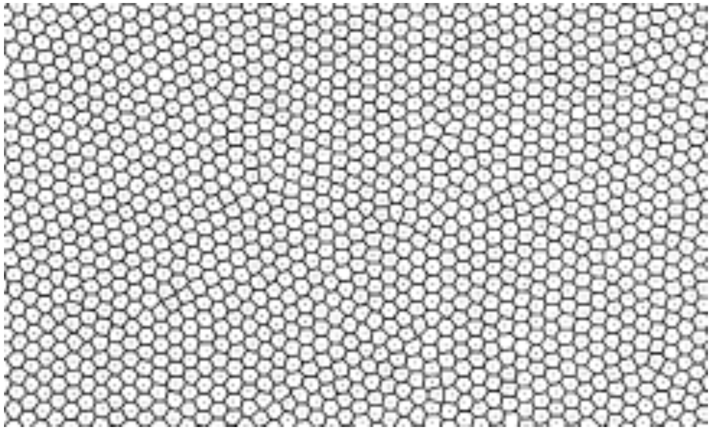
The total energy has minimum at

$$R = R_f \propto 1 / h^{2/(4-d)}$$

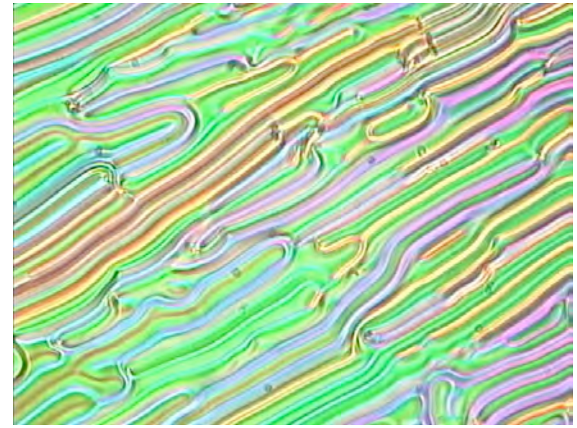


Destruction of the long-range order by however weak random field in less than four dimensions: Larkin-Imry-Ma domains. In superconductors the critical current goes down when Imry-Ma correlation length increases.

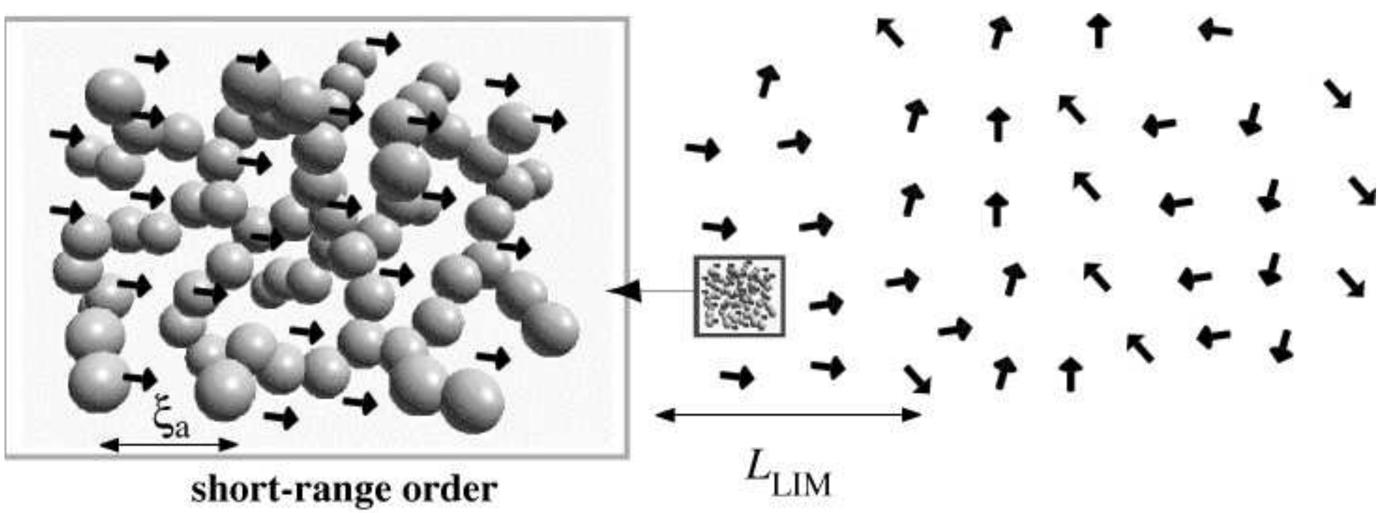
Imry-Ma domains in various systems



Magnetic bubbles (Seshadri & Westervelt 1990s)

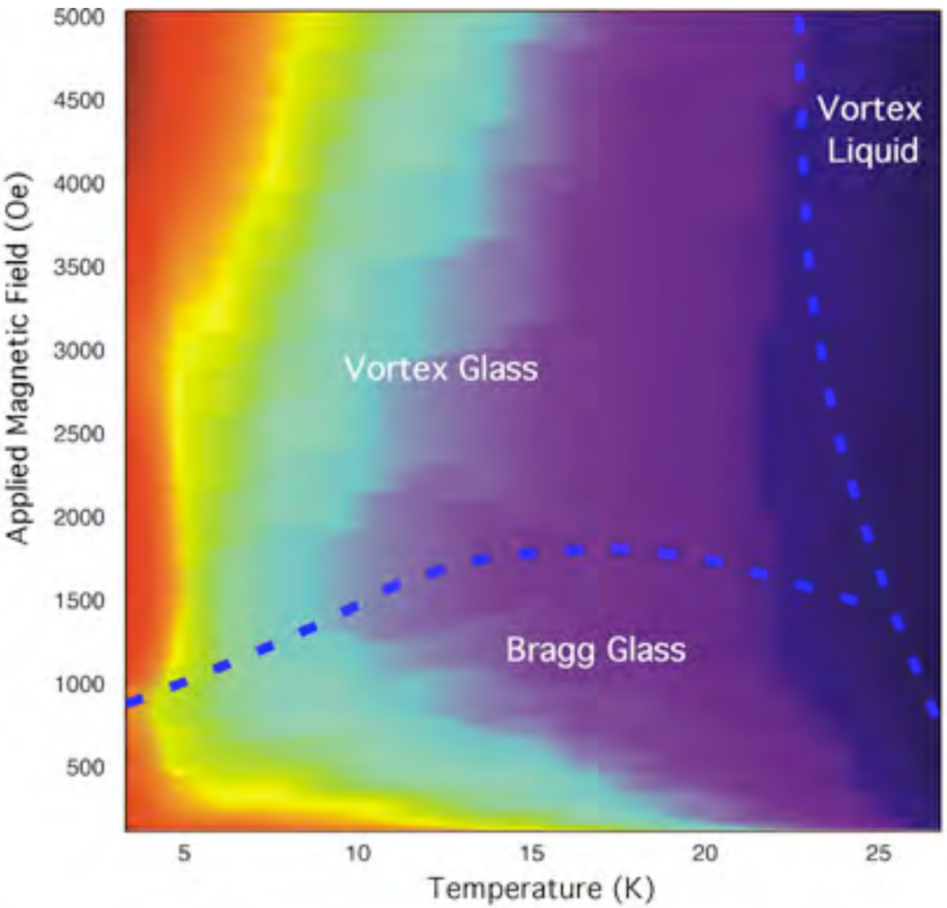


Liquid crystals (Bellini et al. 1998)



Liquid ^{3}He-A in aerogel (Volovik et al. 2008, Li et al. 2013)

Bragg Glass (Nattermann, Giamarchi & Le Doussal 1990s)



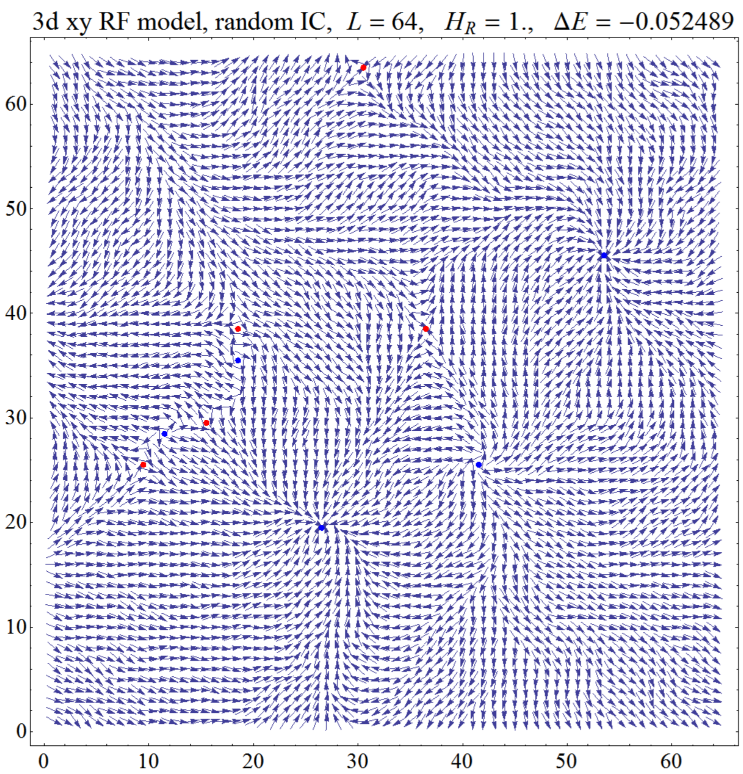
Vortex-free low-temperature phase with the power-law decay of correlations at distances greater than R_f:

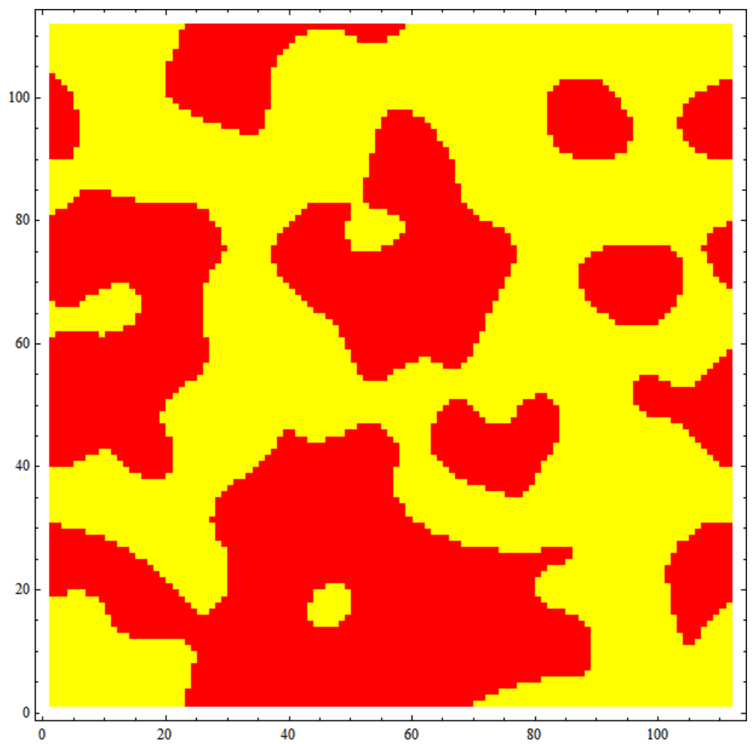
$$\text{SCs}: \left\langle e^{i\vec{G}\cdot[\vec{u}(\vec{r}_1-\vec{r}_2)}\right\rangle_{3d} \propto \frac{1}{|\vec{r}_1-\vec{r}_2|}$$

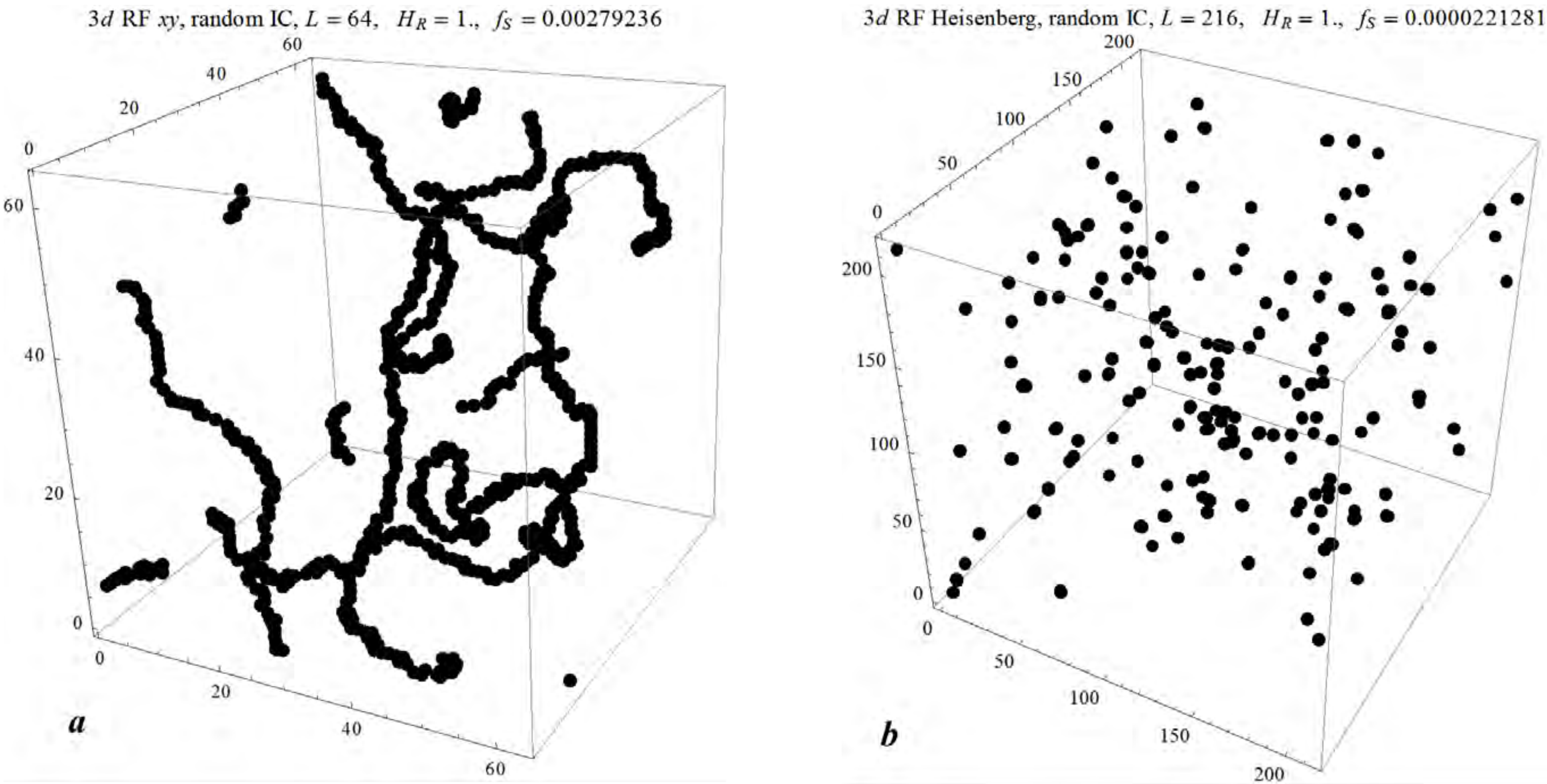
$$\text{Spins}: \left\langle \vec{s}(\vec{r}_1)\cdot\vec{s}(\vec{r}_2)\right\rangle_{3d} \propto \frac{1}{|\vec{r}_1-\vec{r}_2|}$$

Random-Field Spin Model

$$H = -\frac{1}{2}\sum_{ij} J_{ij}\vec{s}_i \cdot \vec{s}_j - \sum_i \vec{h}_i \cdot \vec{s}_i - \vec{H} \cdot \sum_i \vec{s}_i$$

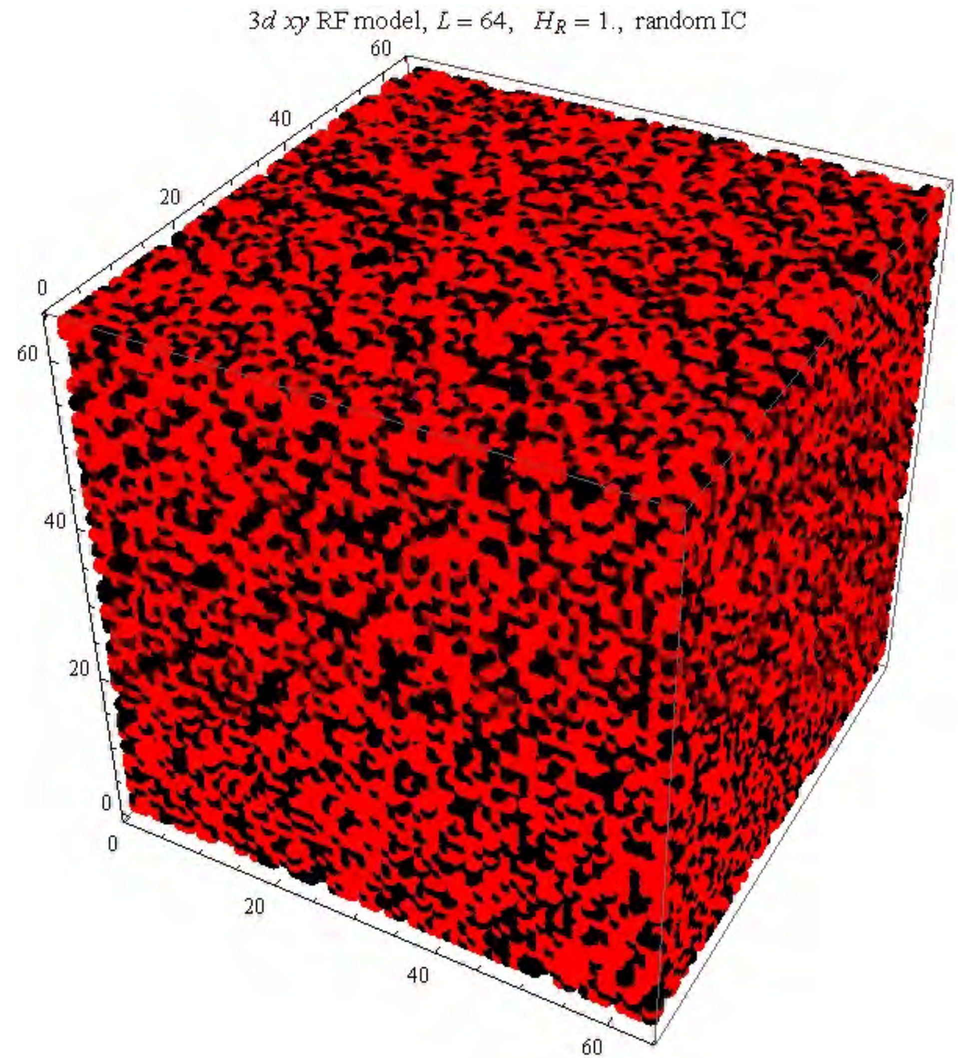


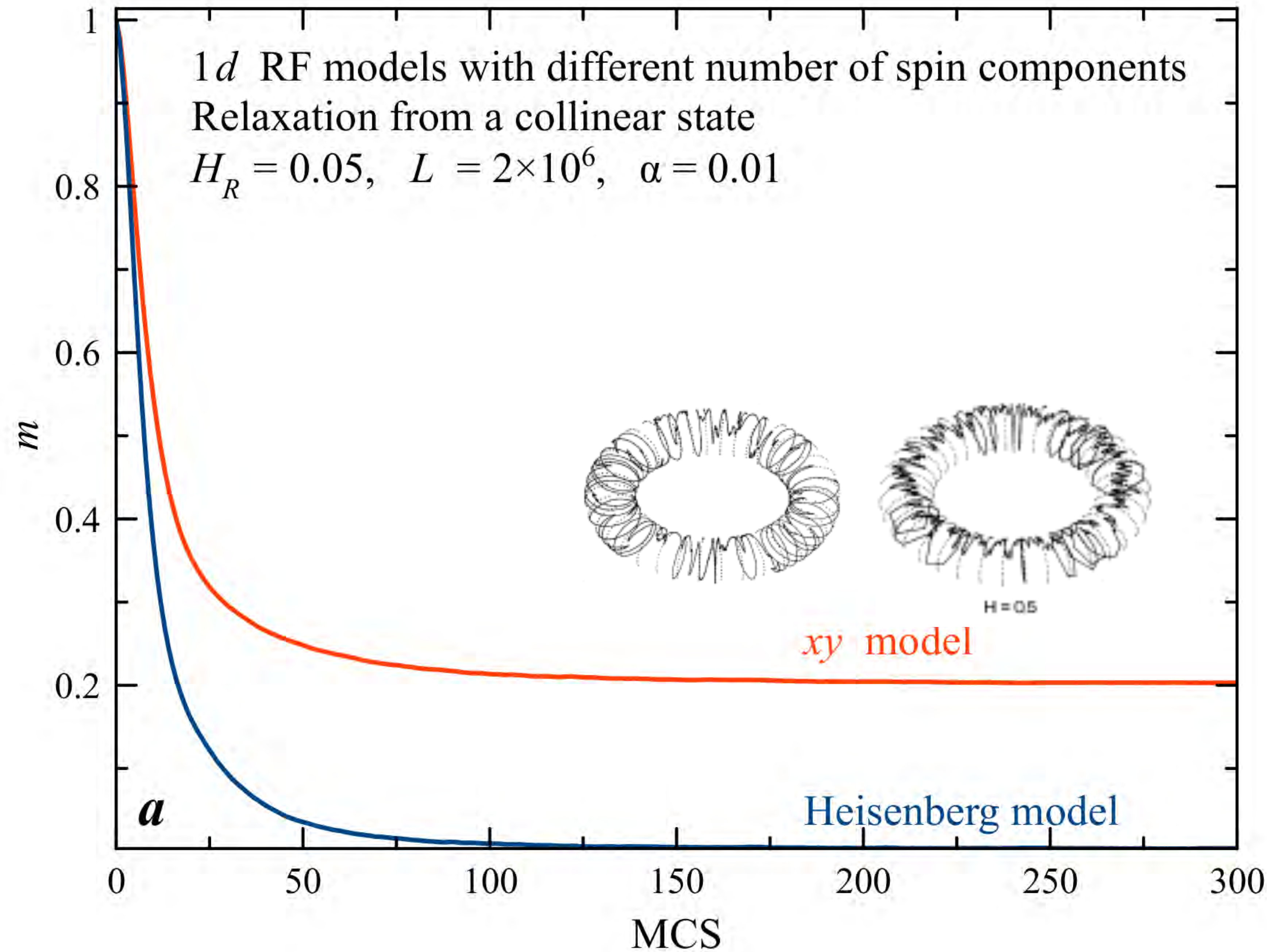




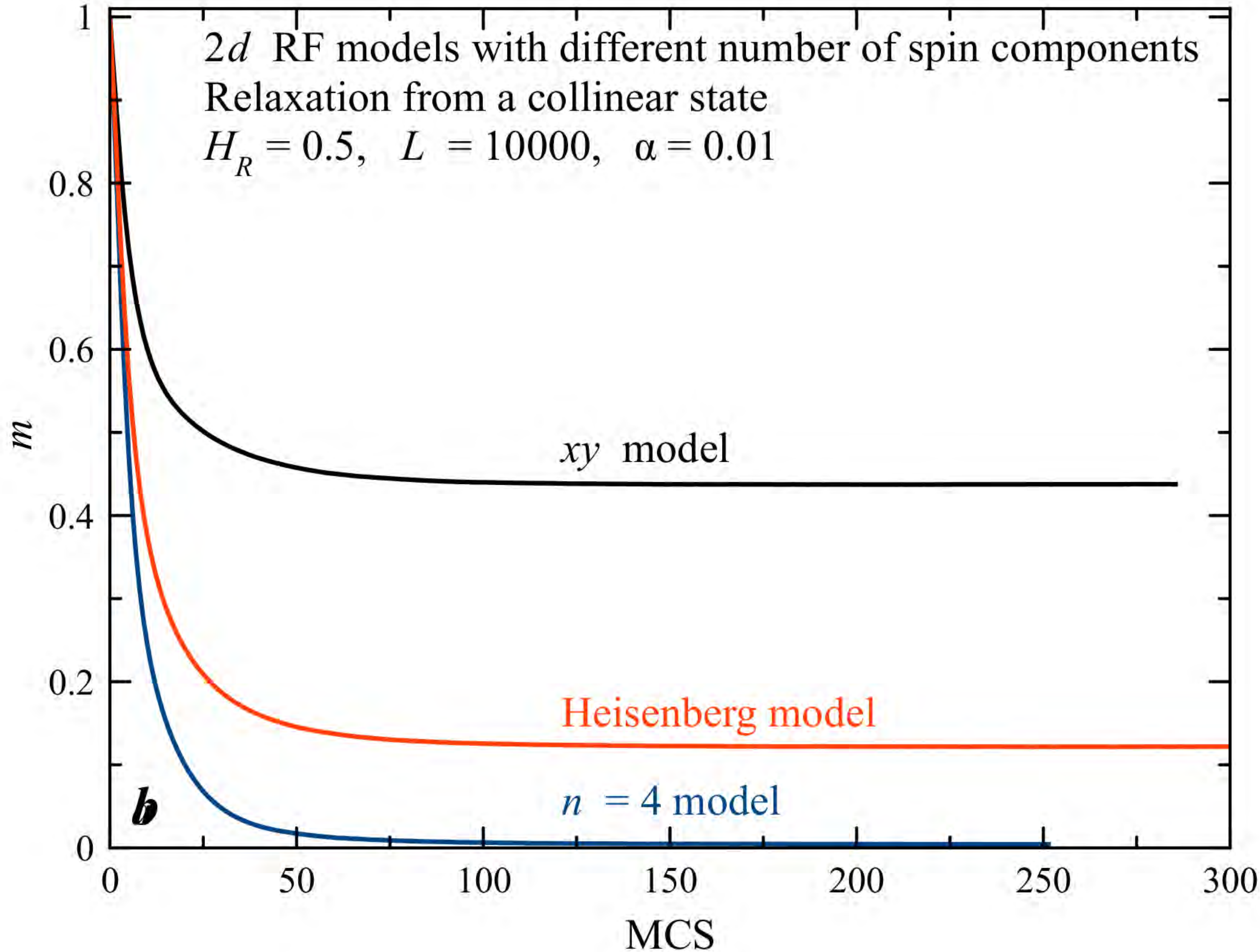
3d RF xy, random IC, L = 64, H_R = 1., f_S = 0.00279236
a
3d RF Heisenberg, random IC, L = 216, H_R = 1., f_S = 0.0000221281
b

Numerical Results - Random Initial Conditions

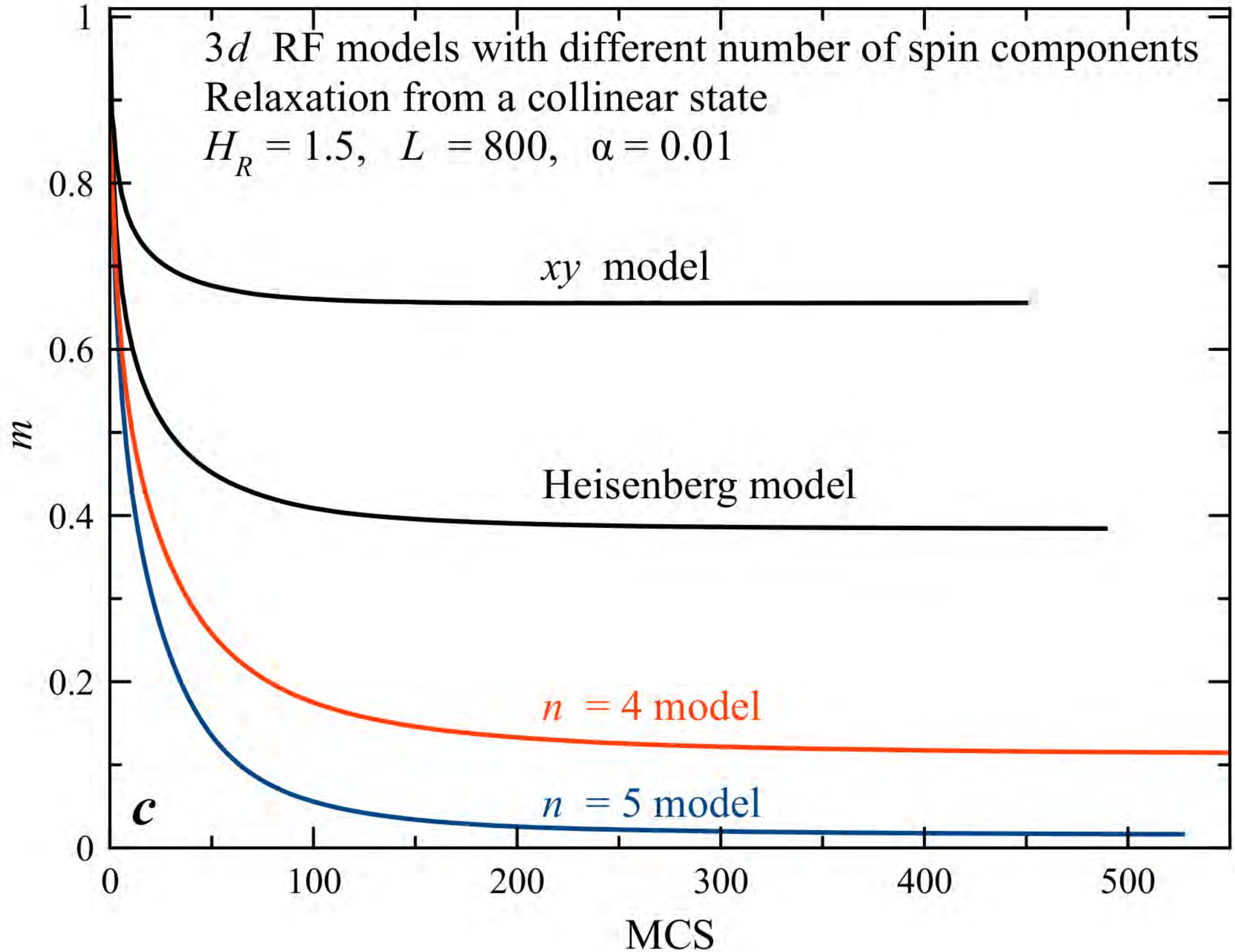




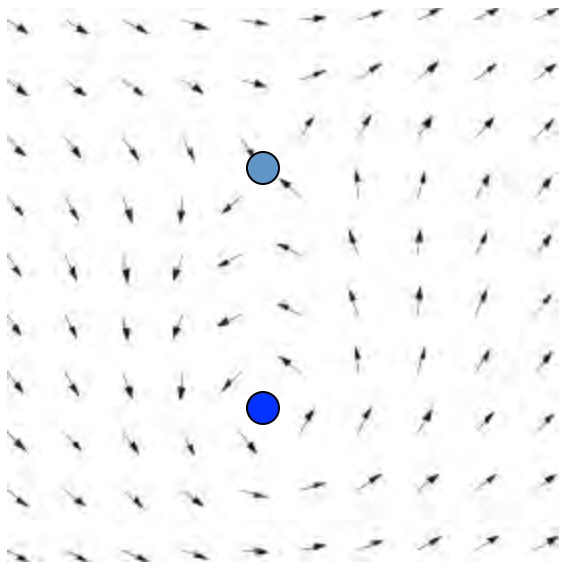
1
0.8
0.6
0.4
0.2
m
1d RF models with different number of spin components
Relaxation from a collinear state
H_R = 0.05, L = 2×10^6, α = 0.01
H=05
xy model
Heisenberg model
a
0
50
100
150
200
250
300
MCS



2d RF models with different number of spin components
Relaxation from a collinear state
H_R = 0.5, L = 10000, α = 0.01
xy model
Heisenberg model
n = 4 model
b
m
MCS
1
0.8
0.6
0.4
0.2
0
0
50
100
150
200
250
300


3d RF models with different number of spin components
Relaxation from a collinear state
$H_R = 1.5$, $L = 800$, $\alpha = 0.01$
xy model
Heisenberg model
n = 4 model
n = 5 model
c
m
MCS
0
0.2
0.4
0.6
0.8
1
0
100
200
300
400
500

Vortices in 2d



vortex-antivortex pair

$$\vec{s} = (s\cos\varphi, s\sin\varphi)$$

$$\vec{r} = (x, y) = (r\cos\phi, r\sin\phi)$$

$$H = -\frac{1}{2} J \sum_{ij} \vec{s}_i \cdot \vec{s}_j \approx Js^2 \int dx\, dy (\vec{\nabla}\varphi)^2$$

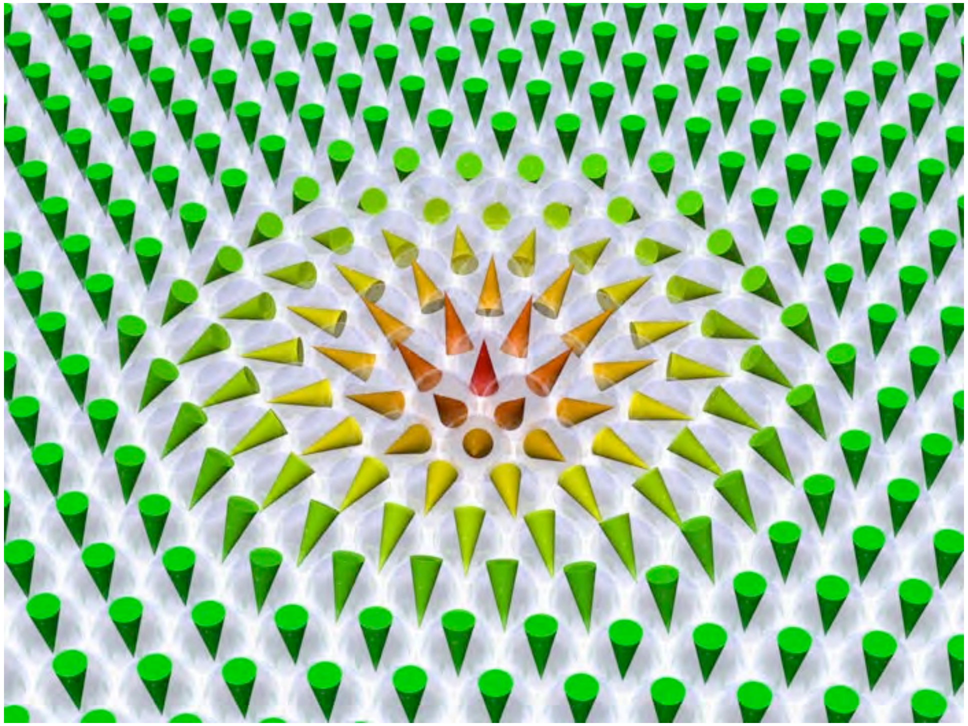
$$\oint d\vec{l} \cdot \vec{\nabla}\varphi = 2\pi n_v, \quad n_v = 0, \pm 1, \pm 2, \ \ldots$$

$$n_v = 1: \qquad \varphi = \arctan(y / x)$$

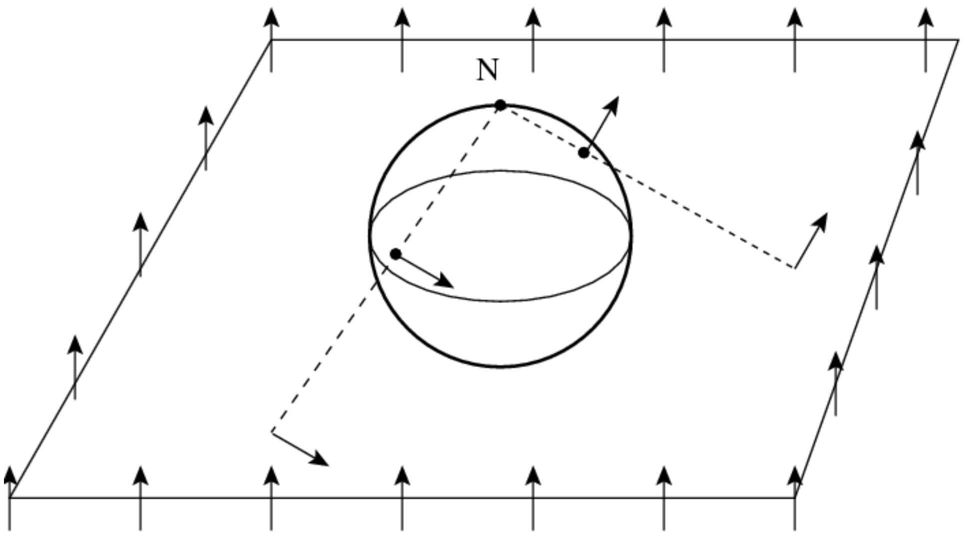
Kosterlitz-Thouless transition: $T_{KT} = \pi J s^2$

Skyrmions in 2d

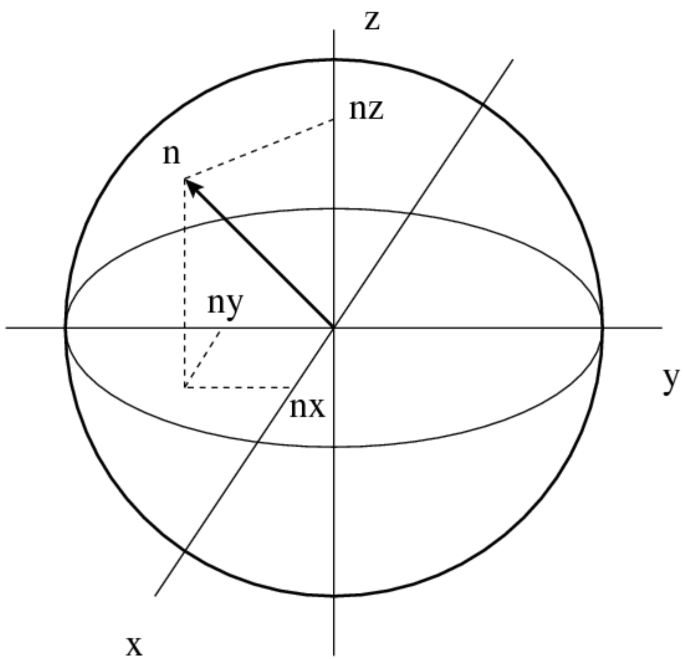
$$H = \frac{1}{2}\int dx\,dy\left(\frac{\partial\vec{n}}{\partial x}\cdot\frac{\partial\vec{n}}{\partial x}+\frac{\partial\vec{n}}{\partial y}\cdot\frac{\partial\vec{n}}{\partial y}\right), \qquad \vec{n}^2 = n_x^2 + n_y^2 + n_z^2 = 1$$



Mapping of the Order Parameter onto Geometrical Space



XY - space

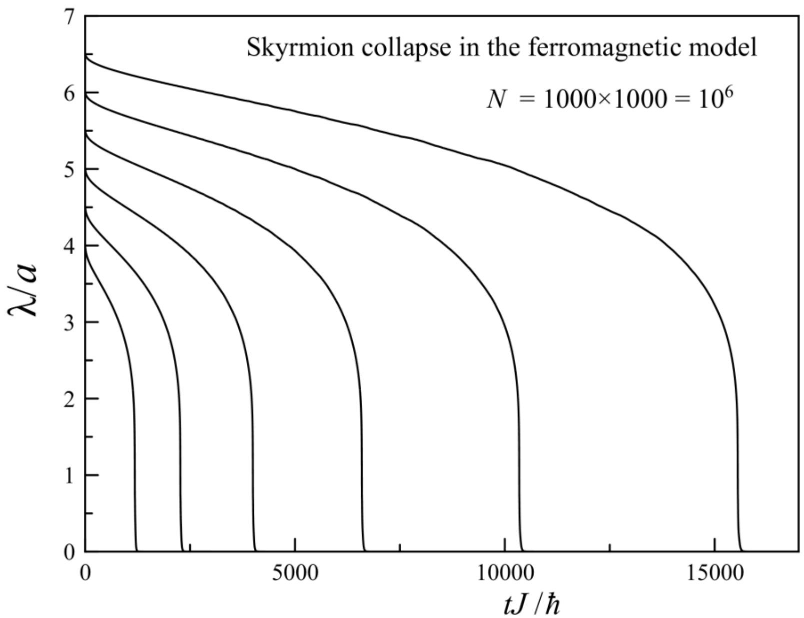


$\vec{n}$ - space

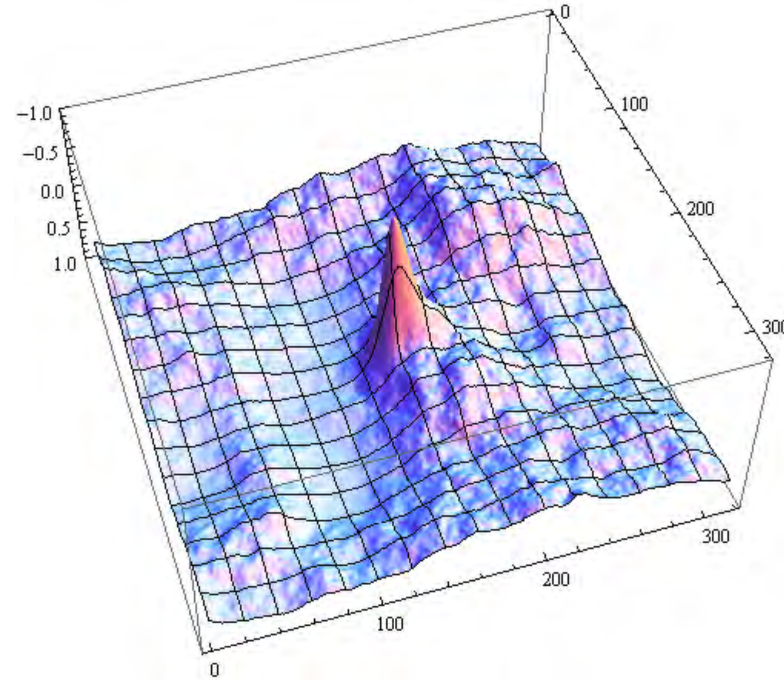
Skyrmion topological charge: $$Q = \frac{1}{4\pi}\int dx\,dy\;\vec{n}\cdot\left(\frac{\partial\vec{n}}{\partial x}\times\frac{\partial\vec{n}}{\partial y}\right)$$

General: n-component fixed-length vector field in d dimensions

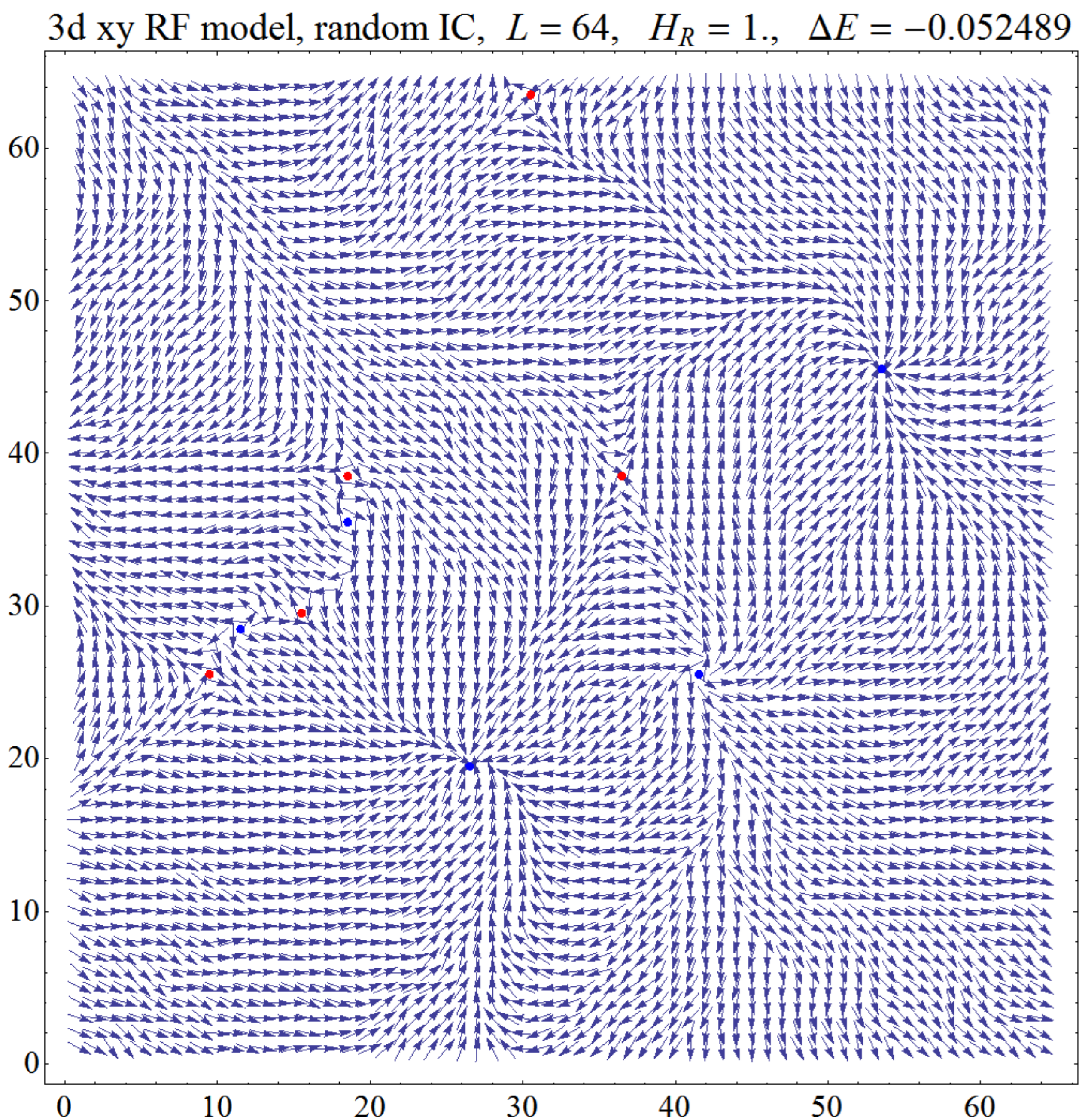
Topological objects exist at $n \le d+1$



Skyrmion collapse in the lattice model
(Liufei Cai, EC, and D. Garanin, PRB-2012)

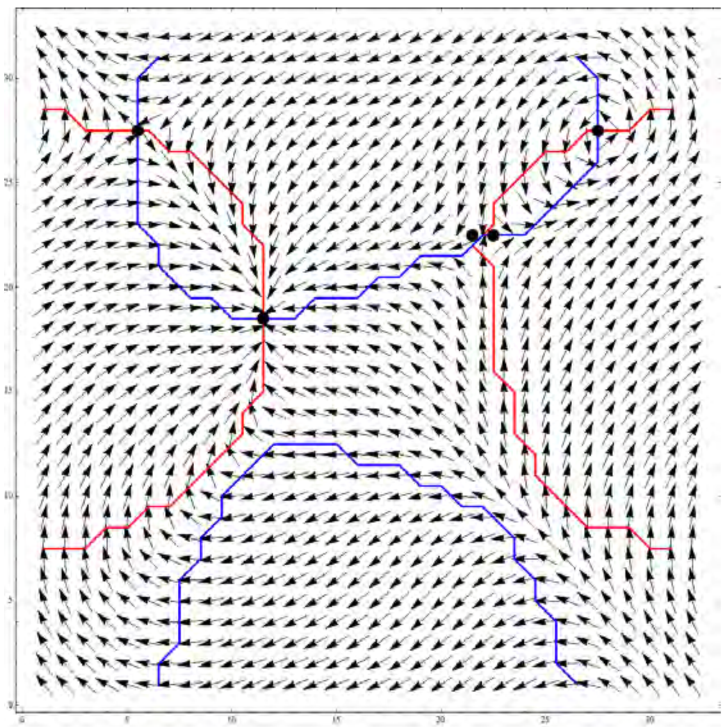


Skyrmion pinned by the random field


3d xy RF model, random IC, L = 64, H_R = 1., ΔE = −0.052489
0
10
20
30
40
50
60
0
10
20
30
40
50
60

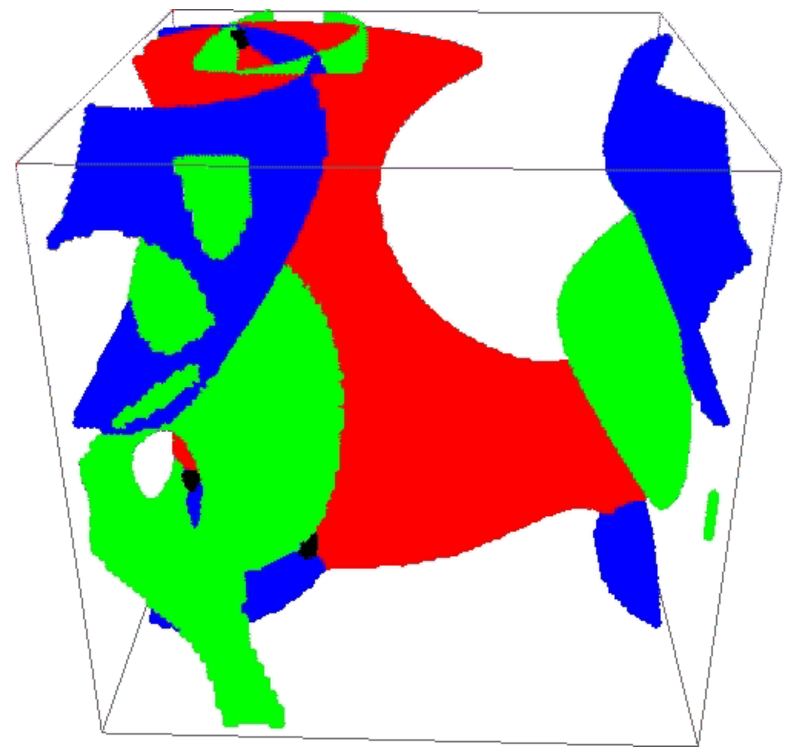
Numerical construction of Imry-Ma domains: $\vec{S}(\vec{r}) \propto \left\langle \vec{h}(\vec{r}) \right\rangle_{V_f}$

For $n < d+1$ singularities appear where $\left\langle h_i \right\rangle = 0$ for all $i = 1, 2, \ldots, n$



XY spins in a $2d$ random-field model.

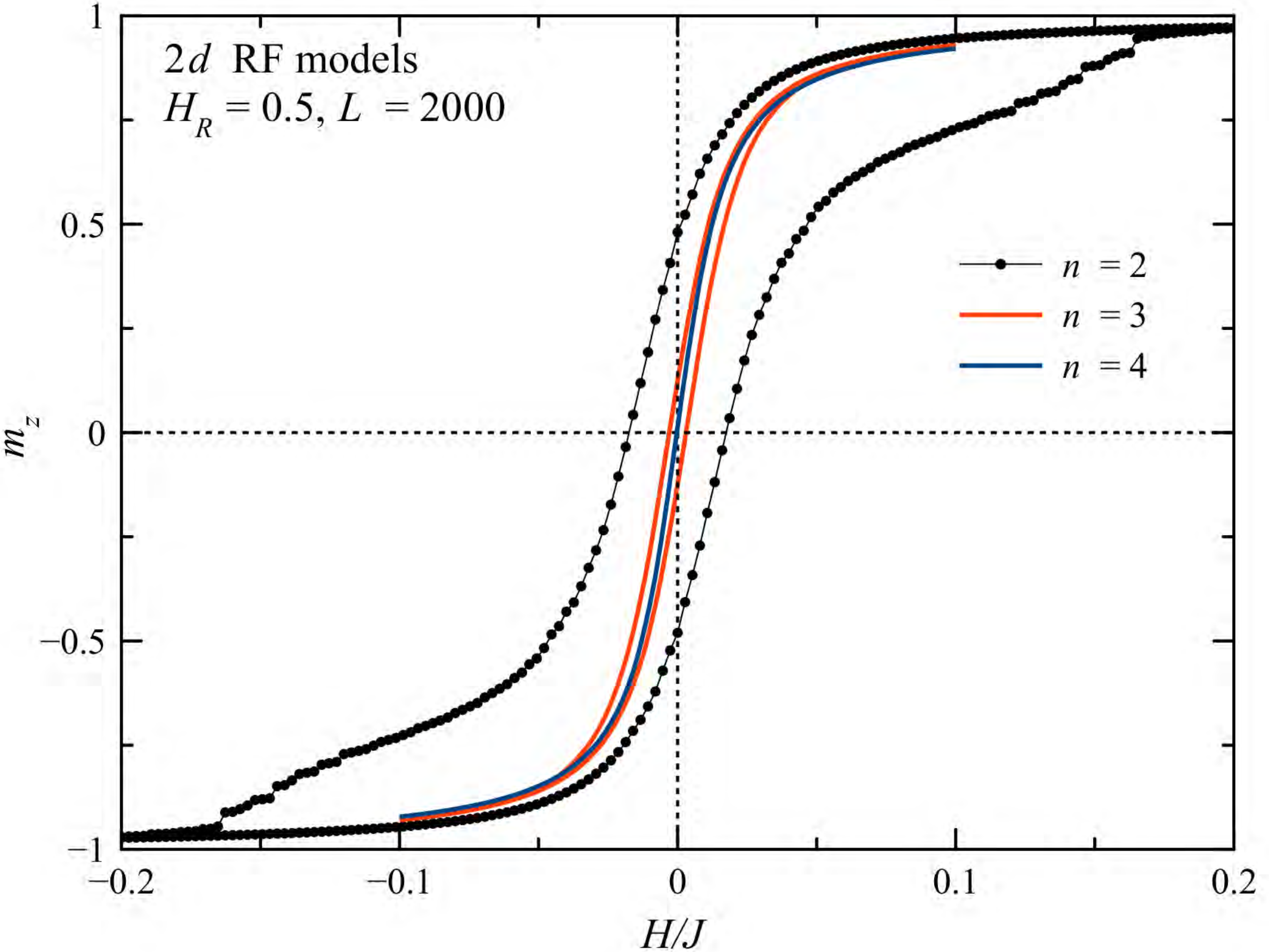
Red lines: $\left\langle h_x \right\rangle = 0$, Blue lines: $\left\langle h_y \right\rangle = 0$



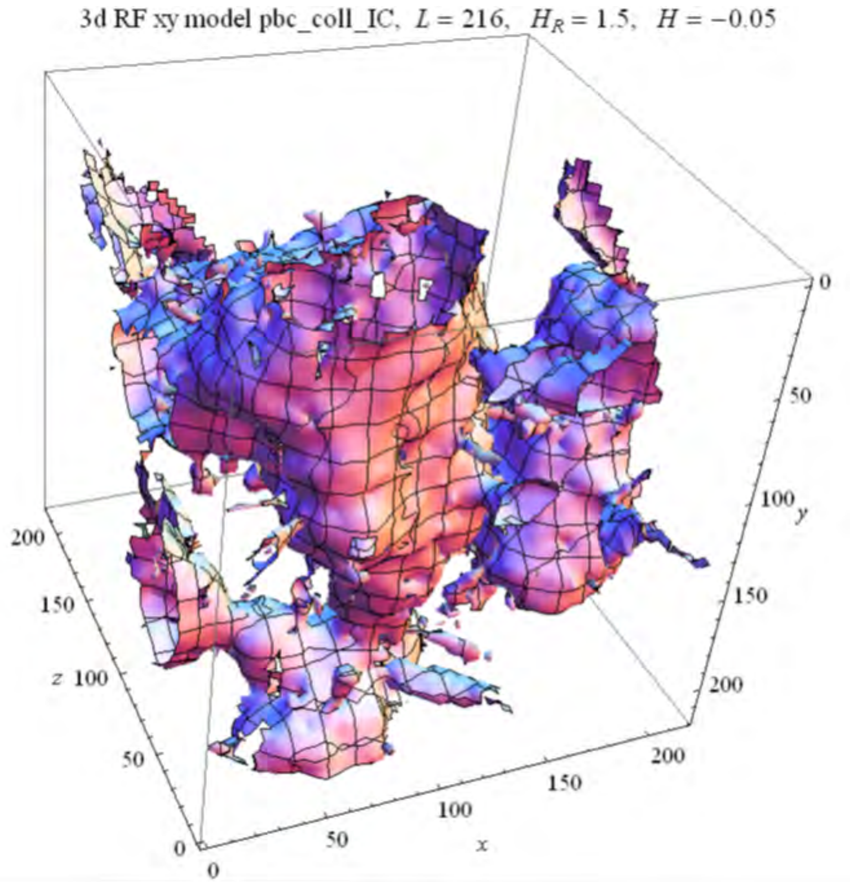
Random-field Heisenberg spin model in $3d$

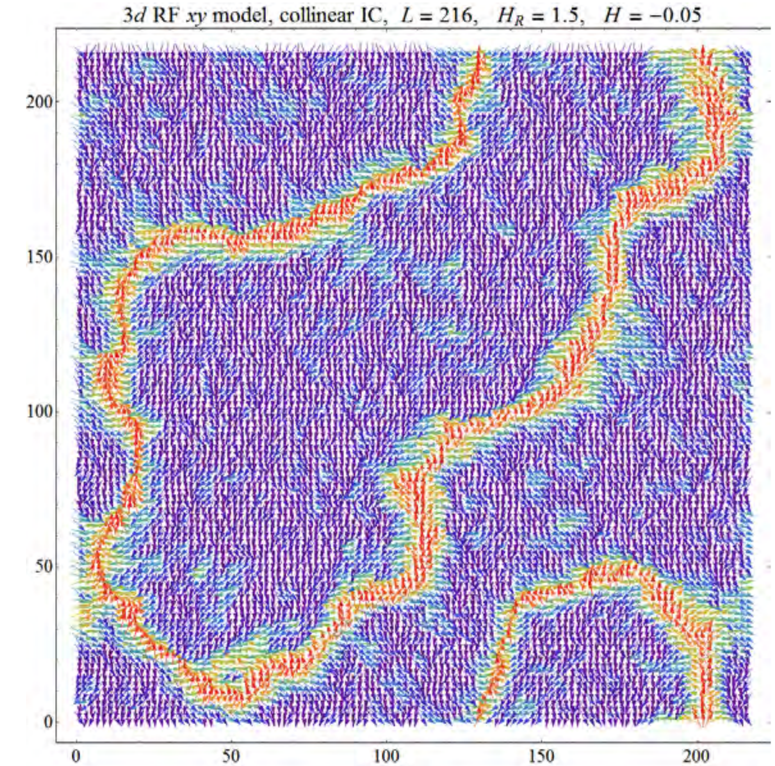
Red: $\left\langle h_x \right\rangle = 0$, Blue: $\left\langle h_y \right\rangle = 0$, Green: $\left\langle h_z \right\rangle = 0$

(Tom Proctor, Dmitry Garanin, and EC, PRL 2014)

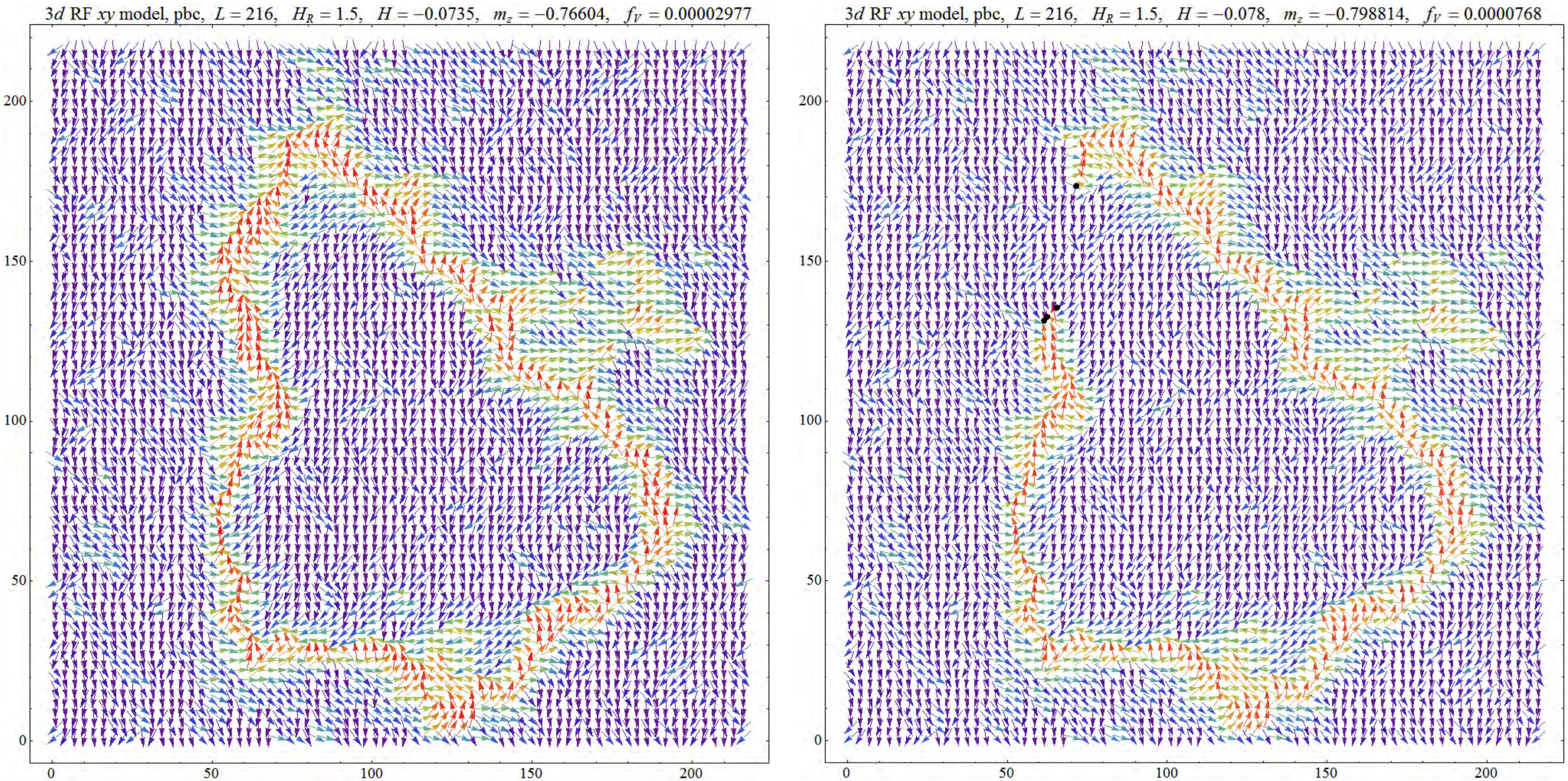


2d RF models
$H_R = 0.5$, $L = 2000$
$n = 2$
$n = 3$
$n = 4$
m_z
H/J
1
0.5
0
−0.5
−1
−0.2
−0.1
0
0.1
0.2

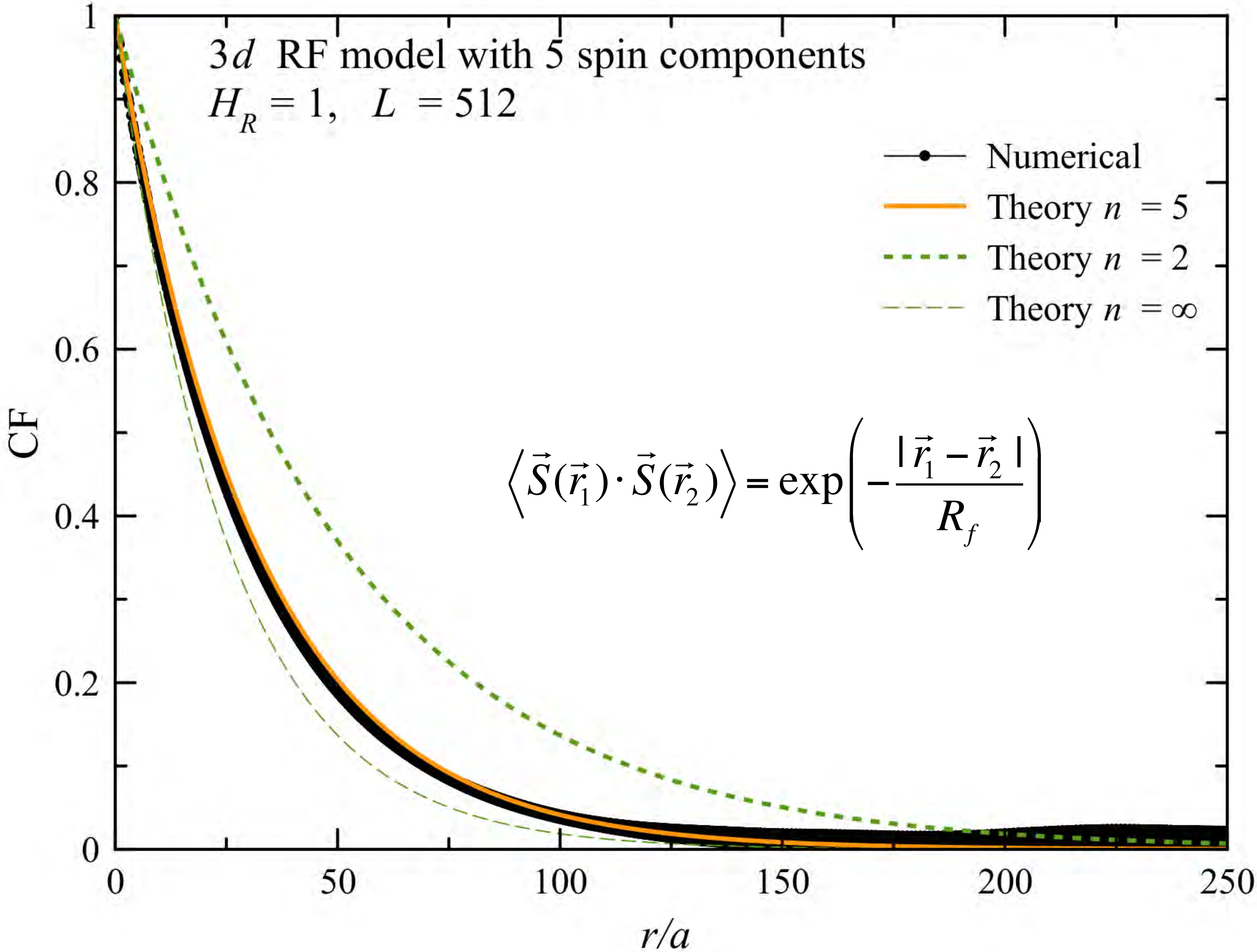




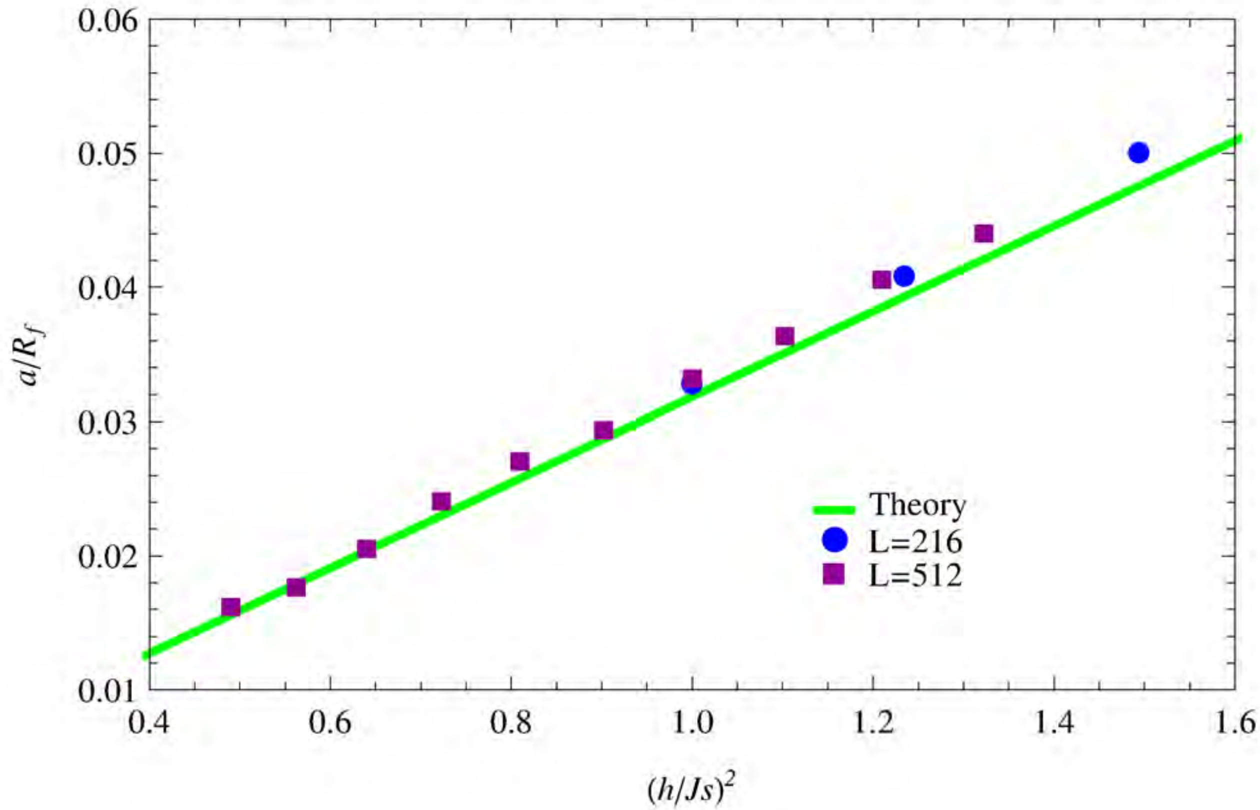
Spin membranes



Rapture of spin membranes by vortices



3d RF model with 5 spin components
$H_R = 1, \quad L = 512$
Numerical
Theory $n = 5$
Theory $n = 2$
Theory $n = \infty$
$\langle \vec{S}(\vec{r}_1) \cdot \vec{S}(\vec{r}_2) \rangle = \exp\left(-\frac{|\vec{r}_1 - \vec{r}_2|}{R_f} \right)$
CF
r/a
0
0.2
0.4
0.6
0.8
1
0
50
100
150
200
250



Analytical result for the correlation length:

$$\frac{R_f}{a} = 8\pi\left(1-\frac{1}{n}\right)^{-1}\left(\frac{Js}{h}\right)^2$$

CONCLUSIONS

In the presence of the weak random field the behavior of the N-component fixed-length order parameter in D dimensions is controlled by topology.

1) At **$N < D + 1$** the system possesses pinned singularities. It exhibits irreversible glassy behavior, with the final state depending on the initial condition.

2) At **$N = D + 1$** the system possesses pinned nonsingular topological objects and exhibits weak metastability.

3) At **$N > D + 1$** topological objects are absent and the system exhibits reversible dynamics with exponential decay of correlations.