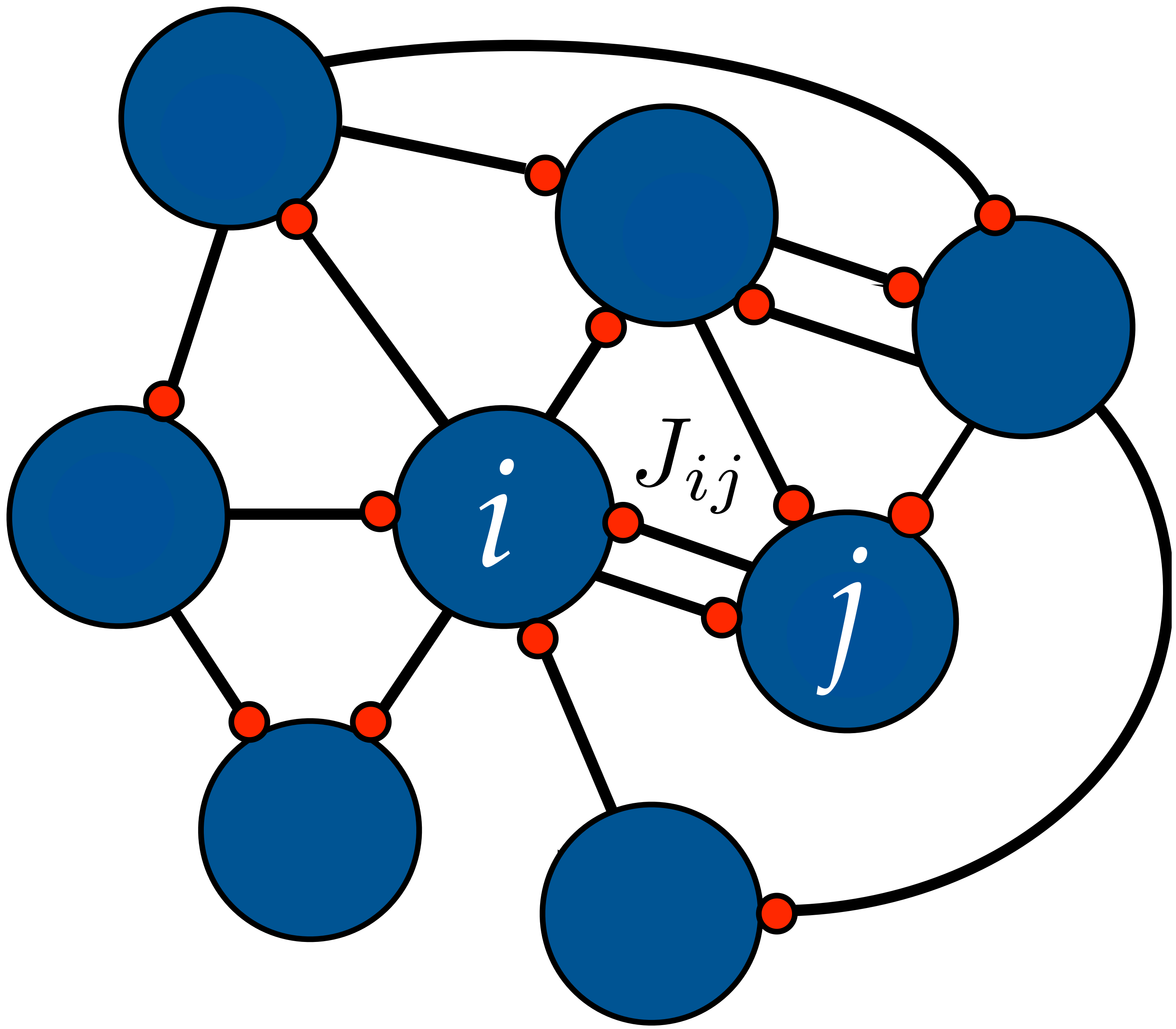
Harnessing the Dynamics of Random Networks

Harnessing the Dynamics of Random Networks

New Results in Random Matrix Theory
from the Analysis of Neuronal Networks

$$\frac{dx_i}{dt} = -x_i + \sum_{j=1}^{N} J_{ij} \tanh(x_j)$$



$i = 1, 2, \ldots, N$

$$\frac{dx_i}{dt} = -x_i + \sum_{j=1}^{N} J_{ij} \tanh(x_j)$$

$$\boxed{\langle J_{ij} \rangle_J = 0 \quad \text{and} \quad \langle J_{ij}^2 \rangle_J = \frac{g^2}{N}}$$

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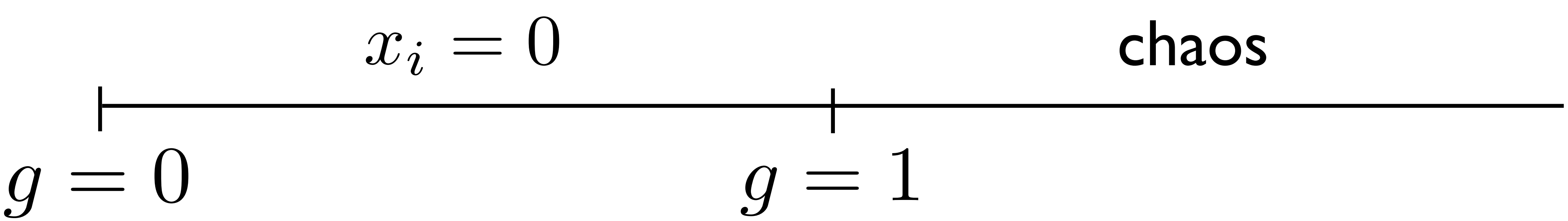

Sompolinsky, Crisanti & Sommers, 1988

$$\frac{dx_i}{dt} = -x_i + \sum_{j=1}^{N} J_{ij} \tanh(x_j)$$

$$x_i = 0 \text{ for } i = 1, \ldots, N$$

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stability

$$M_{ij} = -\delta_{ij} + J_{ij}$$

$$\frac{dx_i}{dt} = -x_i + \sum_{j=1}^{N} J_{ij} \tanh(x_j)$$

$$x_i = 0 \text{ for } i = 1, \ldots, N$$

stability

$$M_{ij} = -\delta_{ij} + J_{ij}$$

compute eigenvalues

$$\langle J_{ij} \rangle_J = 0 \quad \text{and} \quad \langle J_{ij}^2 \rangle_J = \frac{g^2}{N}$$

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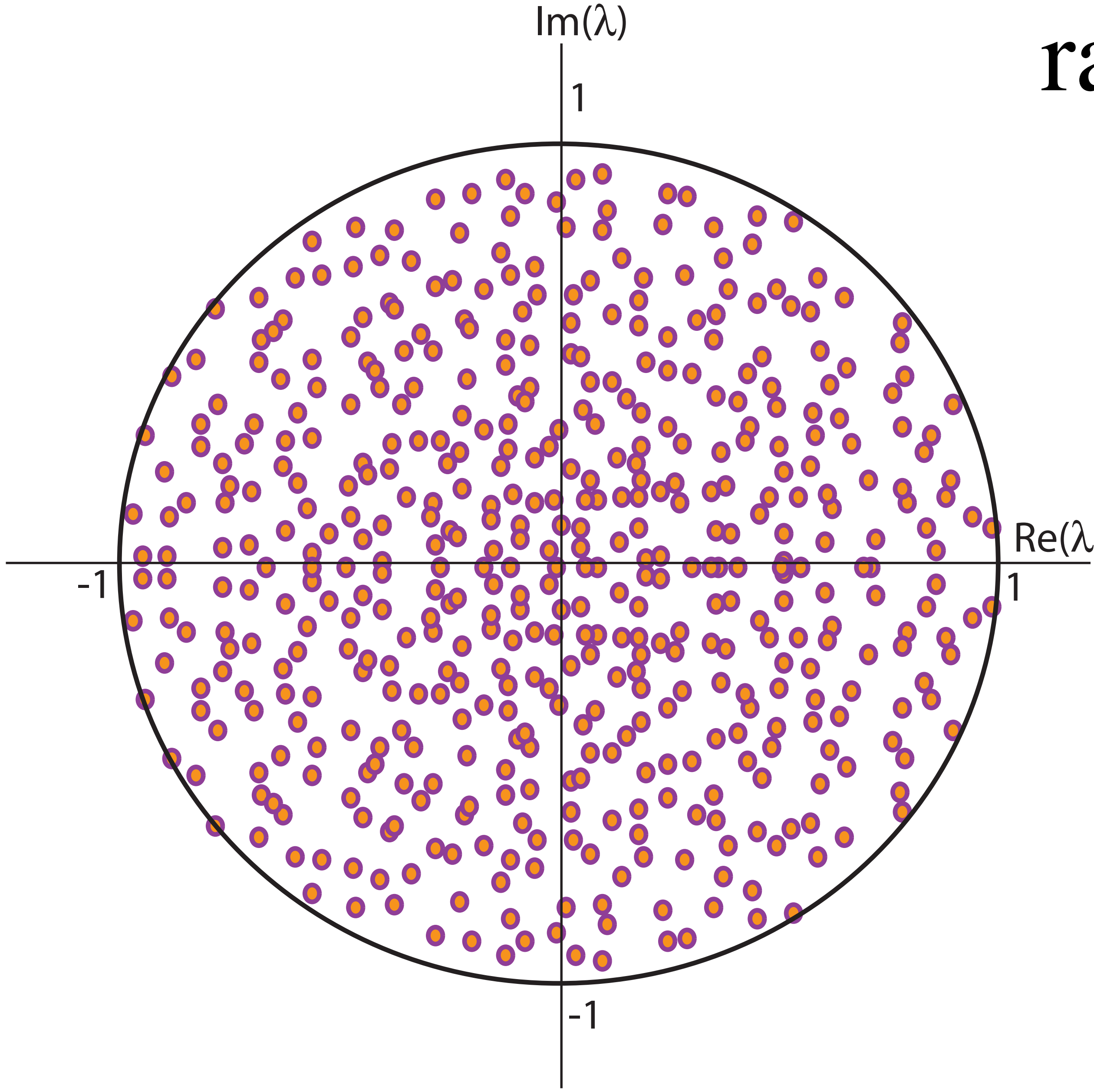
radius = g

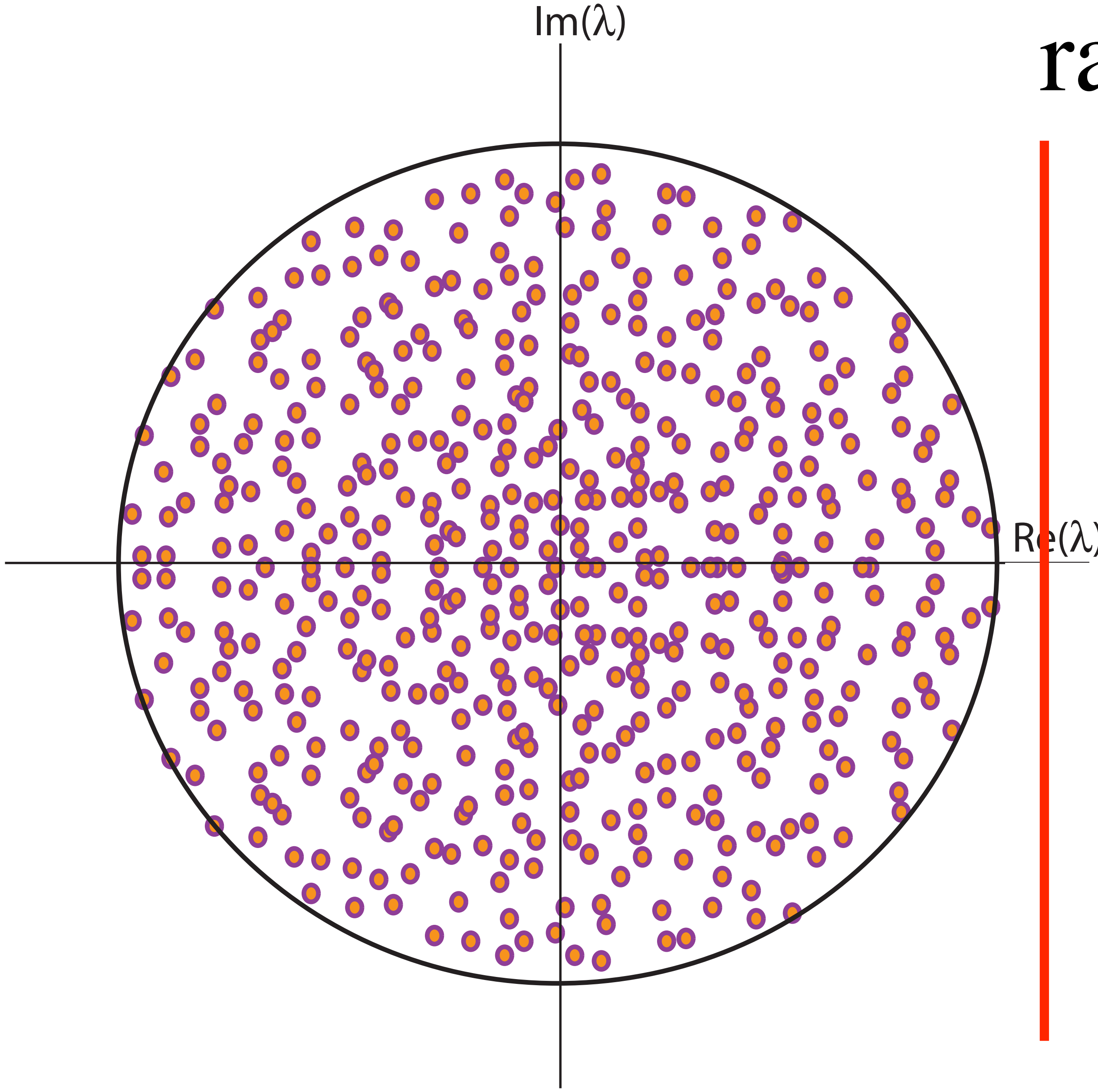


Ginibre, 1965

Girko, 1985

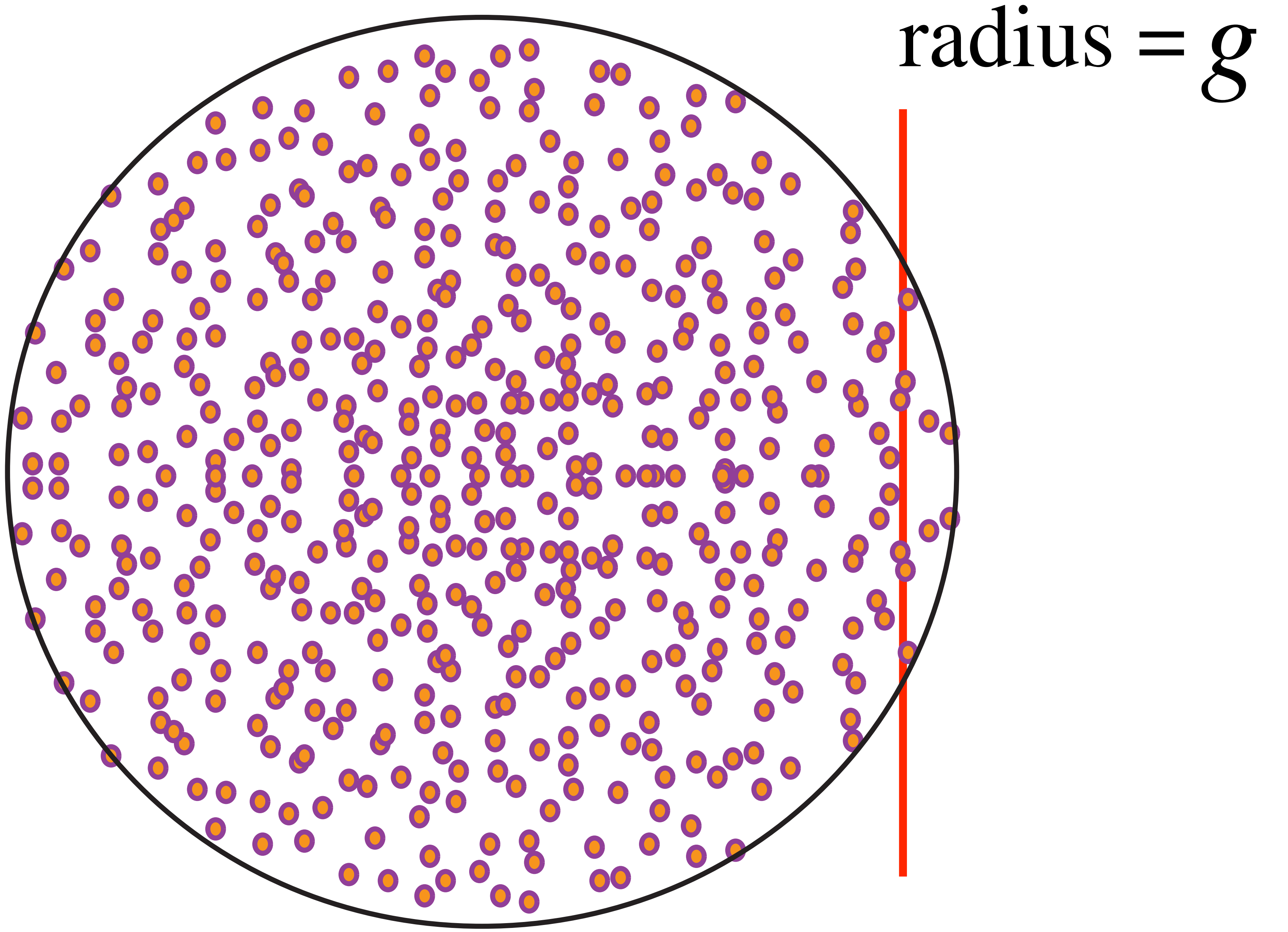
Tao & Vu, 2010

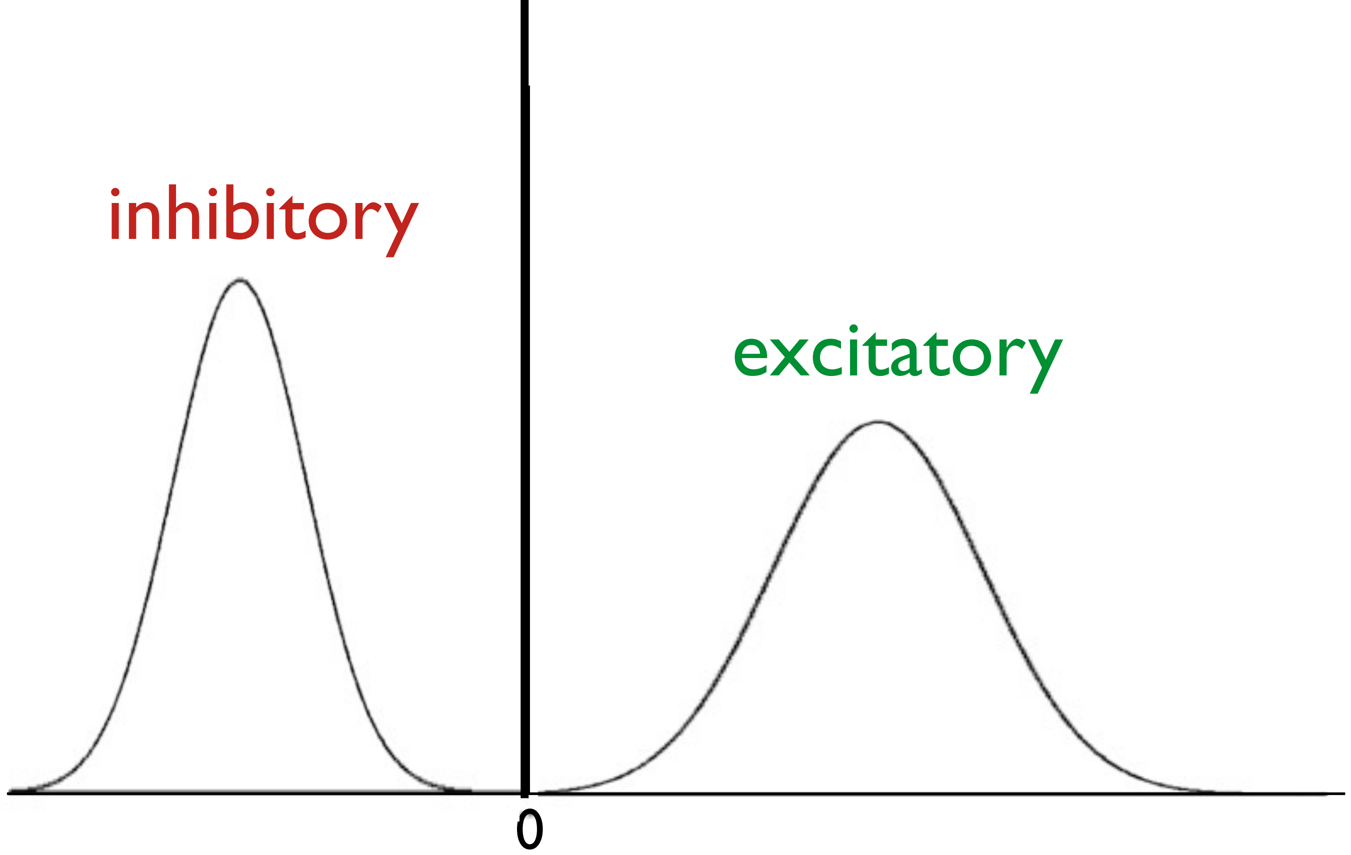
$$\langle J_{ij} \rangle_J = 0 \quad \text{and} \quad \langle J_{ij}^2 \rangle_J = \frac{g^2}{N}$$



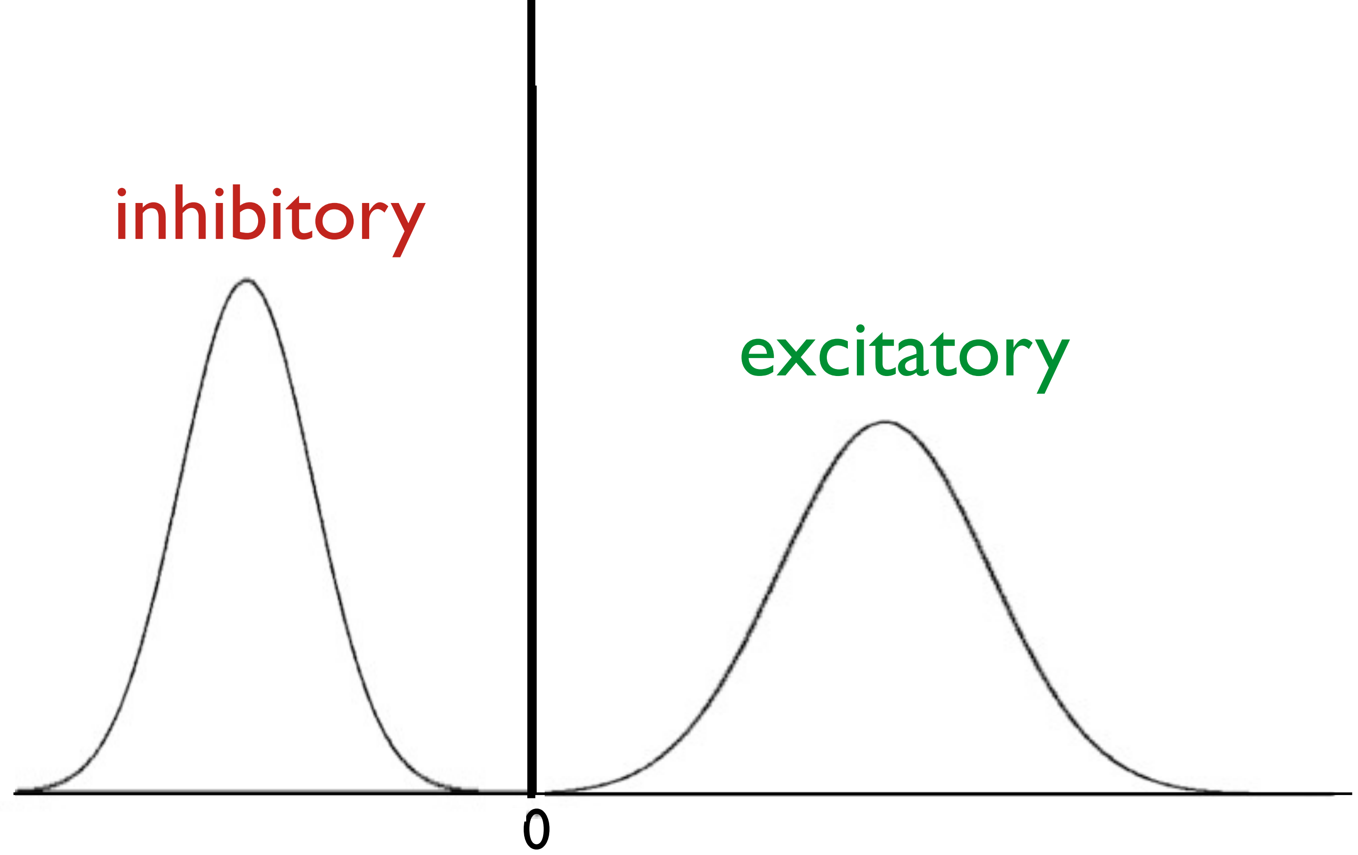
radius = g

$$\langle J_{ij} \rangle_J = 0 \quad \text{and} \quad \langle J_{ij}^2 \rangle_J = \frac{g^2}{N}$$



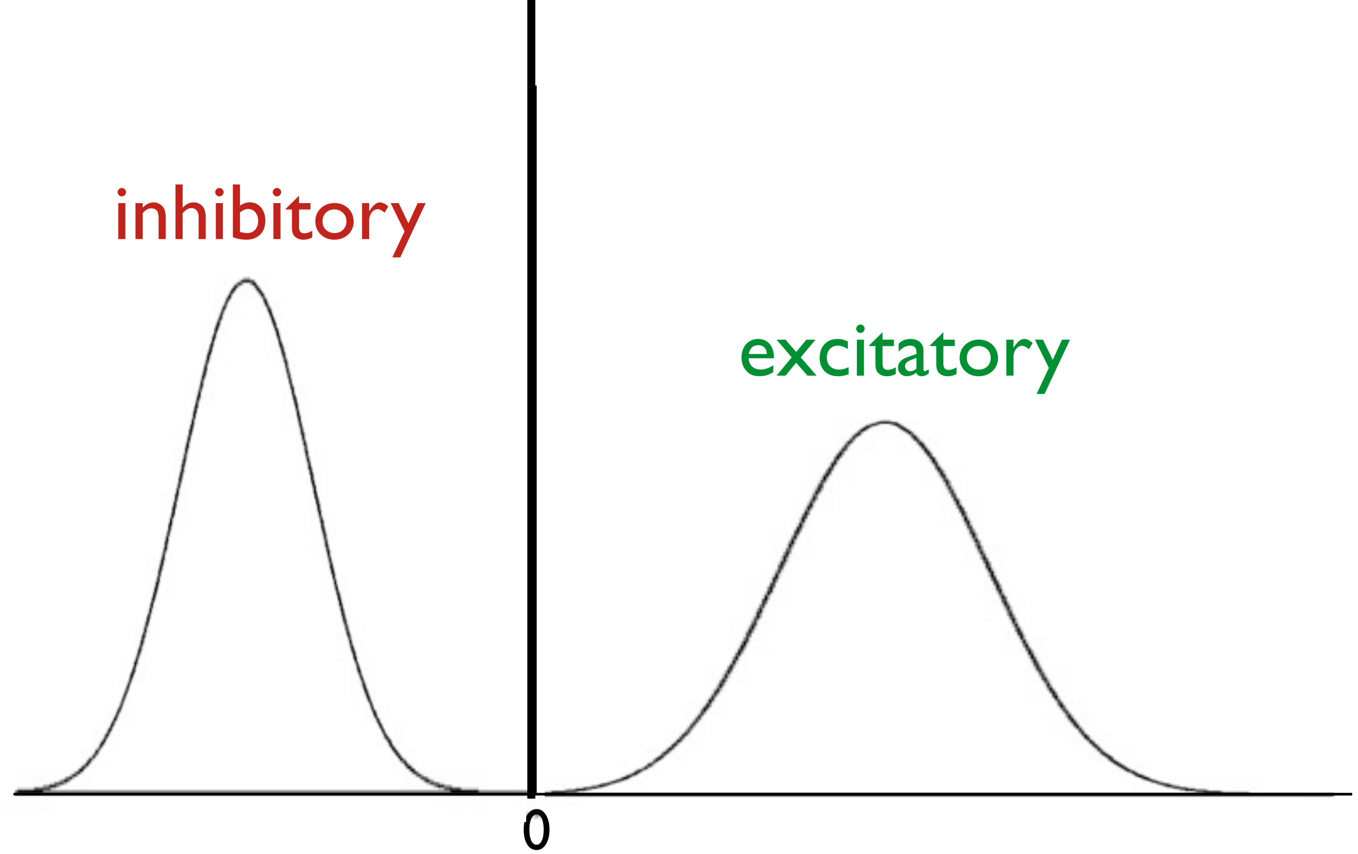


inhibitory
excitatory
0



with excitatory or
inhibitory synapses

$$\begin{pmatrix} + & - & + \\ - & \cdot & \cdot \\ + & \cdot & + \\ - & & \cdot \end{pmatrix}$$

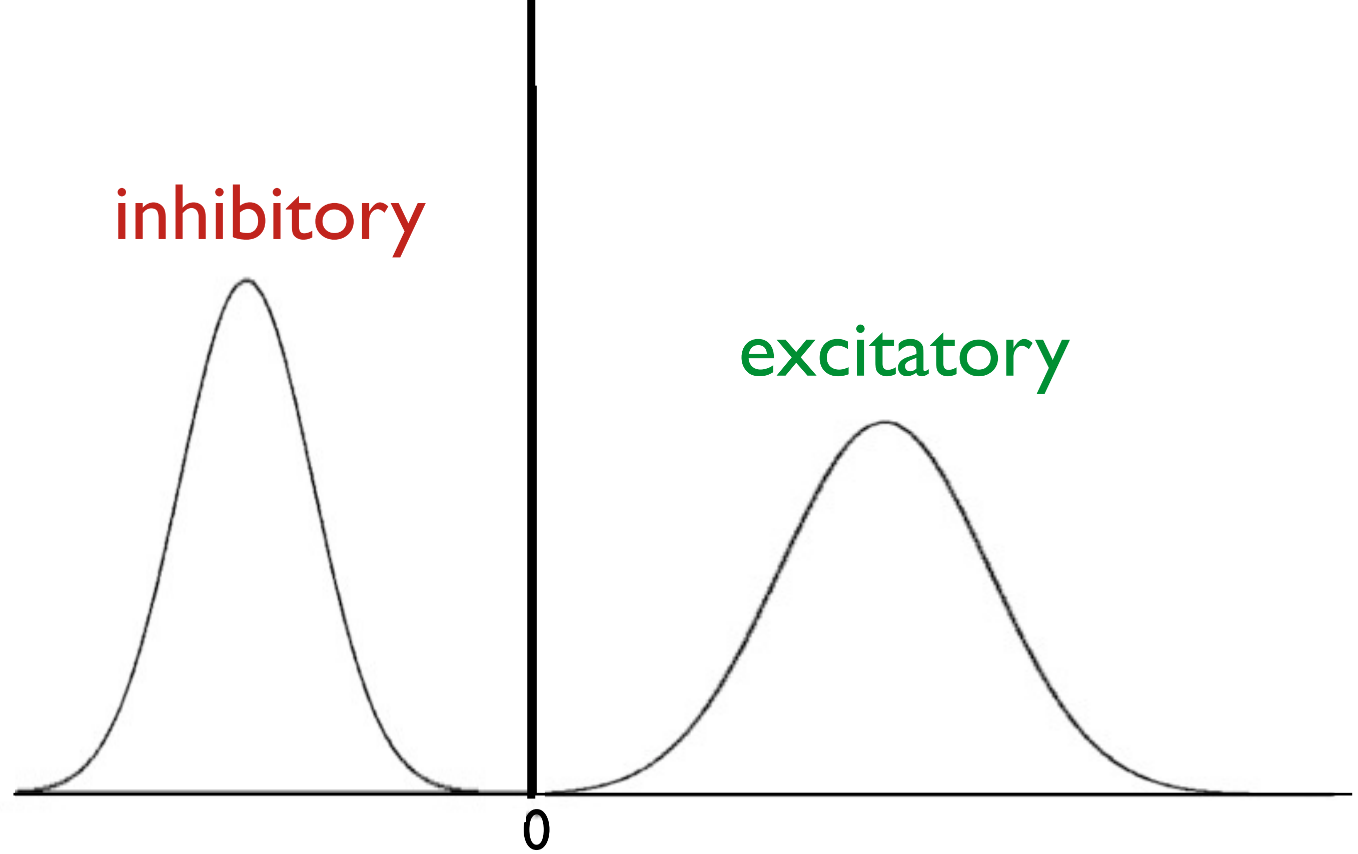


with excitatory or inhibitory synapses

$$\begin{pmatrix} + & - & + \\ - & \cdot & \cdot \\ + & \cdot & + \\ - & & \cdot \end{pmatrix}$$

with excitatory or inhibitory neurons

$$\begin{pmatrix} + & & - \\ + & \cdot & - \\ + & \cdot & - \\ + & & - \end{pmatrix}$$



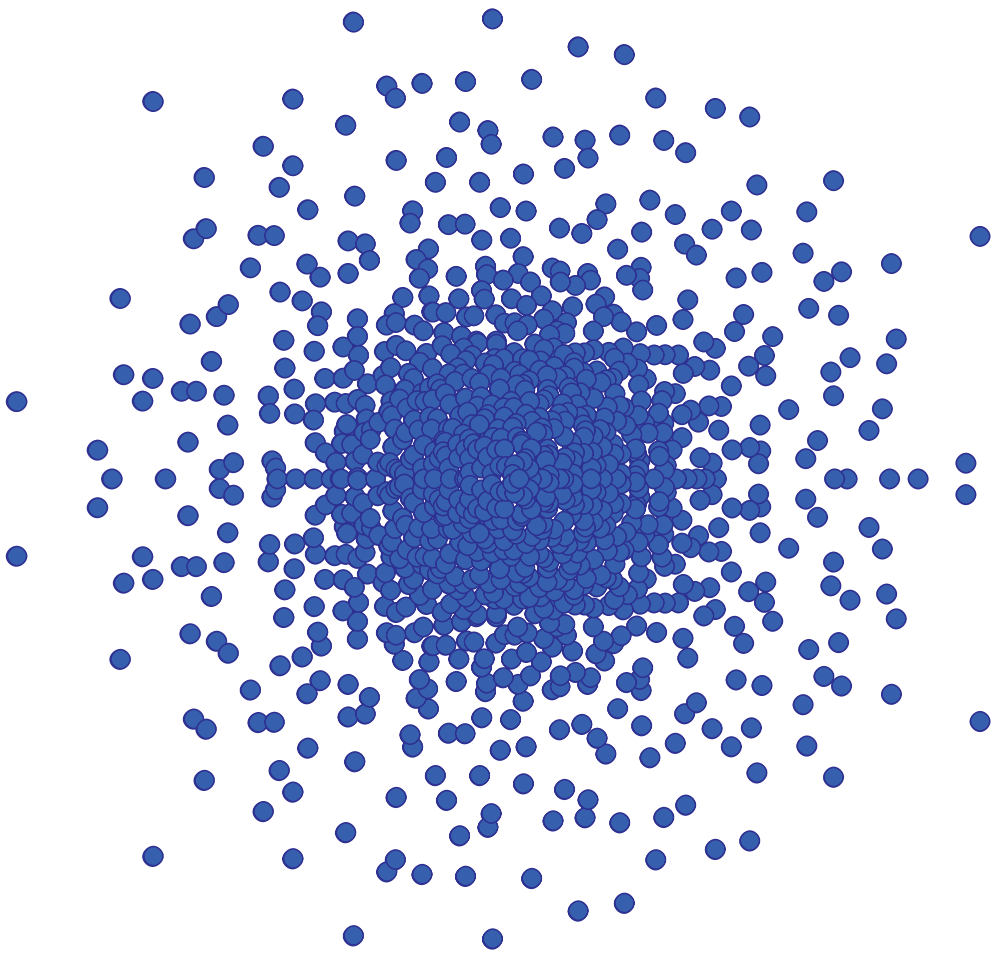
with excitatory or inhibitory synapses

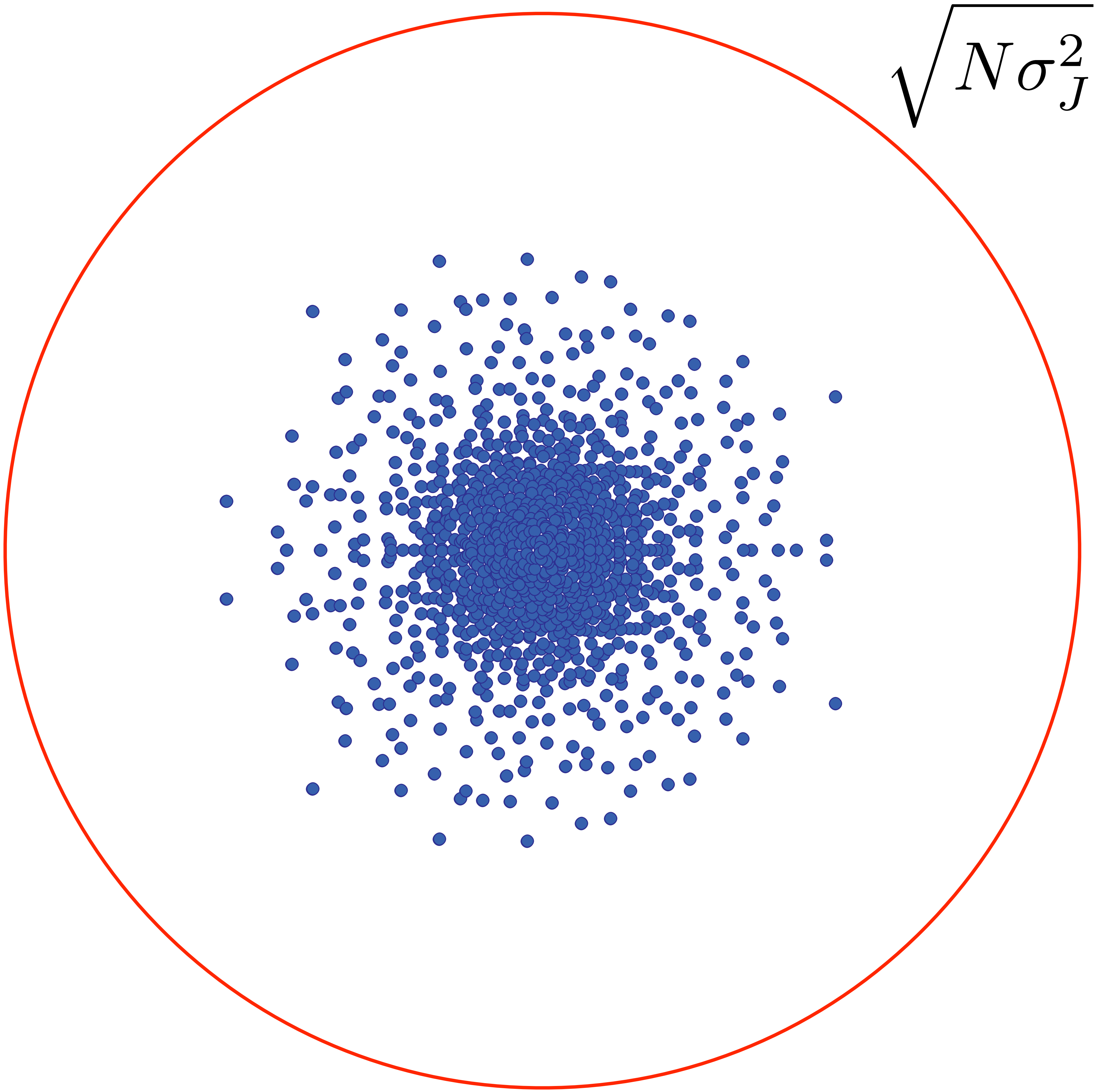
$$\begin{pmatrix} + & - & + \\ - & \cdot & \cdot \\ + & \cdot & + \\ - & & \cdot \end{pmatrix}$$

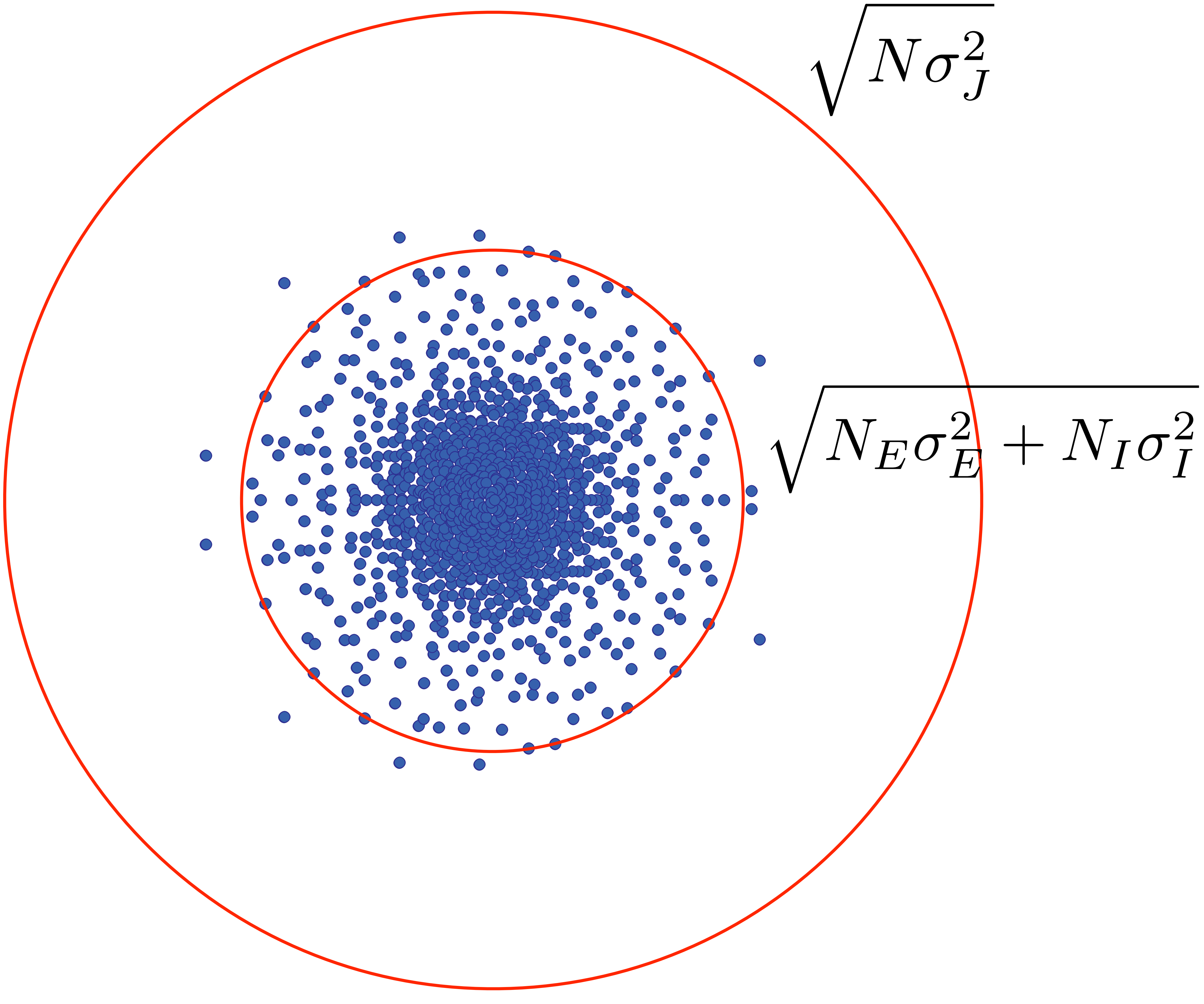
with excitatory or inhibitory neurons

$$\begin{pmatrix} + & & - \\ + & \cdot & - \\ + & \cdot & - \\ + & & - \end{pmatrix}$$

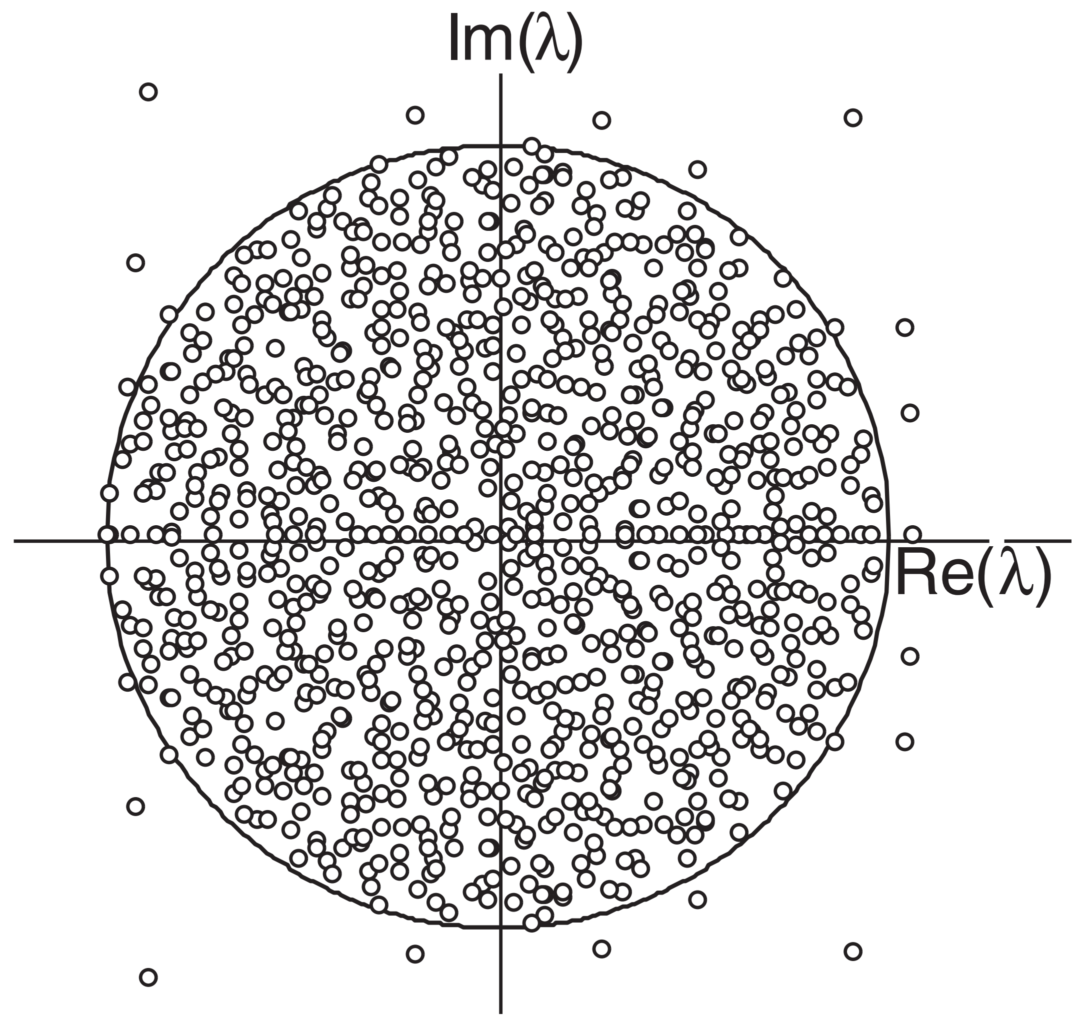
Kanaka Rajan




$\sqrt{N\sigma_J^2}$

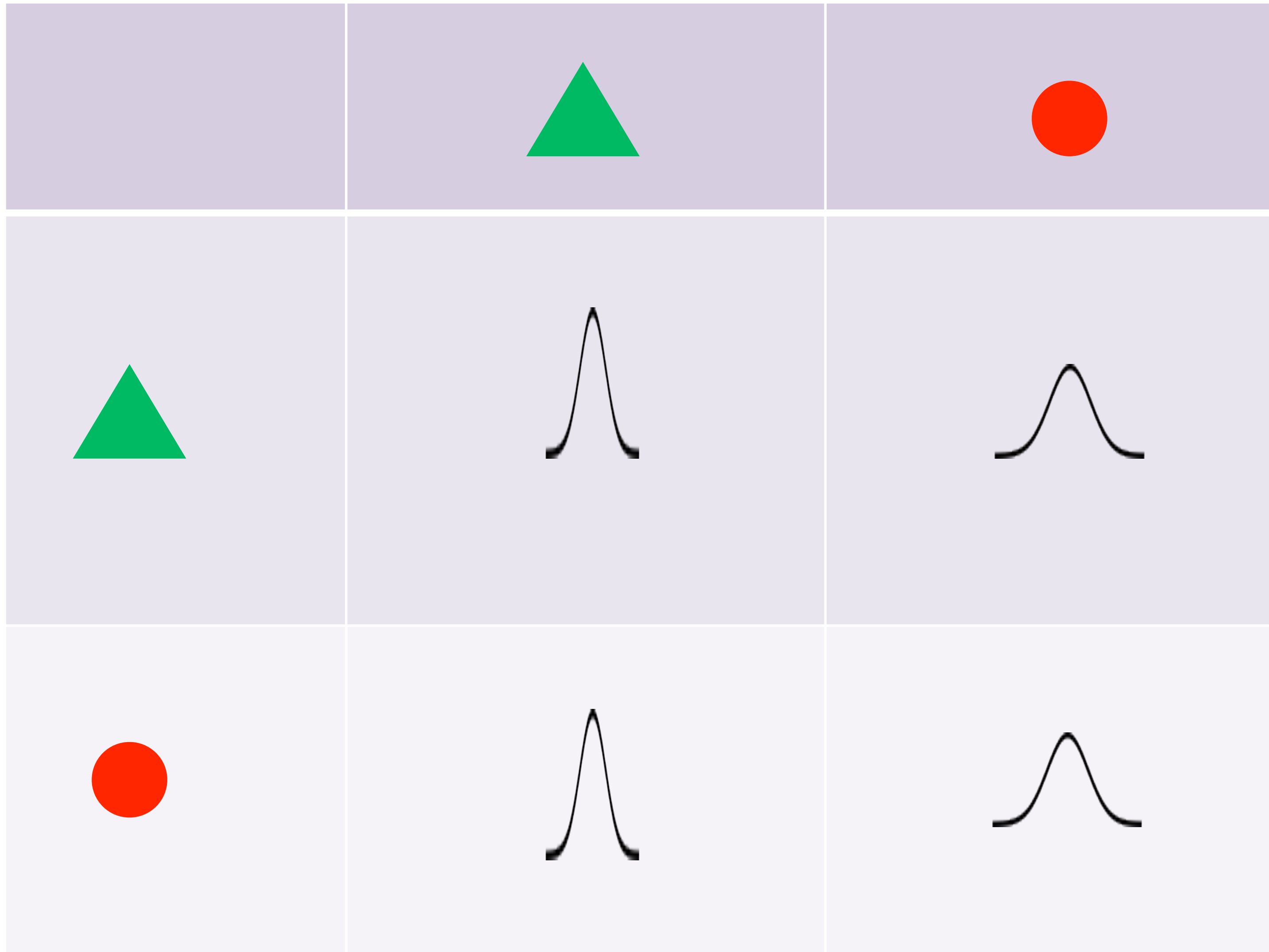


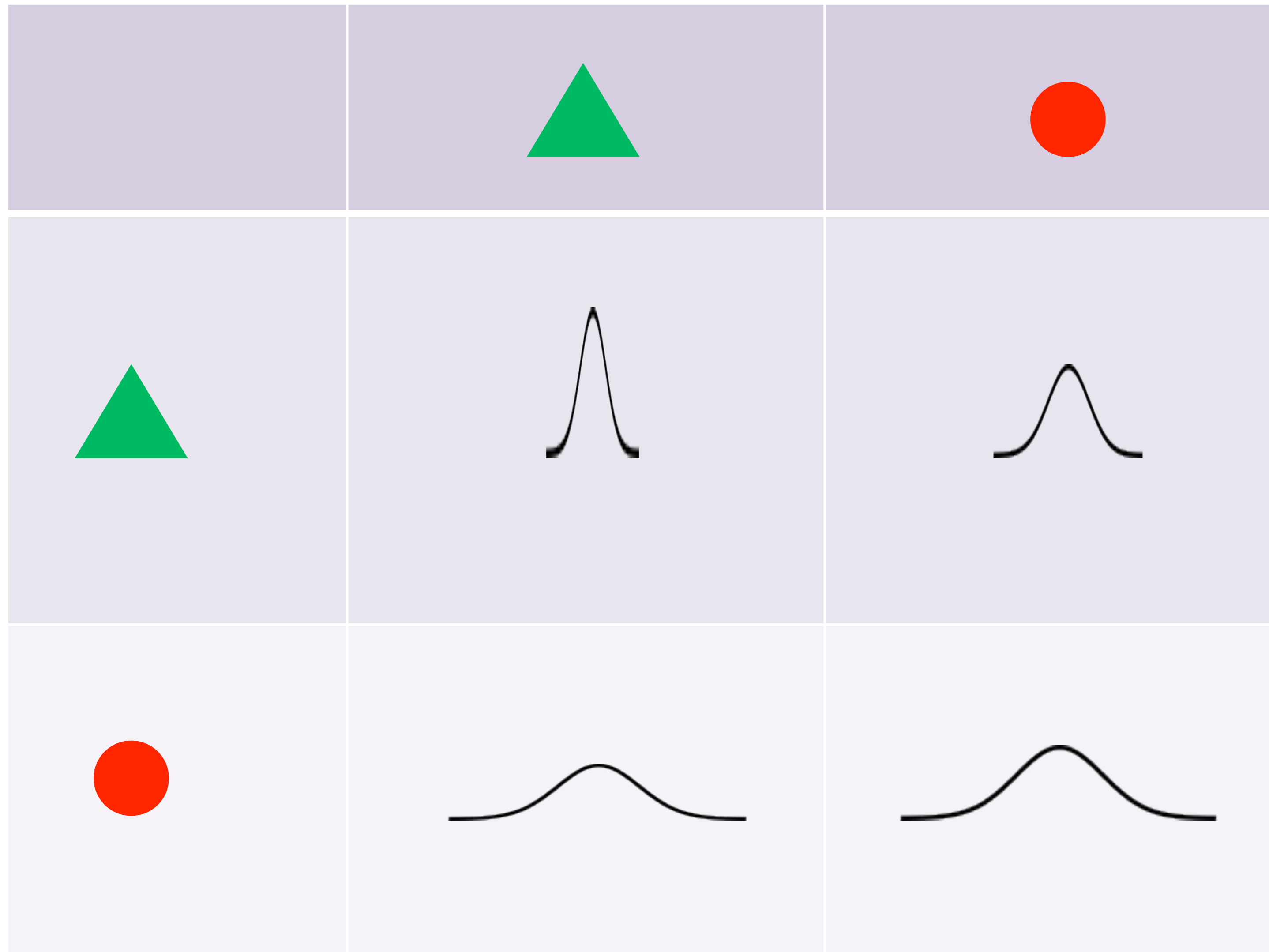
$\sqrt{N\sigma_J^2}$
$\sqrt{N_E\sigma_E^2 + N_I\sigma_I^2}$



Terrance Tao, 2010

http://terrytao.wordpress.com/2010/12/22/
outliers-in-the-spectrum-of-iid-matrices-with-bounded-rank-permutations/

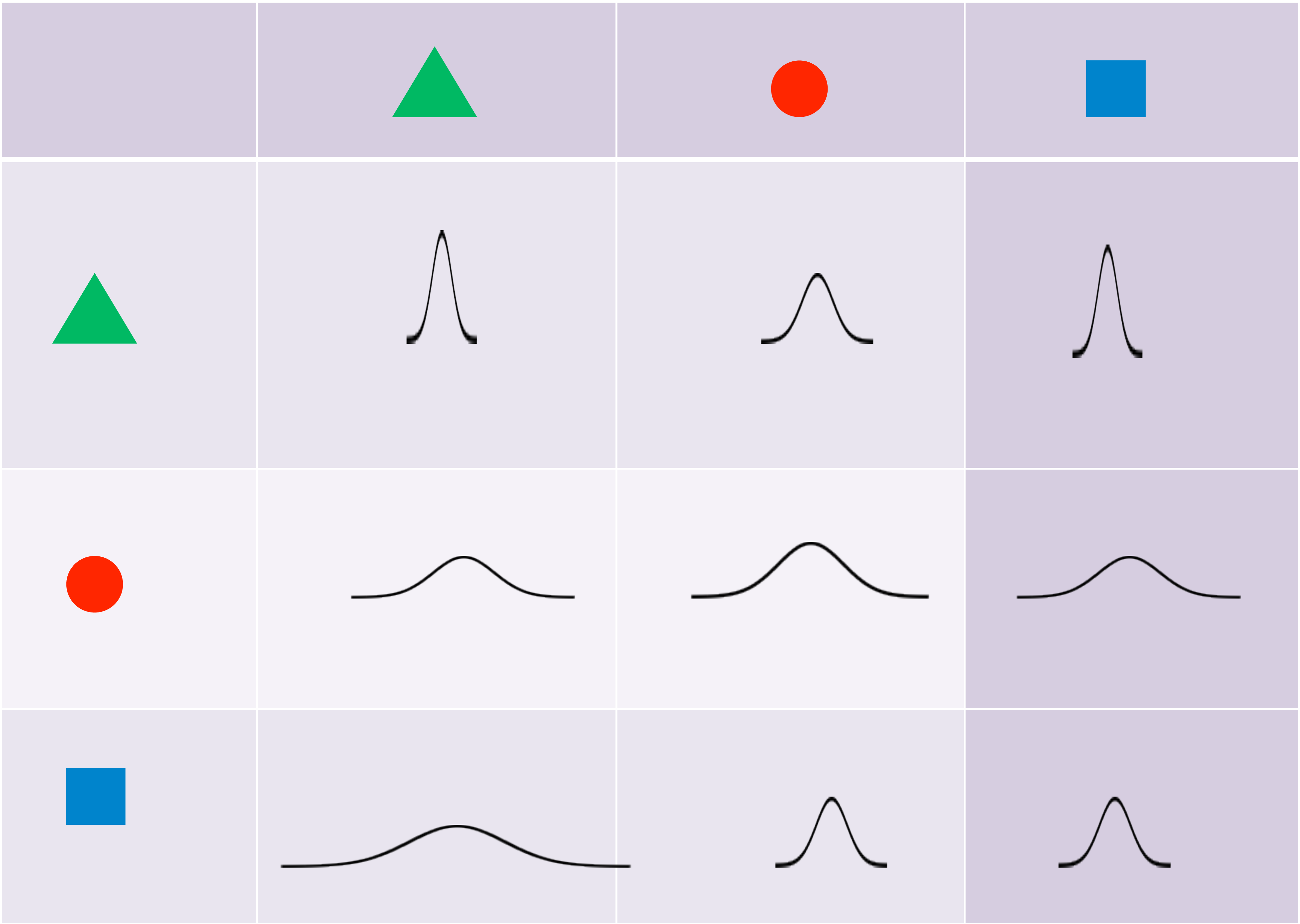


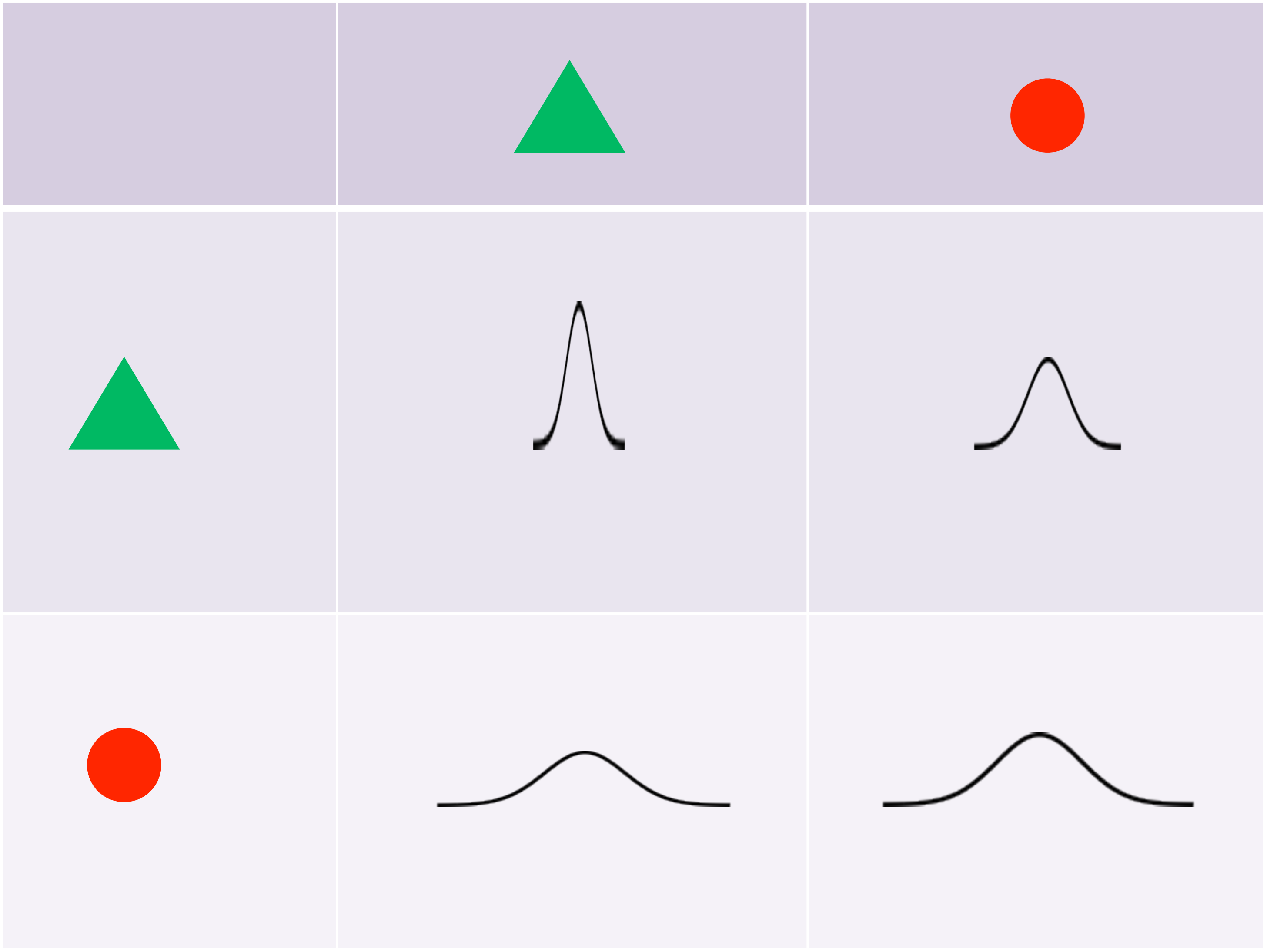


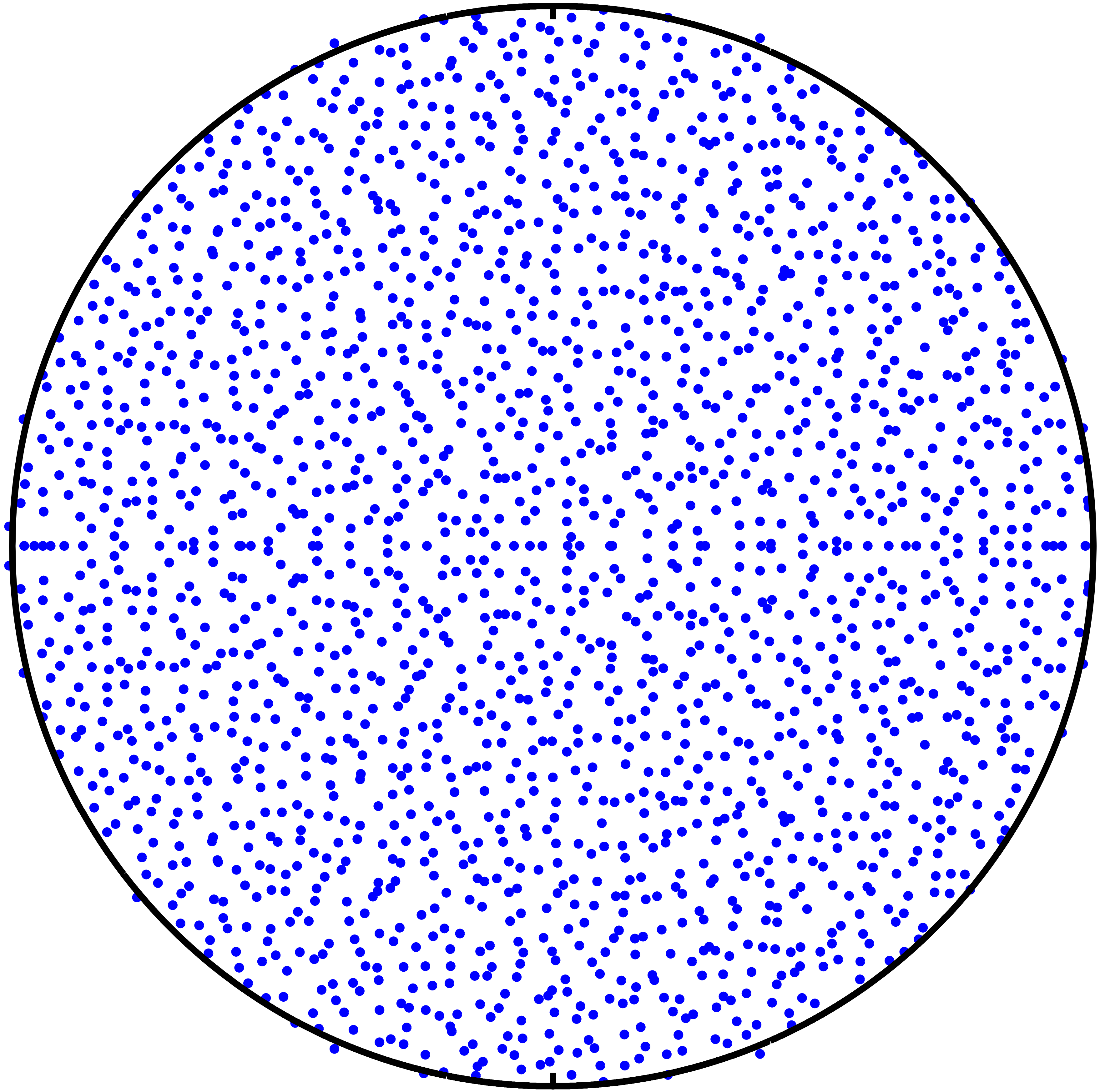
Merav Stern

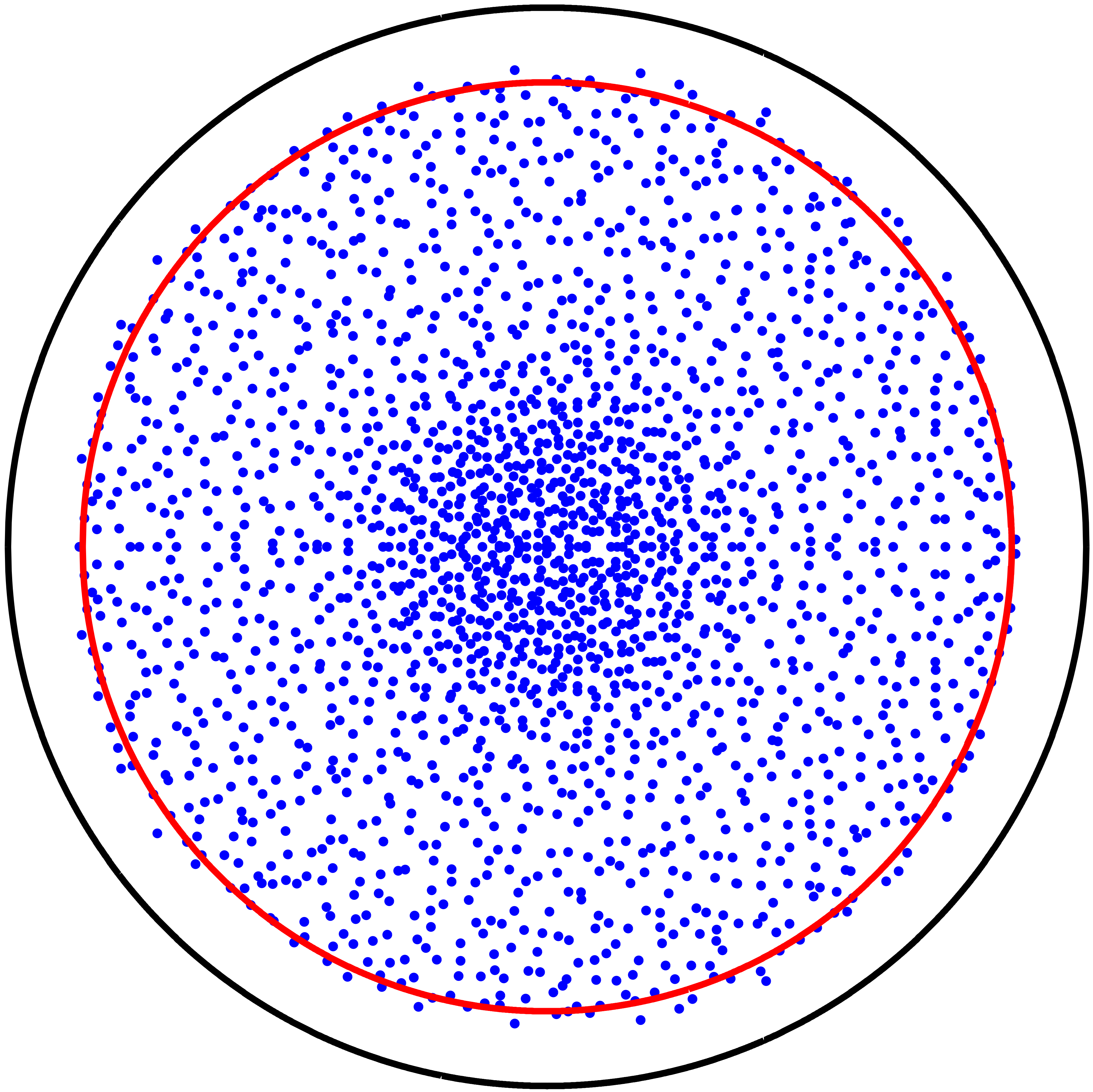
Yonatan Aljadeff

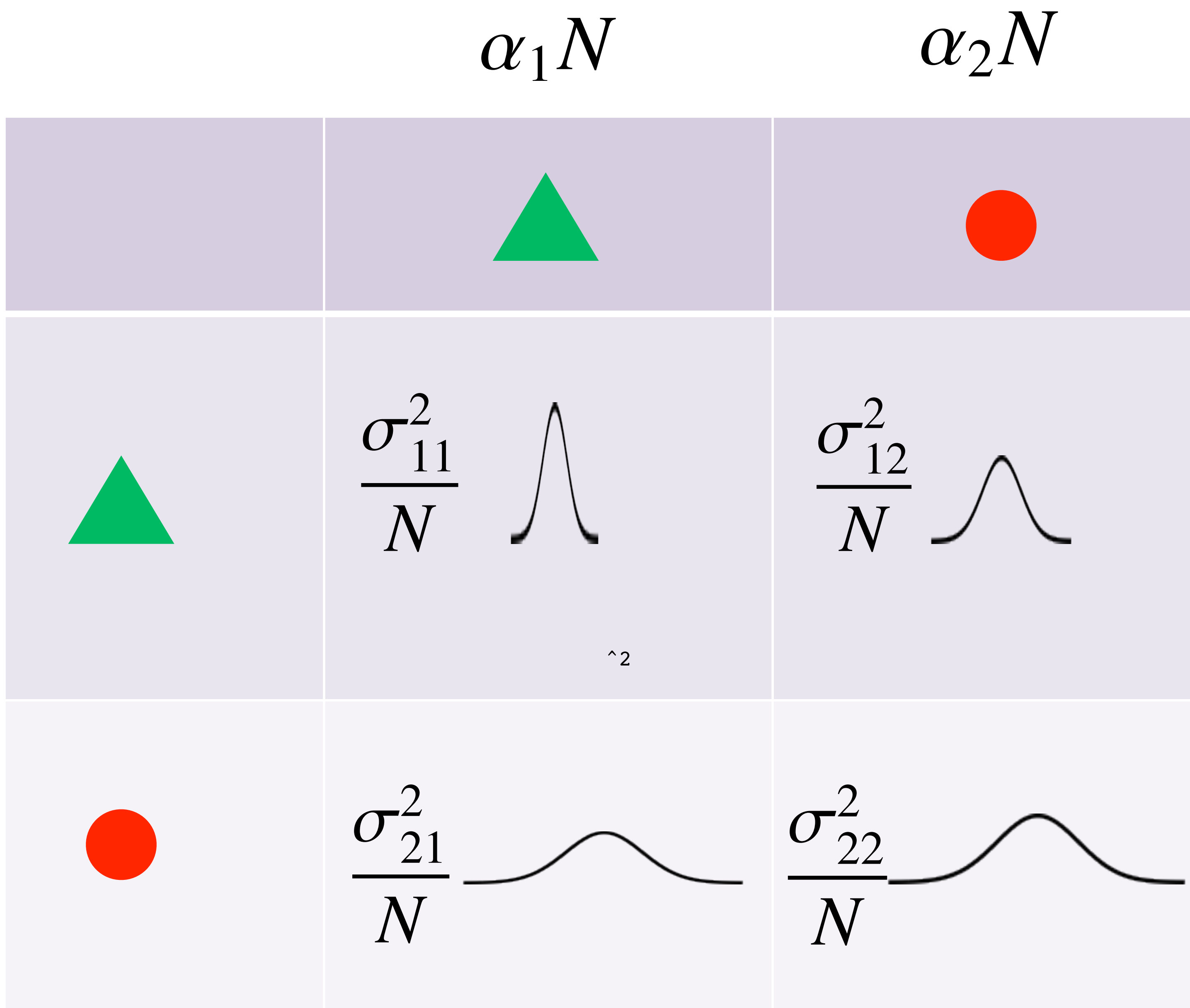
Tatyana Sharpee









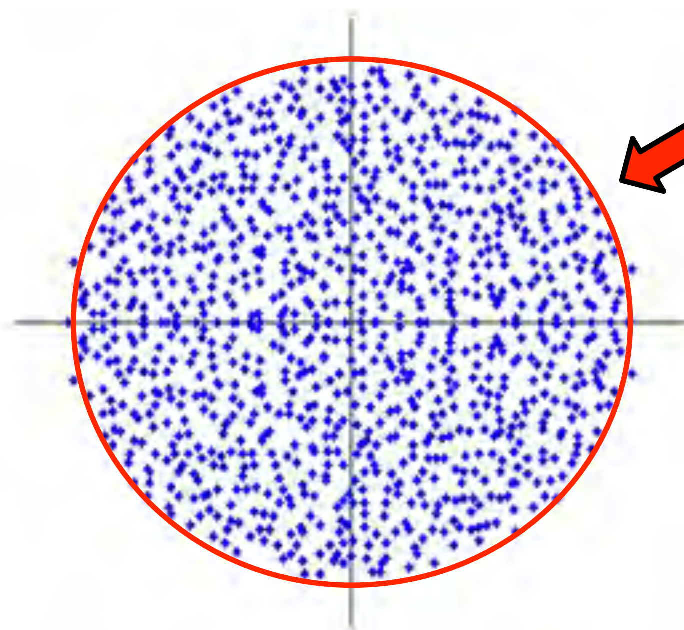

$\alpha_1 N$
$\alpha_2 N$
$\frac{\sigma_{11}^2}{N}$
$\frac{\sigma_{12}^2}{N}$
^2
$\frac{\sigma_{21}^2}{N}$
$\frac{\sigma_{22}^2}{N}$

$$\begin{pmatrix} \alpha_1\sigma_{11}^2 & \alpha_2\sigma_{12}^2 \\ \alpha_1\sigma_{21}^2 & \alpha_2\sigma_{22}^2 \end{pmatrix}$$

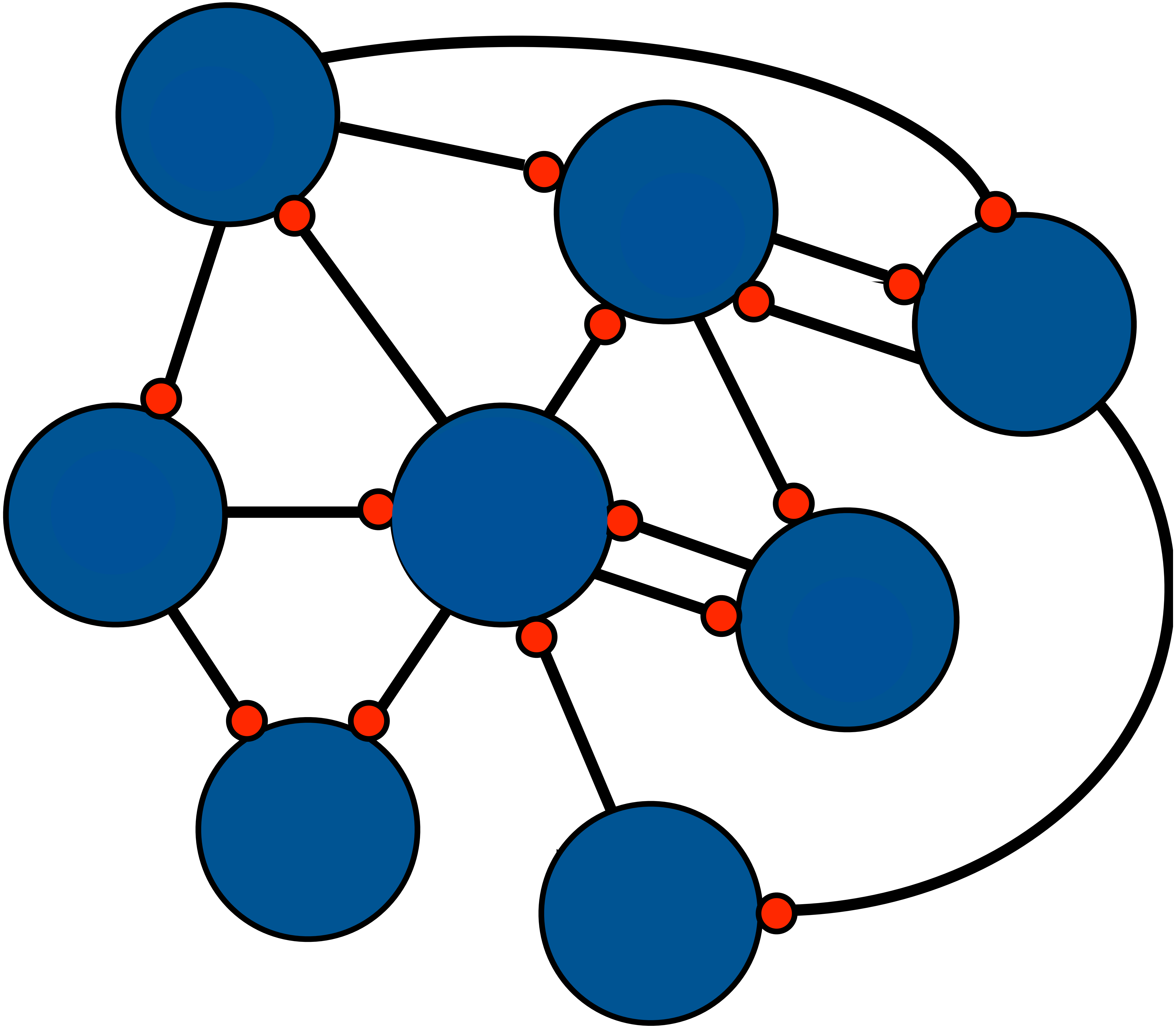
$$\begin{pmatrix} \alpha_1\sigma_{11}^2 & \alpha_2\sigma_{12}^2 \\ \alpha_1\sigma_{21}^2 & \alpha_2\sigma_{22}^2 \end{pmatrix} \qquad \Lambda_1 \dots \Lambda_D$$

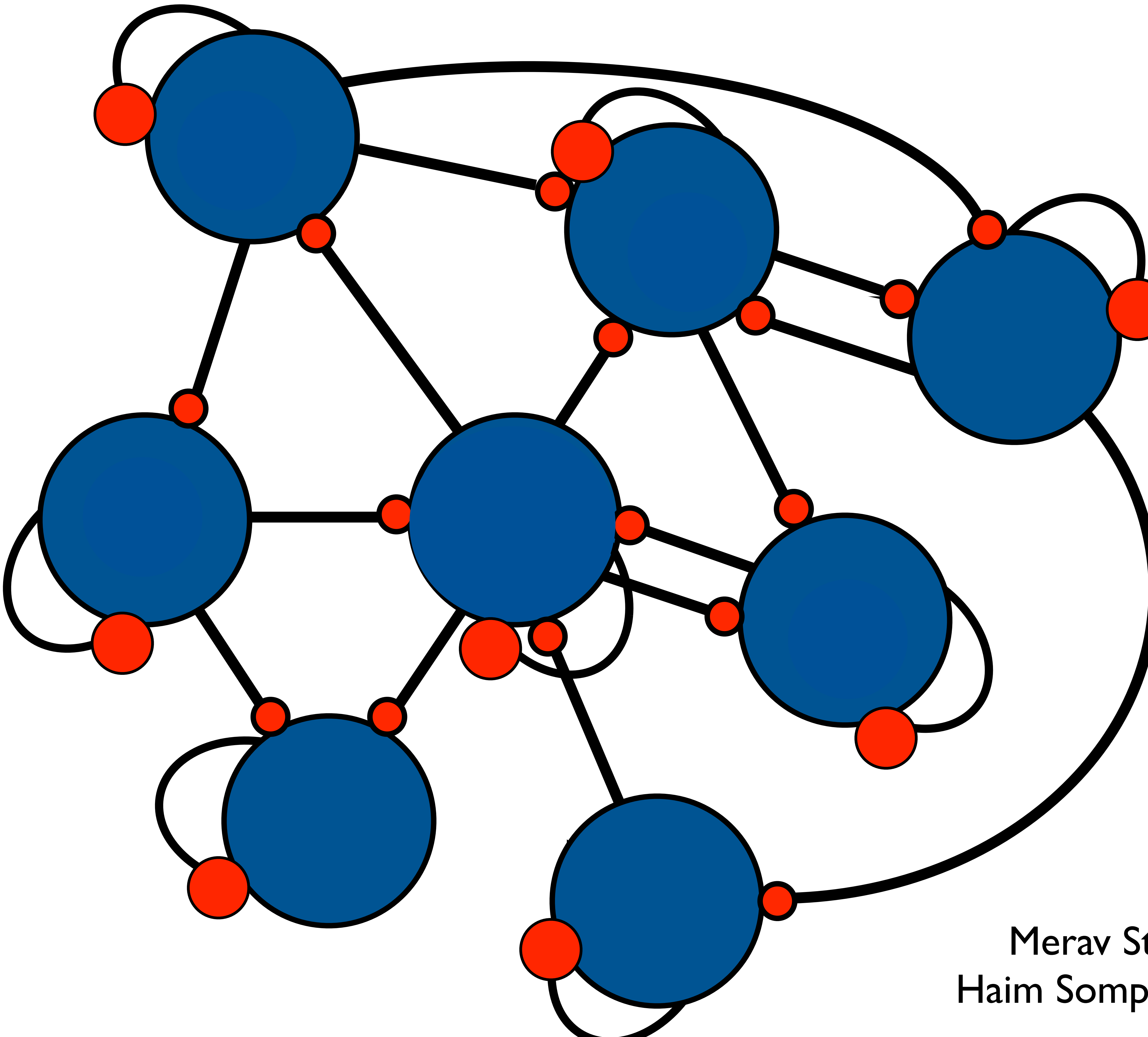
$$\begin{pmatrix} \alpha_1\sigma_{11}^2 & \alpha_2\sigma_{12}^2 \\ \alpha_1\sigma_{21}^2 & \alpha_2\sigma_{22}^2 \end{pmatrix}$$

$\Lambda_1 \ldots \Lambda_D$

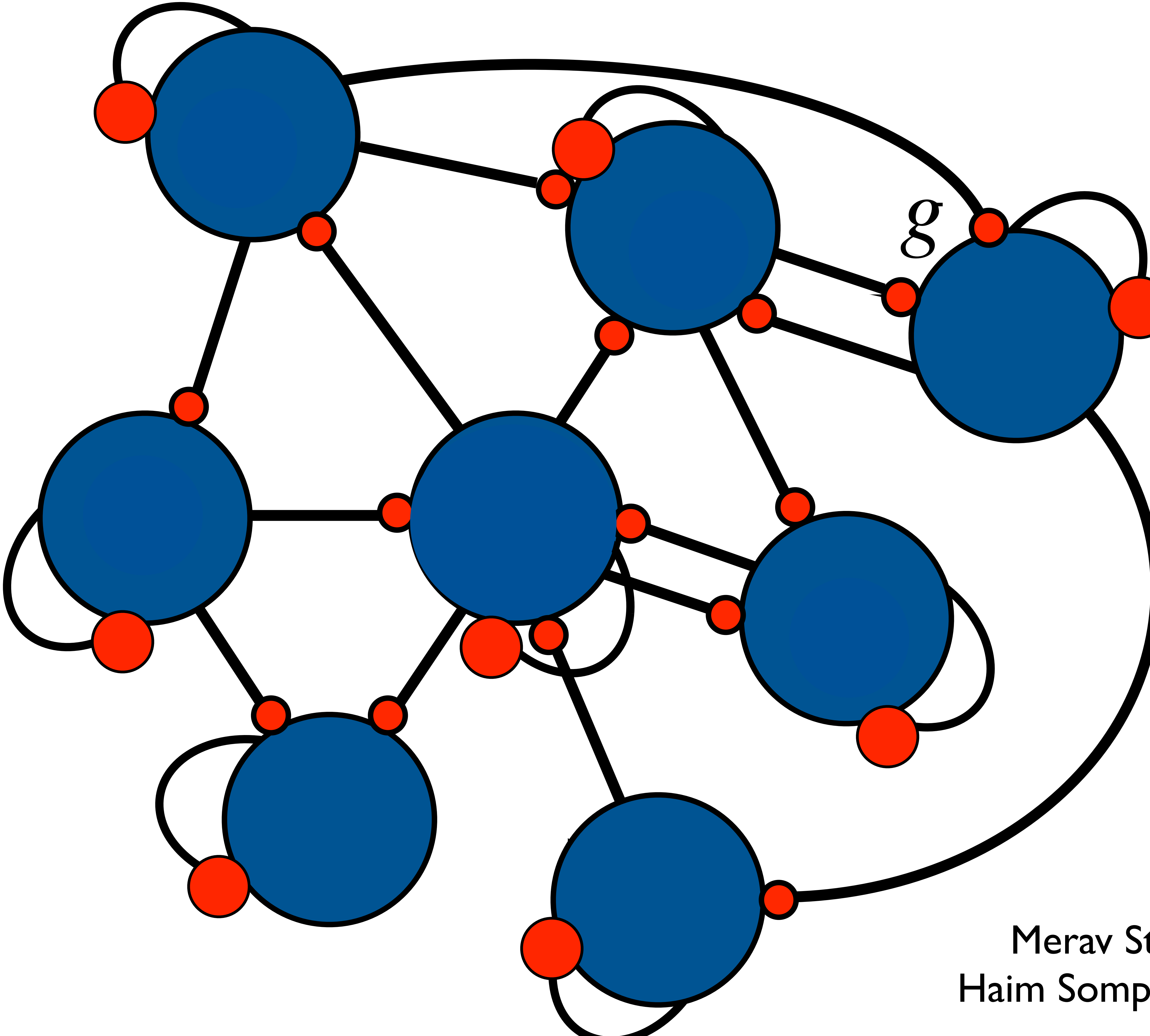


$\sqrt{\Lambda_1}$





Merav Stern
Haim Sompolinsky



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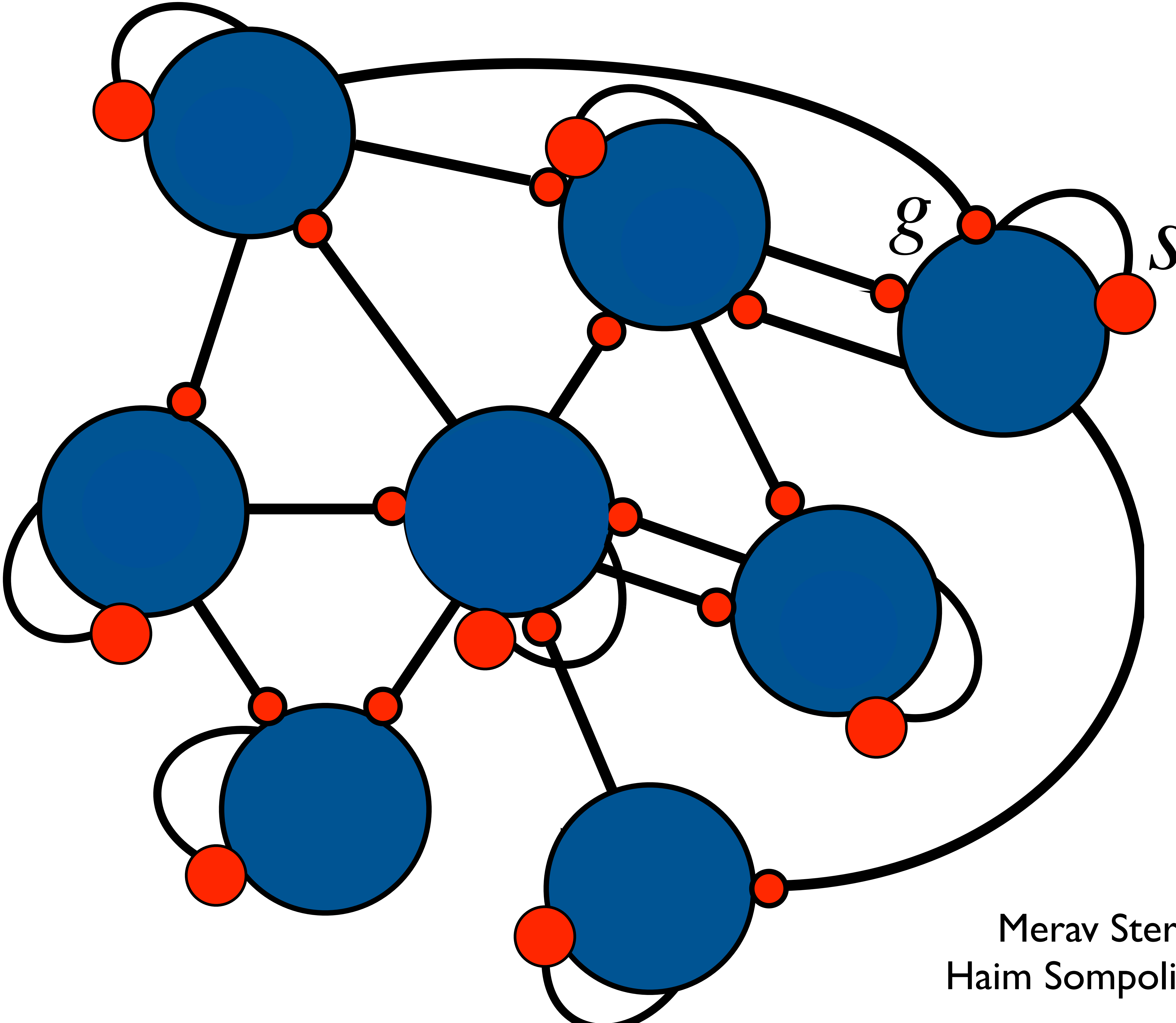

Merav Stern
Haim Sompolinsky

$$\frac{dx_i}{dt} = -x_i + s\tanh(x_i) + \sum_{j=1}^{N} J_{ij}\tanh(x_j)$$

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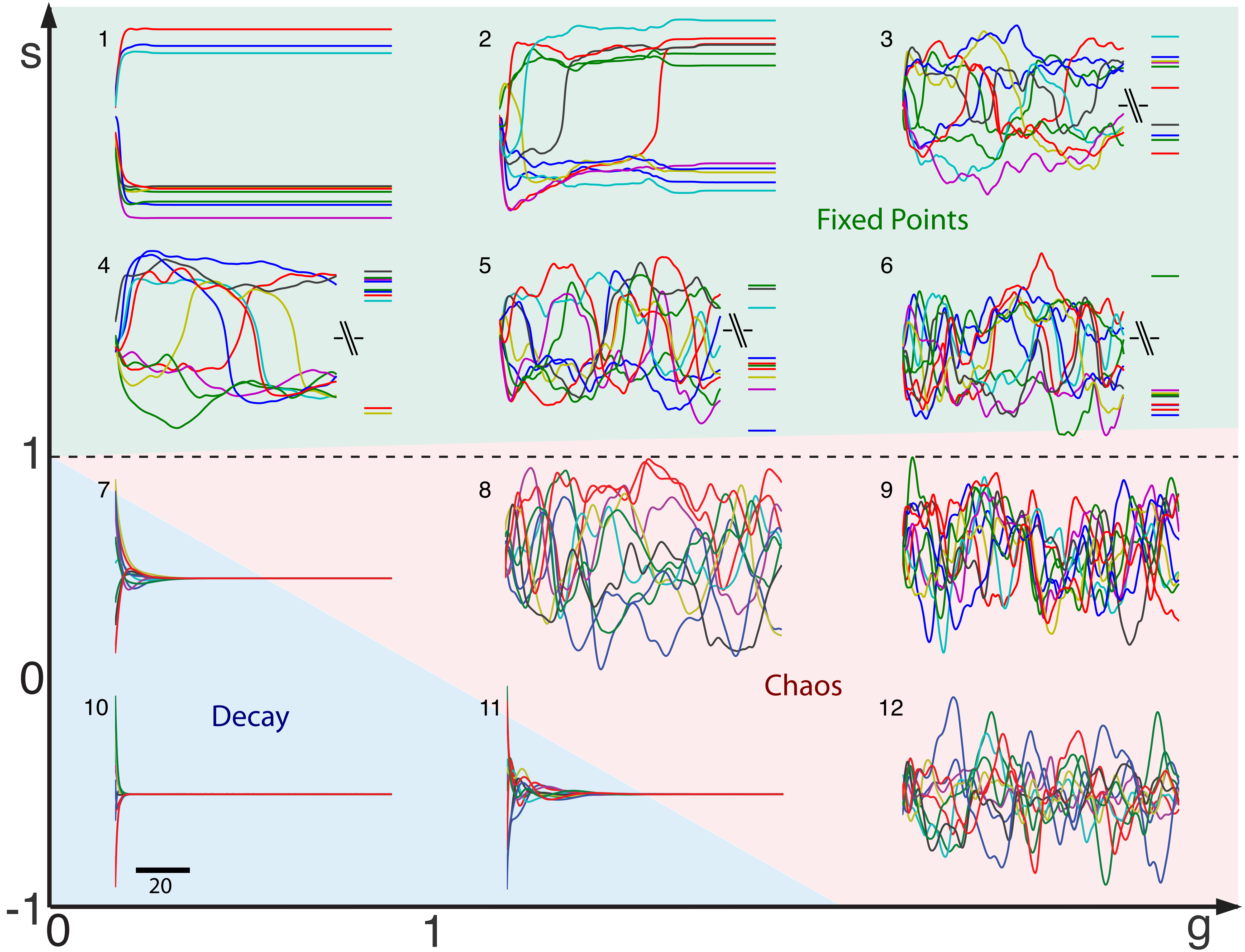

s
1
0
-1
0
1
g
1
2
3
4
5
6
7
8
9
10
11
12
Fixed Points
Chaos
Decay
20

$$\frac{dx_i}{dt} = -x_i + s\tanh(x_i) + \sum_{j=1}^{N} J_{ij}\tanh(x_j)$$

$$\frac{dx_i}{dt} = -x_i + s\tanh(x_i) + \sum_{j=1}^{N} J_{ij}\tanh(x_j)$$

$$x_i - s\tanh(x_i) = \sum_{j=1}^{N} J_{ij}\tanh(x_j)$$

$$\frac{dx_i}{dt} = -x_i + s\tanh(x_i) + \sum_{j=1}^{N} J_{ij}\tanh(x_j)$$

$$x_i - s\tanh(x_i) = \sum_{j=1}^{N} J_{ij}\tanh(x_j)$$

$$x_i - s\tanh(x_i) = \eta_i$$

$$\frac{dx_i}{dt} = -x_i + s\tanh(x_i) + \sum_{j=1}^{N} J_{ij}\tanh(x_j)$$

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$$x_i - s\tanh(x_i) = \eta_i \qquad \langle \eta \rangle = 0$$

$$\frac{dx_i}{dt} = -x_i + s\tanh(x_i) + \sum_{j=1}^{N} J_{ij}\tanh(x_j)$$

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$$x_i - s\tanh(x_i) = \eta_i \qquad \langle\eta\rangle = 0 \quad \langle\eta^2\rangle = g^2\langle\tanh^2(x)\rangle$$

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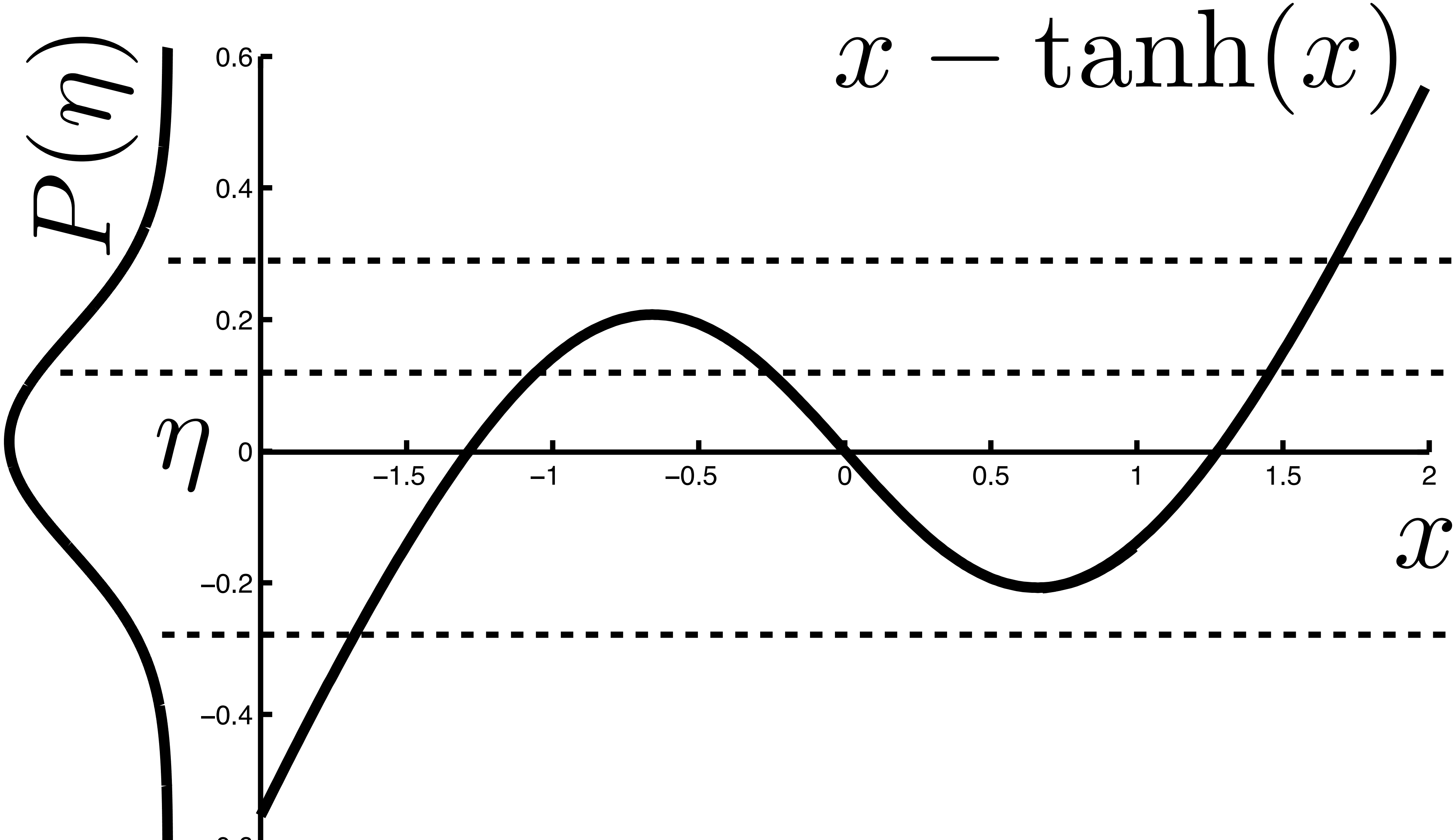
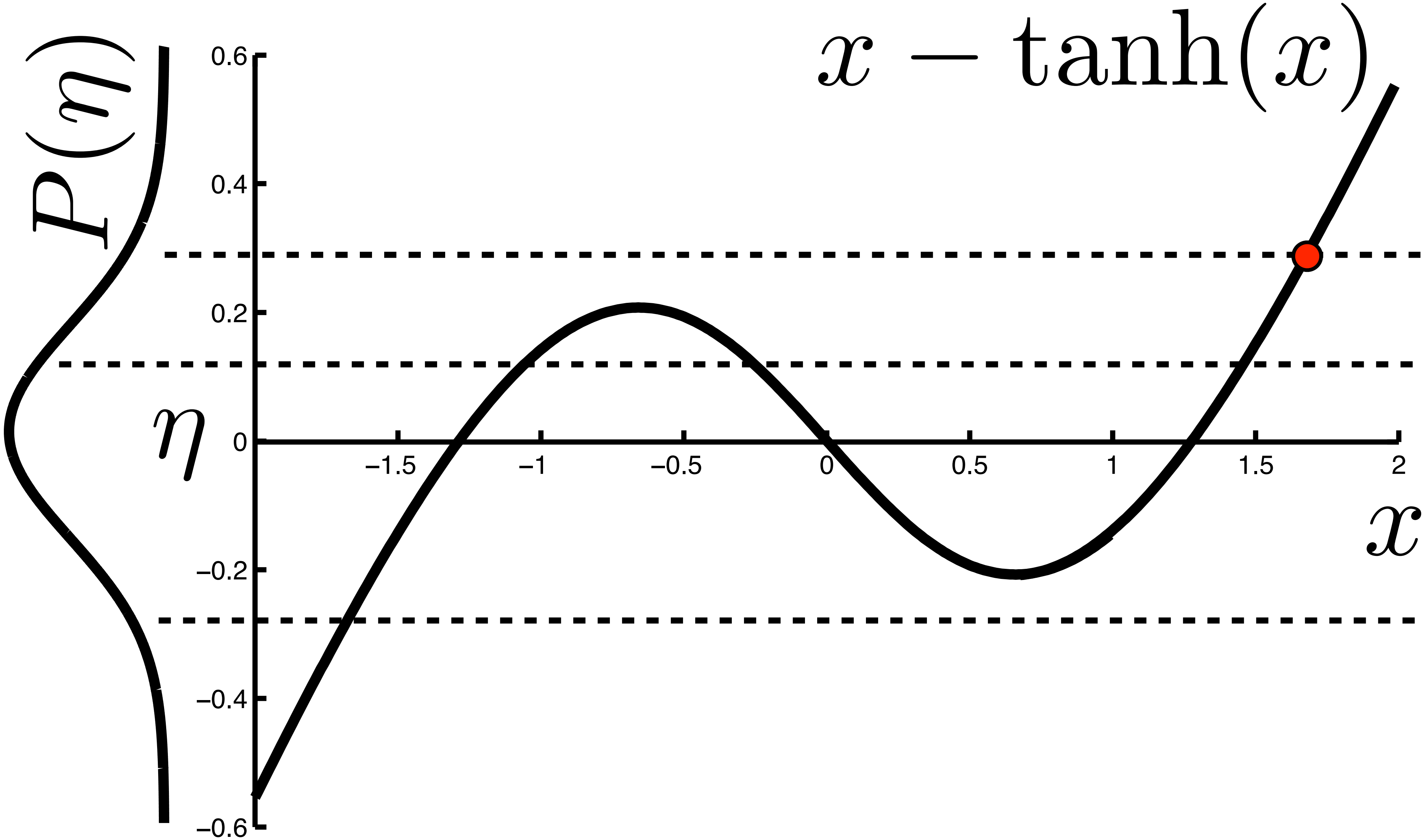
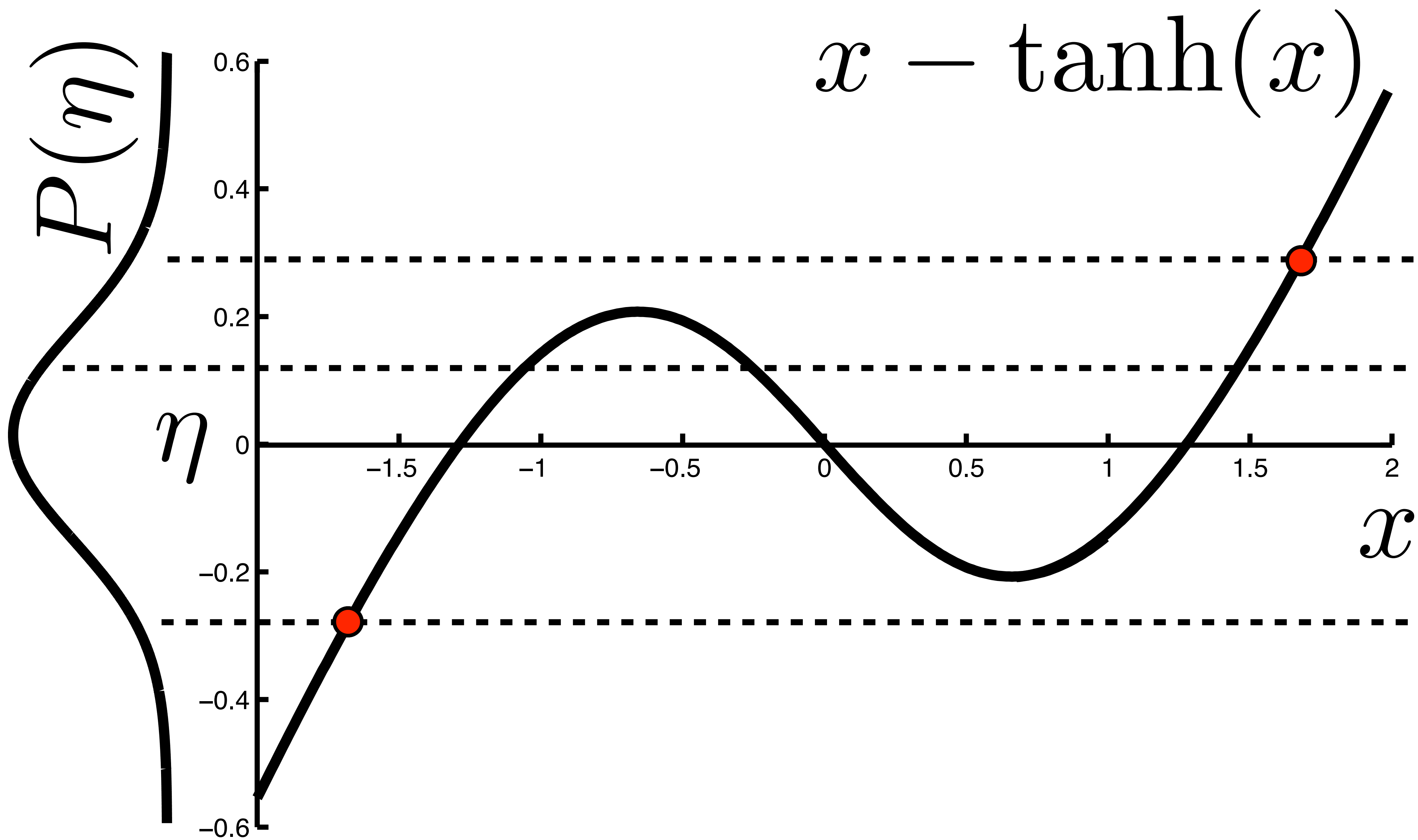


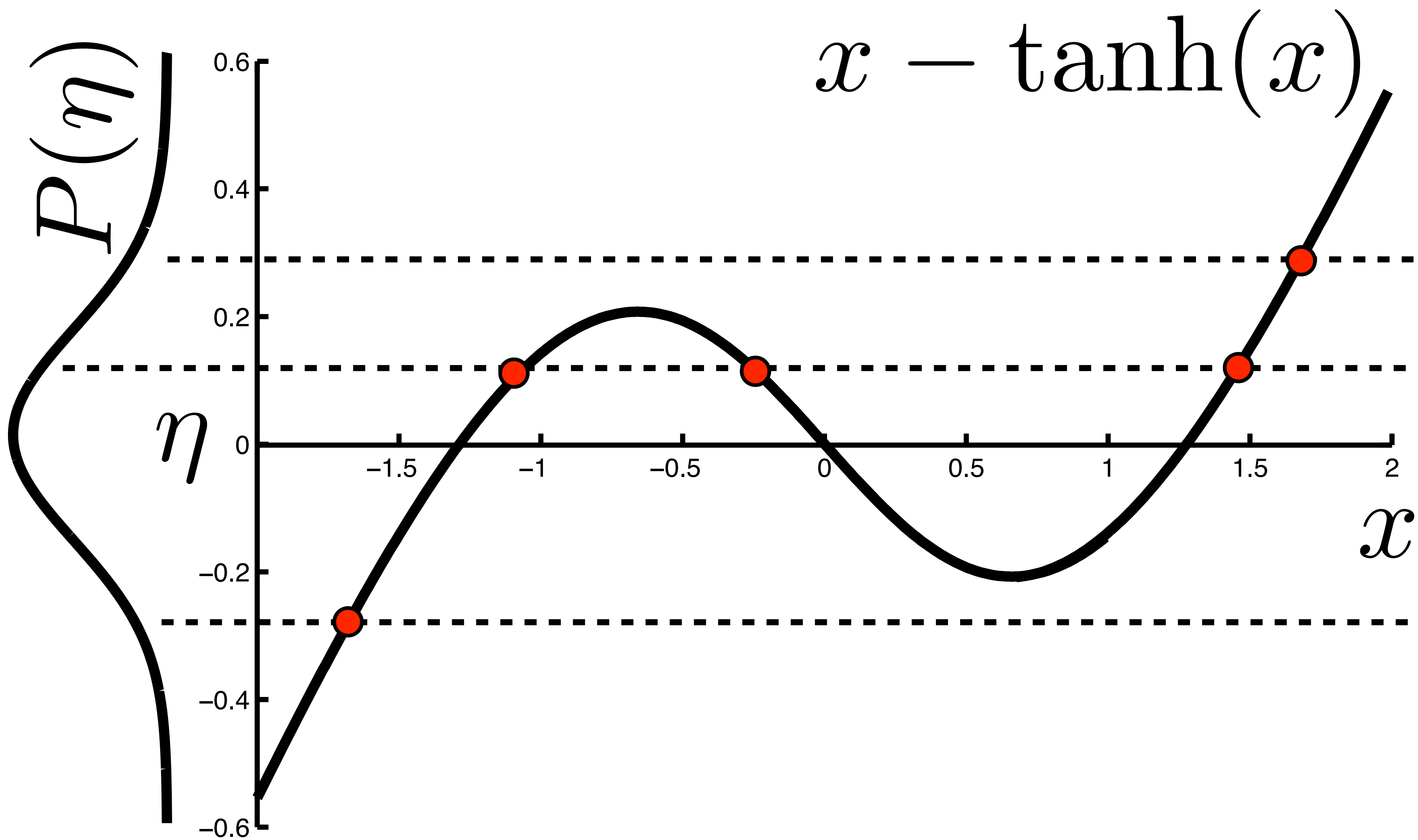
$x - \tanh(x)$
$P(\eta)$
η
x
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−1.5
−1
−0.5
0
0.5
1
1.5
2



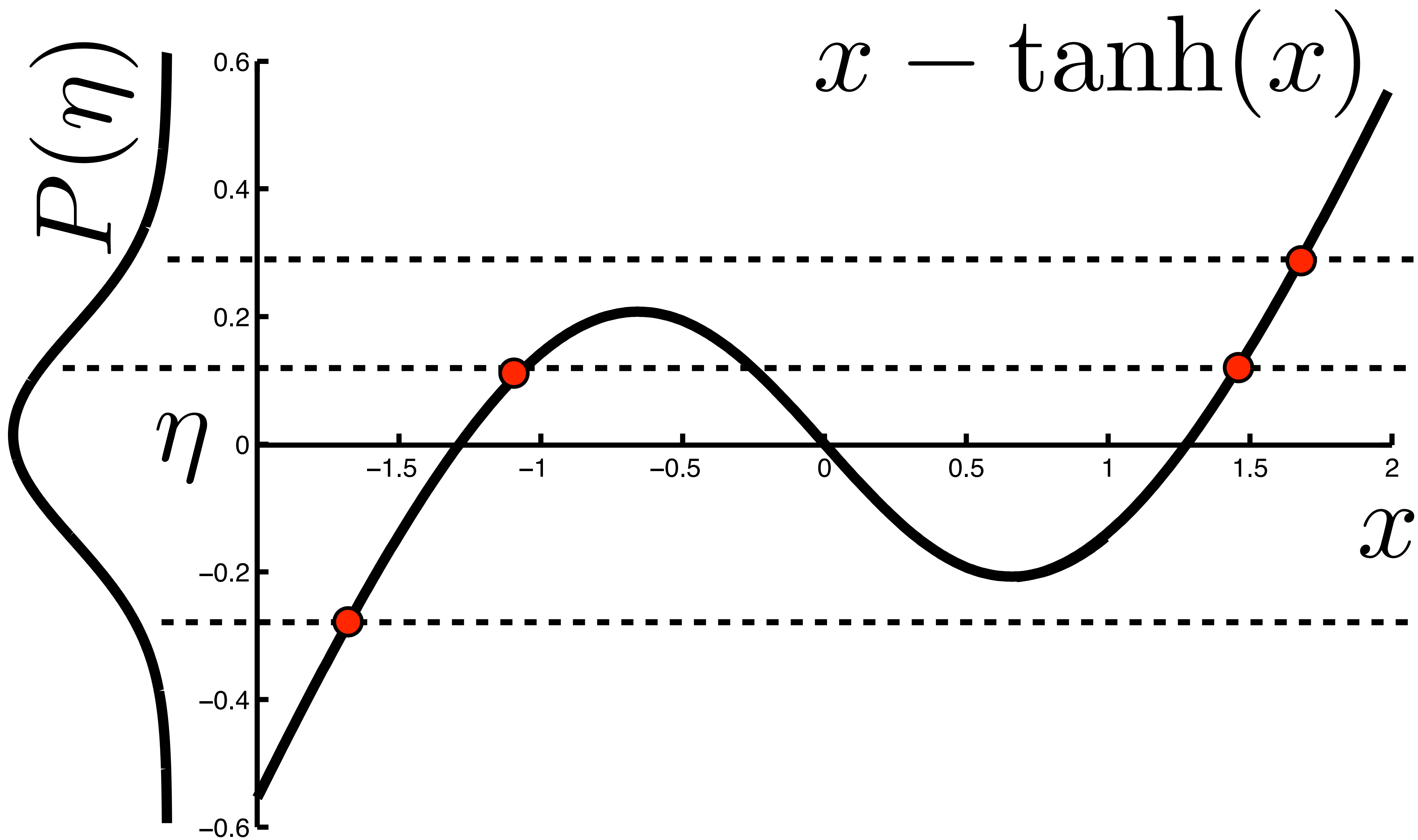
$x - \tanh(x)$
$P(\eta)$
η
x
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−1.5
−1
−0.5
0
0.5
1
1.5
2



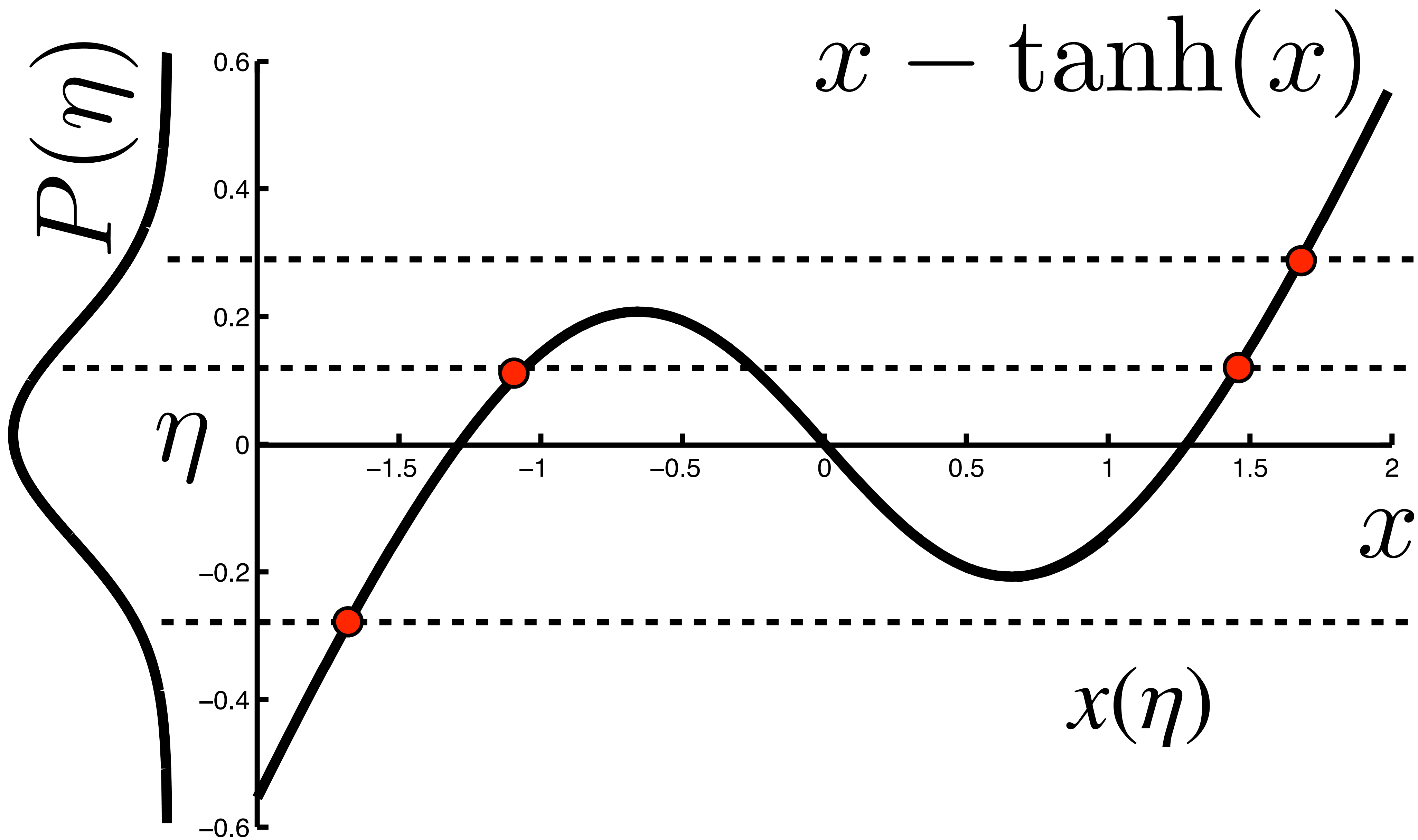
$x - \tanh(x)$
$P(\eta)$
η
x
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−1.5
−1
−0.5
0
0.5
1
1.5
2



$x - \tanh(x)$
$P(\eta)$
η
x
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−1.5
−1
−0.5
0
0.5
1
1.5
2

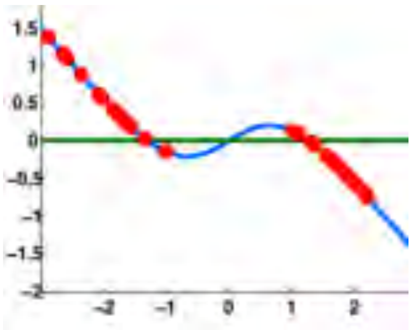


$x - \tanh(x)$
$P(\eta)$
η
x
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−1.5
−1
−0.5
0
0.5
1
1.5
2



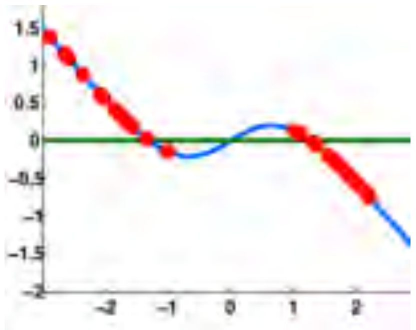
$x - \tanh(x)$
$P(\eta)$
η
x
$x(\eta)$
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−1.5
−1
−0.5
0
0.5
1
1.5
2

Stability Analysis



$$\frac{dx_i}{dt} = -x_i + s\tanh(x_i) + \sum_{j=1}^{N} J_{ij}\tanh(x_j)$$

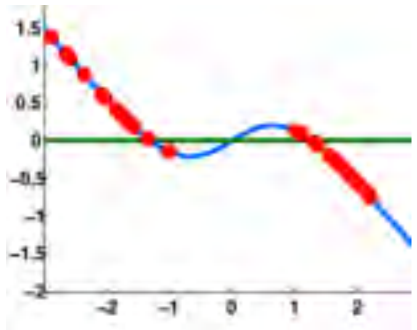
Stability Analysis



$$\frac{dx_i}{dt} = -x_i + s\tanh(x_i) + \sum_{j=1}^{N} J_{ij}\tanh(x_j)$$

$$M_{ij} = \delta_{ij}\left(-1 + s(1 - \tanh^2(x_i))\right) + J_{ij}\left(1 - \tanh^2(x_j)\right)$$

Stability Analysis

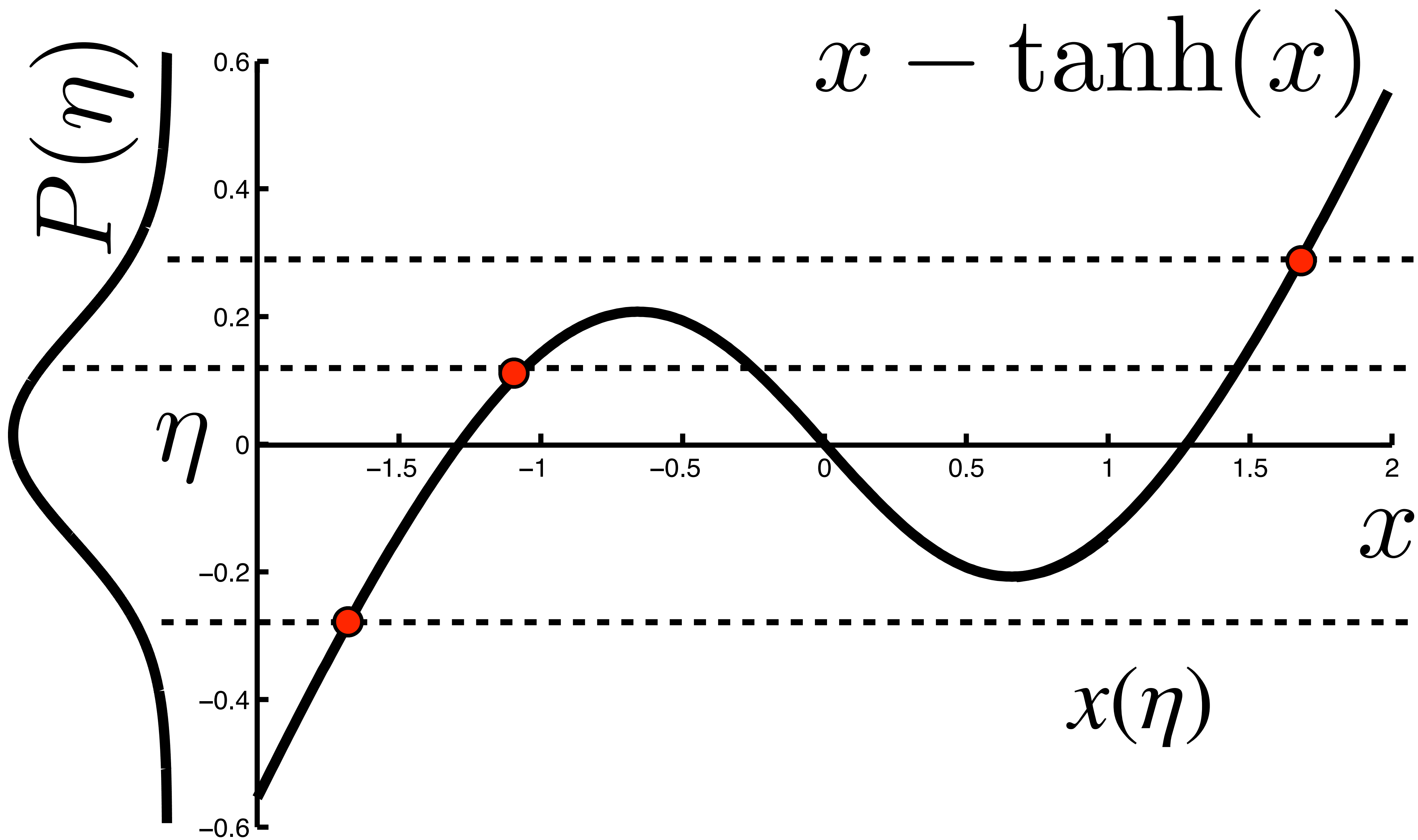


$$\frac{dx_i}{dt} = -x_i + s\tanh(x_i) + \sum_{j=1}^{N} J_{ij}\tanh(x_j)$$

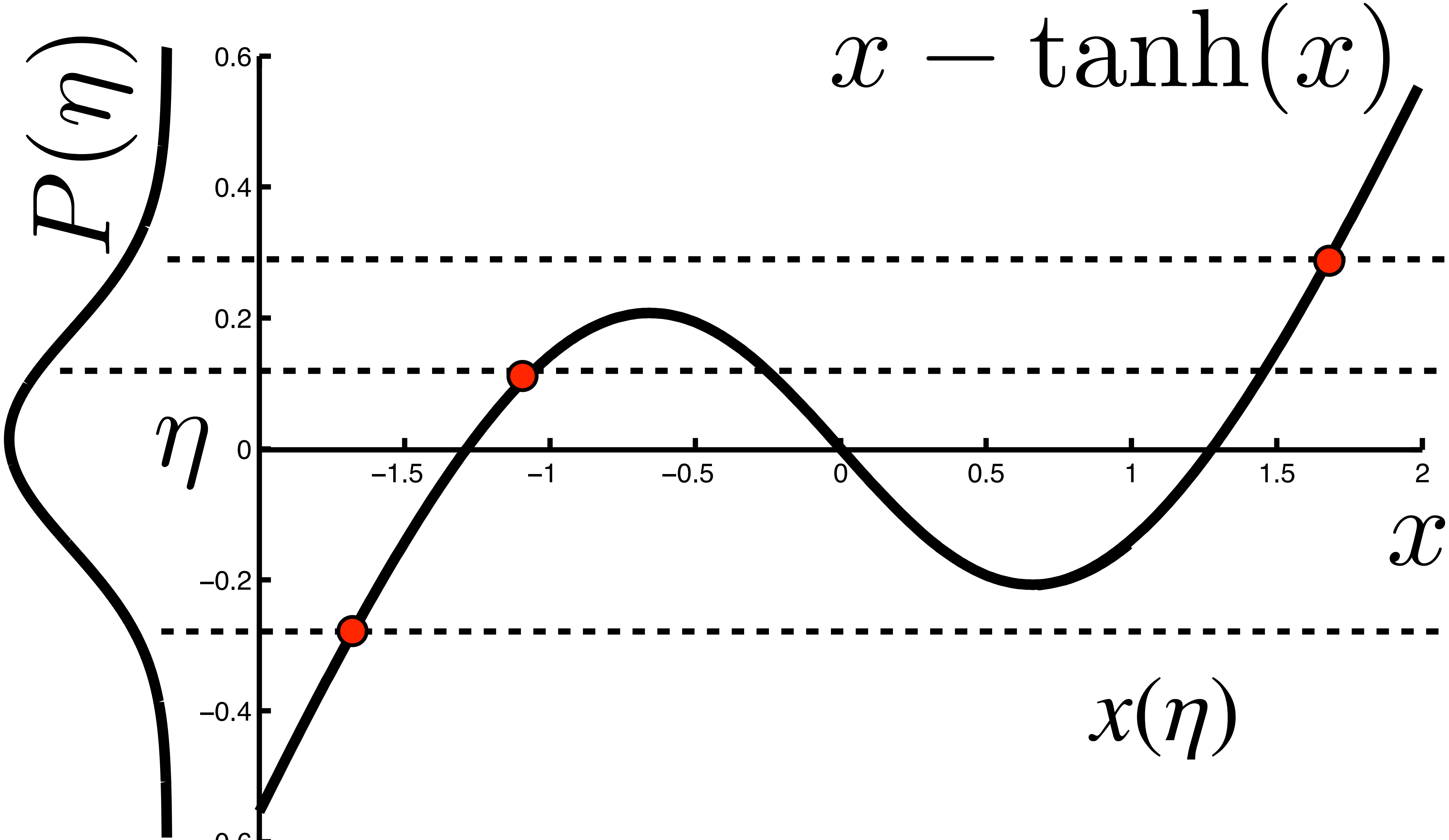
$$M_{ij} = \delta_{ij}\left(-1 + s(1 - \tanh^2(x_i))\right) + J_{ij}\left(1 - \tanh^2(x_j)\right)$$

$$\left\langle \left(\frac{g\left[1 - \tanh^2\left(x(\eta)\right)\right]}{\left|z + 1 - s\left[1 - \tanh^2\left(x(\eta)\right)\right]\right|} \right)^2 \right\rangle > 1$$

Ahmadian, Fumarola, Miller (2013)

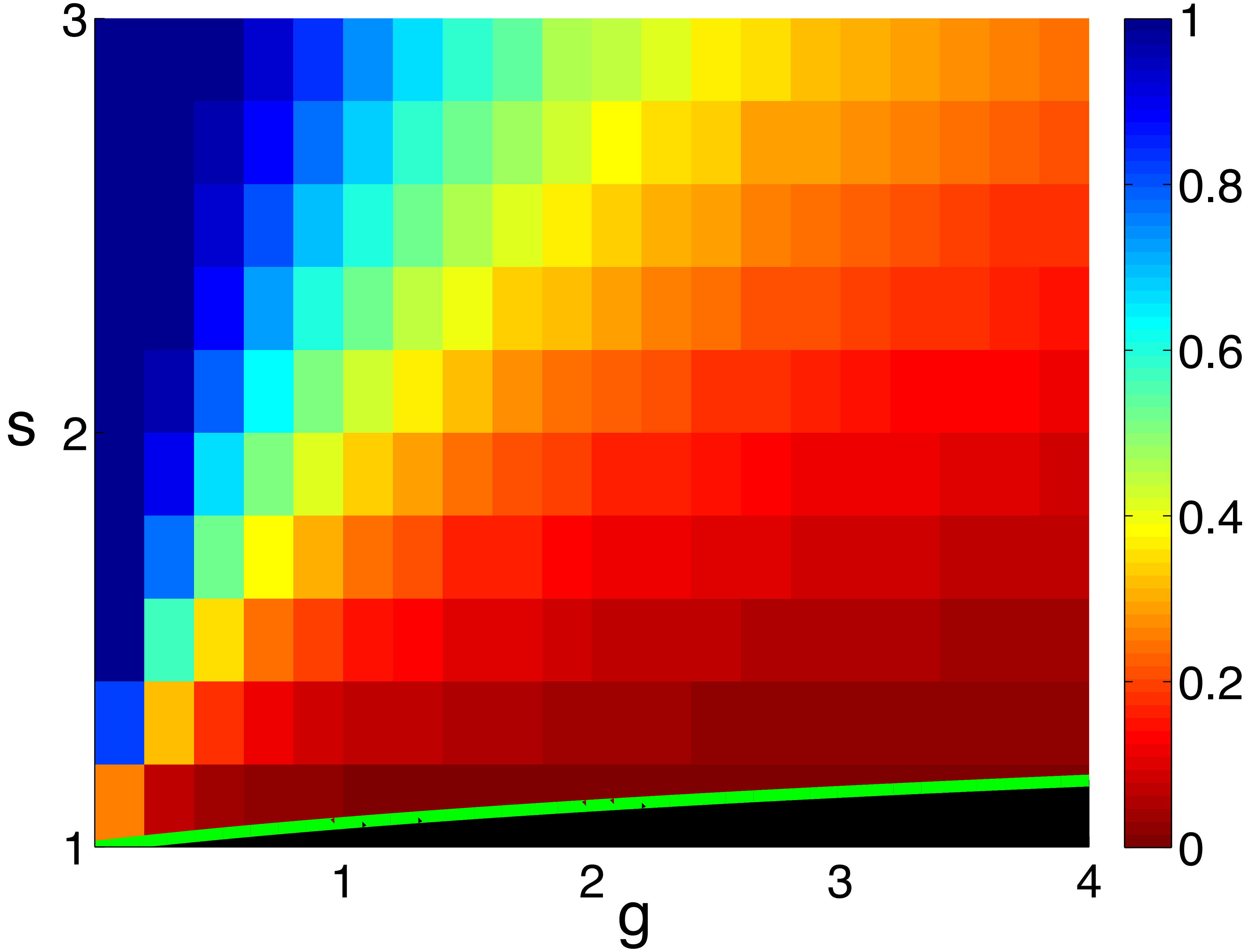


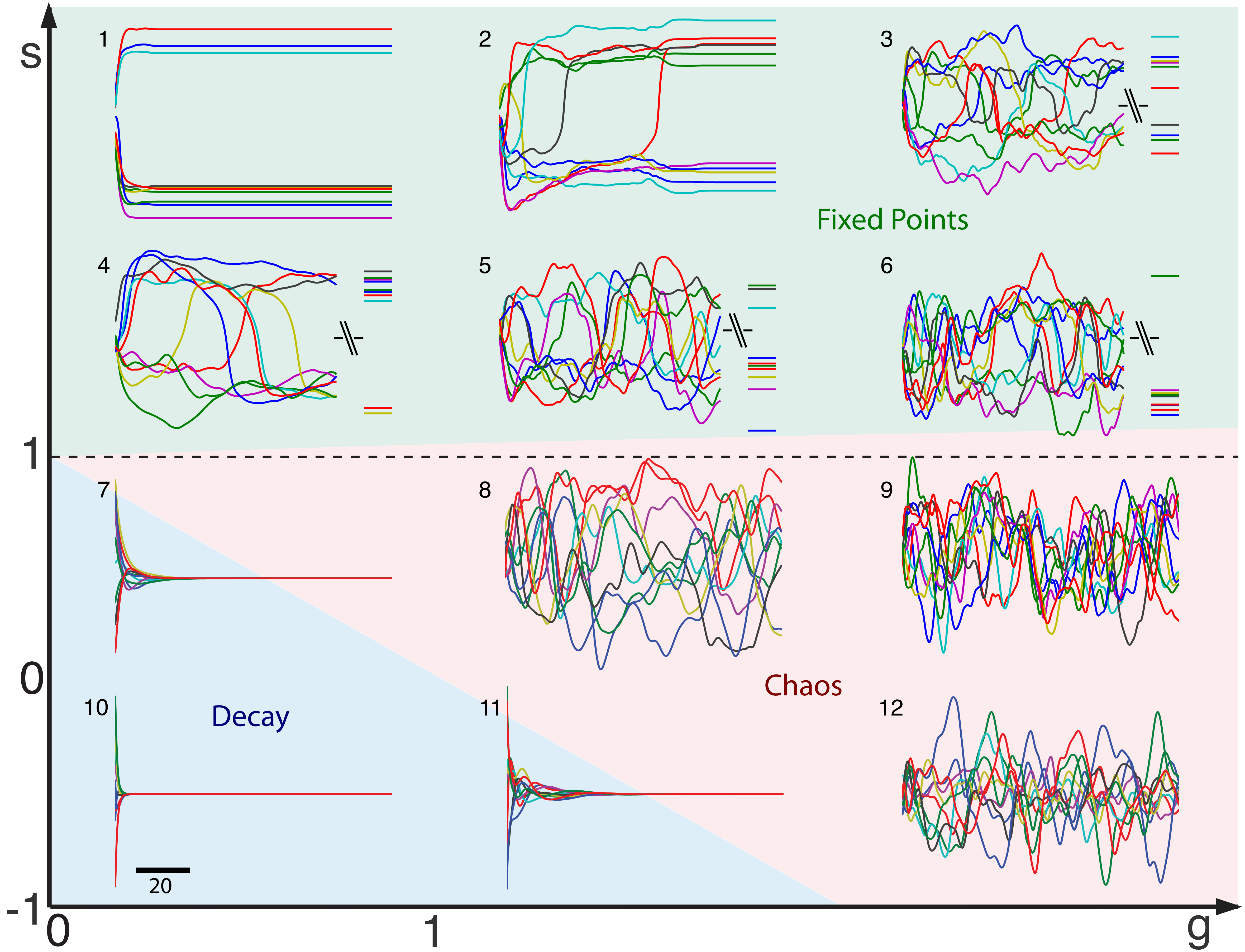
$x - \tanh(x)$
$P(\eta)$
η
x
$x(\eta)$
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−1.5
−1
−0.5
0
0.5
1
1.5
2



$$\left\langle \left(\frac{g}{\cosh^2(x(\eta)) - s} \right)^2 \right\rangle_\eta < 1$$

number of fixed points = 2^{fN}





s
1
0
-1
0
1
g
1
2
3
4
5
6
7
8
9
10
11
12
Fixed Points
Chaos
Decay
20