Flux Noise in Superconducting Qubits

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Sue Coppersmith’s recent work on quantum computing and quantum information

- Quantum dot hybrid qubit: hybrid between charge and spin silicon qubits (with Mark Eriksson, Mark Freisen, et al.)
- Quantum algorithms.
Introduction to Superconductivity

Superconductivity was discovered in 1911 by Kamerlingh Onnes.
• Zero electrical resistance
Meissner Effect

• Magnetic field expelled. Superconducting surface current ensures $B=0$ inside the superconductor.
Flux Quantization

$$\Phi = \int \vec{B} \cdot d\vec{A} = n\Phi_o$$

where the “flux quantum” $\Phi_o$ is given by

$$\Phi_o = \frac{hc}{2e}$$

$$= 2 \times 10^{-7} \text{ gauss cm}^2$$
Type I Superconductors

Type I superconductors expel the magnetic field totally, but if the field is too big, the superconductivity is destroyed.
Type II Superconductors

For intermediate field strengths, there is partial field penetration in the form of vortex lines of magnetic flux.
Each vortex contains 1 flux quantum $\Phi_0 = \frac{hc}{2e}$. The superconducting order parameter goes to zero at the center of a flux quantum. The core of the vortex has normal electrons.
Explanation of Superconductivity

BCS Theory
(Bardeen-Cooper-Schrieffer)

- Electrons are paired into Cooper pairs.
Explanation of Superconductivity

Ginzburg-Landau Order Parameter

\[ \psi = |\psi| e^{i\theta} \]

Think of this as a wavefunction describing all the electrons. Phase \( \theta \) wants to be spatially uniform ("phase rigidity").
Josephson Effect

If we put 2 superconductors next to each other separated by a thin insulating layer, the phase difference $(\theta_2 - \theta_1)$ between the 2 superconductors will cause a current of superconducting Cooper pairs to flow between the superconductors. Current flow without batteries! This is the Josephson effect.

\[ J = J_o \sin(\theta_2 - \theta_1) = J_o \sin \delta \] where $J_o$ is the critical current density and $\delta$ is the phase difference.
Josephson Junction Washboard Potential

\[ J_o \sin \delta = \frac{2e}{\hbar} \frac{\partial U_J}{\partial \delta} \Rightarrow U_J = \frac{\hbar J_o}{2e} - \frac{\hbar J_o}{2e} \cos \delta \]

\[ U_{tot} = \frac{\hbar J_o}{2e} - \frac{\hbar J_o}{2e} \cos \delta - \frac{\hbar J_{ext}}{2e} \delta \]

Washboard potential tilts with application of external current.
SQUIDs
(~ 2 slit device for superconducting wave functions)

- SQUID is a Superconducting QUantum Interference Device.
- DC SQUID is a loop with 2 Josephson junctions.
- Phase difference around the loop proportional to magnetic flux through loop.
- Current through the SQUID is modulated by the magnetic flux through loop.
- SQUID(s) are sensitive detectors of the amount of magnetic flux $\Phi$ through the loop.
- SQUID(s) can be used as qubits (quantum bits).
Why is Quantum Computing Useful?

- Parallel computation of exponentially-large states
- Factorization of large numbers into prime numbers (Shor) (cryptography)  
  Exponential speedup of algorithm
- Fast search algorithms (Grover) \{ n^{1/2} vs. n \}
- Adiabatic algorithms for minimization (Farhi)
- Simulation of quantum systems (Feynman)
- Other? (Quantum Information Theory)
Challenge: **Coupling vs. Decoherence**

Experimental challenge:
- Couple qubits to each other, and control, & measure,
- Avoid coupling qubits to noise and dissipation

Qubit is a quantum bit

\[ \Psi = \cos \frac{\theta}{2} |0\rangle + \sin \frac{\theta}{2} e^{i\phi} |1\rangle \]

Qubit wavefunction
Feynmann (1985): “it seems that the laws of physics present no barrier to reducing the size of computers until bits are the size of atoms, and quantum behavior holds sway.”
Josephson Junction Qubit Taxonomy

**Phase**
States are a linear combination of 2 different energy states in one well of JJ potential.

**Flux**
2 different flux states, e.g., up and down states of a SQUID.

**Charge**
2 different charge states, e.g., \( n \) and \( (n + 1) \) Cooper pairs in a Cooper pair box.

Potential & wavefunction

Phase difference \( \delta \)

Flux \( \Phi \)

Charge \( Q \)
Quantum Computing and Qubits

Josephson junctions can be used to construct qubits.

- Major Advantage: scalability using integrated circuit (IC) fabrication technology.
- Major Obstacle: Noise and Decoherence

\[ \Psi = \cos \frac{\theta}{2} |0\rangle + \sin \frac{\theta}{2} e^{i\phi} |1\rangle \]

Qubit wavefunction

Microscopic Sources of Noise

- Fluctuating charges, e.g., quasiparticles, electron hopping, electric dipoles flipping, two level systems
- Fluctuating magnetic spins
- Fluctuating magnetic vortices
Flux Noise Is a Major Source of Noise and Decoherence in SQUIDs

Flux noise looks like fluctuating vortices or fluxoids in the SQUID, but that is not the source of flux noise at low temperatures. Magnetic spins on the surface of the SQUID are the source.
Noise Spectrum

Noise comes from fluctuations of some type. For example, let $\delta M(t)$ be a fluctuation at time $t$. The autocorrelation function is

$$\psi_M(t) = \langle \delta M(t) \delta M(0) \rangle$$

The noise spectral density is proportional to the Fourier transform:

$$S_M(\omega) = 2\psi_M(\omega) = 2 \int dt e^{i\omega t} \psi_M(t)$$

$1/f$ noise dominates at low frequencies, and corresponds to

$$S_M(\omega) \sim \frac{1}{\omega}$$

(Actually “$1/f$ noise” refers to $S(f) \sim 1/f^\alpha$ where $\alpha$ is approximately 1 and $f = \text{frequency}$.)
1/f Noise Requires Distribution of Relaxation Times

Fluctuations in magnetization $M$ lead to noise. We use the relaxation time approximation:

$$\frac{d\delta M}{dt} = -\frac{\delta M}{\tau} \Rightarrow \delta M = \delta M_0 e^{-t/\tau}$$

where $1/\tau$ is the relaxation rate.

$$\psi_M(t) = \langle \delta M(t)\delta M(t = 0) \rangle$$

The Fourier transform is a sum of Lorentzians:

$$\psi_M(\omega) = A \int_{-D}^{D} \frac{\tau(\varepsilon,T)}{1 + \omega^2 \tau^2(\varepsilon,T)} g(\varepsilon,T) d\varepsilon$$

If we use $g(\varepsilon,T) = g_o$ for the density of states and $\tau = \tau_o e^{\varepsilon/kT}$ the noise spectral density is given by

$$S_M(\omega) = 2\psi_M(\omega) \sim \frac{1}{\omega}$$
1/f Flux Noise in SQUIDs

[Wellstood et al., APL 50 772 (’87)]

1/f^α with 0.58 < α < 0.80

“Universal” 1/f flux noise

Independent of: inductance materials geometry

Not due to fluctuating vortices (seen in wires too thin to have a vortex)

Mechanism was unknown
Paramagnetic Susceptibility

Paramagnetism: Magnetization $M$ is proportional to the magnetic field $H$

$$M = \chi H$$

Curie Susceptibility: $\chi \sim \frac{1}{T}$

- Consider a toroidal current loop (SQUID) with spins on the surface.
- Current produces $B$ field that polarizes spins.
- Polarized spins contribute to $M$ and flux $\Phi$.
- Flux $\Phi = LI \leftrightarrow$ Magnetization $M = \chi H$.

$$\Phi \leftrightarrow M, \ L \leftrightarrow \chi, \ I \leftrightarrow H$$
 Flux Noise in SQUIDs

• Noise \( \sim (1/f)^\alpha \) where 0.5 < \( \alpha < 1 \).
• 1/f flux noise in SQUIDs is produced by fluctuating magnetic impurities.
• Paramagnetic impurities produce flux \( \sim 1/T \) on Al, Nb, Au, Re, Ag, etc.

Bluhm et al. PRL (2009)
Sendelbach et al. PRL (2008)
Evidence Indicates Spins Reside on Surface

- Flux noise scales with surface area of the metal in the SQUID.
- Magnetic impurities in the bulk superconductor would be screened.
- Weak localization dephasing time $\tau_\phi$ grows as $T$ decreases (Bluhm et al.). If spin impurities in the bulk limited $\tau_\phi$, $\tau_\phi$ would saturate at low $T$ (Birge et al.).
- Concentration $\sim 5 \times 10^{17}/m^2$ implies a spacing of $\sim 1$ nm between impurities if spin moment is $1 \mu_B$.
- May be due to electrons localized at the metal-insulator interface with magnetic moments (Choi et al.).
- Lee et al. proposed adsorbed neutral OH are the spins but spin reorientation barrier $\sim 600$ K.
Where do the spins causing flux noise come from?

- Oxygen ($O_2$) molecules adsorbed on the surface.
- Consistent with flux noise independent of material and scaling with surface area.

Molecular oxygen is paramagnetic. $O_2$ molecule has 2 unpaired electron spins in the triplet state ($S=1$) with magnetic moment $= 2\mu_B$. 
Are $O_2$ molecules adsorbed on the surface responsible for flux noise?

- $O_2$ molecules have $S = 1$ (mag. moment $= 2\mu_B$)
- Oxygen weakly coupled to sapphire substrate ($Al_2O_3$)
- $O_2$ molecules sit on top of $Al$ atoms.

Density Functional Theory (DFT) done with Ruqian Wu, Hui Wang, and Jun Hu (UC Irvine)
Low barrier for $\text{O}_2$ magnetic moment spin reorientation

- Magnetic moment perpendicular to chemical bond.
- Low barrier (~10 mK) for oxygen spin reorientation on $\text{Al}_2\text{O}_3$.
- Thermal fluctuations of spin could produce flux noise.

![Diagram](image)
Could spin fluctuations produce 1/f flux (magnetization) noise?

- Need a distribution of spin relaxation times to get 1/f noise.
- Spin interactions would give such a distribution.
- Density functional theory indicates that oxygen molecule spins interact ferromagnetically.

Exchange coupling energy $J$:
$J(r=4.8 \text{ Å}) = 0.14 \text{ meV} \sim 1.6 \text{ K}$
$J(r=9.6 \text{ Å}) = 0.05 \text{ meV} \sim 0.6 \text{ K}$
Fluctuation Cross Correlation Consistent with Ferromagnetism

- Cross correlation $<\delta \Phi \delta L> \sim <\delta M \delta \chi> \sim <(\delta M)^3>$ is odd under time reversal. ($\Phi = LI \leftrightarrow M = \chi H$, $\Phi \leftrightarrow M$, $L \leftrightarrow \chi$, $I \leftrightarrow H$)
- Large cross correlation seen experimentally implies ferromagnetism or very slow fluctuators (Weissman).
- Correlation could average to zero over very long times if there is time reversal invariance.

(Sendelbach et al. 2009)
Monte Carlo simulations test whether spin fluctuations produce 1/f magnetization noise

Work done with Chuntai Shi

• We model the adsorbed oxygen molecules as XY spins and investigated whether the combination of in-plane spin anisotropy and ferromagnetic interaction could give rise to the 1/f type noise spectra.

• The Hamiltonian

\[ H = -\sum_{\langle i,j \rangle} J_{ij} \left( s_i^x s_j^x + s_i^y s_j^y \right) - A \sum_i \left( s_i^x \right)^2 \]

• Poisson-like distribution \( P(J) \) of ferromagnetic couplings.

• \( A = \) anisotropy energy, i.e., the barrier to spin reorientations

• From DFT, the barrier to spin reorientations is \( A \approx 10 \text{mK} \).
Monte Carlo: $O_2$ spin fluctuations could produce 1/f flux (magnetization) noise

Work done by Chuntai Shi
Experimental Evidence for Adsorbed Oxygen Molecules Producing a Magnetic Signature and Flux Noise
X-Ray Magnetic Circular Dichroism (XMCD)

- Magnetic moment couples to orbital angular momentum via spin-orbit coupling.
- Slight imbalance between $Y_{1,1}$ and $Y_{1,-1}$ in $2\pi^*$ in $O_2$ in B field.
- Detect magnetism by differential absorption of right- ($J_z=-\hbar$) and left- ($J_z=+\hbar$) circularly polarized x-rays which produce transitions from $1s \rightarrow 2\pi^*$ in $O_2$. This gives XMCD.
- x-ray absorption spectrum (XAS) is the sum of both polarizations.

https://www-ssrl.slac.stanford.edu/stohr/xmcd.htm
XMCD and XAS

- No XMCD signal on bare Al detected until air was let into chamber and frozen onto Al sample.
- Strength of signal depends on tilt angle of O\(_2\) molecule:
- Best agreement for tilt 55° from vertical

From Freeland and McDermott groups
XMCD

No XMCD signal on bare Al detected until air was let into chamber and frozen onto Al sample. Strength of signal depends on tilt angle of O\(_2\) molecule:
Best agreement for tilt 55° from vertical.

(A) EXPERIMENT

From Freeland and McDermott groups

(B) THEORY
Recall: Experimental Evidence for Surface Spins

Flux $\sim 1/T$

- $B_{fc} = 500 \text{ mT}$
- $B_{fc} = -500 \text{ mT}$

$\sigma_s \sim 5 \times 10^{17} \text{ m}^{-2}$

Surface spin density

[Sendelbach et al., PRL 100, 227006 (08)]
Surface passivation by ammonia in hermetic enclosure reduces flux (susceptibility) in SQUID.

Ammonia exposed (~10 times decrease in surface spin density).

$B_{fc} = \pm 125\mu T$

From McDermott Lab.
Flux (Susceptibility) vs. Temperature

- Flux through Nb SQUID
- Cooling field = 128 µT
- Air: Curie-like 1/T
- NH₃: χ~constant in T
- NH₃ blocks adsorption of O₂
- NH₃ binds 10x more strongly than O₂.
- ~10 times decrease in surface spin density with NH₃

Magnetic susceptibility greatly reduced when sample exposed to ammonia.

From McDermott Lab
Simulations: Decrease in susceptibility should mean decrease in flux noise

- Let $M$ = magnetization per spin
- Susceptibility:
  \[
  \chi = \frac{N \sigma^2_M}{k_B T} \sim \sigma^2_M \quad \text{where } \sigma^2_M \text{ is the variance of } M
  \]
- Total noise power
  \[
  S_{tot} = \frac{1}{N \tau} \sum_{\omega=0}^{\omega_{max}} S_M(\omega) = \sigma^2_M
  \]
  \[
  \chi \sim \sigma^2_M \sim S_{tot}
  \]
Surface Treatments Reduce Flux Noise

Suppression in $S_F \sim 5$ times

Suppression in $S_F \sim 4$ times

Flux noise experiments on Al-based SQUIDs encapsulated in SiN$_x$ (P. Kumar and R. McDermott)
Conclusions

• Flux noise in SQUIDs is produced by mysterious magnetic impurities on metal surfaces.
• We propose that paramagnetic O$_2$ molecules adsorbed on the surface produce flux noise in SQUIDs.
• Evidence for adsorbed O$_2$ producing magnetic spins (and flux noise) on the SQUID surface
  – Theory: DFT and Monte Carlo simulations.
  – Experiment: Curie-like susceptibility.
  – Experiment: XMCD with and without air exposure.
  – Experiment: Blocking adsorption of O$_2$ with ammonia significantly reduces the magnetic susceptibility.
  – Experiment: Surface treatments with UHV/UV or ammonia reduce flux noise.

THE END