Fowler–Nordheim emission modified by laser pulses in the adiabatic regime

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2 authors:

Alexander Rokhlenko
Rutgers, The State University of New Jersey

Joel Lebowitz
Rutgers, The State University of New Jersey

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A.Rokhlenko and J.L.Lebowitz*

Department of Mathematics, Rutgers University
Piscataway, NJ 08854-8019

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Abstract

We investigate enhanced field emission due to a continuous or pulsed oscillating field added to a constant electric field E at the emitter surface. When the frequency of oscillation, field strength, and property of the emitter material satisfy the Keldysh condition $\gamma < 1/2$ one can use the adiabatic approximation for treating the oscillating field, i.e. consider the tunneling through the instantaneous Fowler-Nordheim barrier created by both fields. Due to the great sensitivity of the emission to the field strength the average tunneling current can be much larger than the current produced by only the constant field.

We carry out the computations for arbitrary strong constant electric fields, beyond the commonly used Fowler-Nordheim approximation which exhibit in particular an important property of the wave function inside the potential barrier where it is found to be monotonically decreasing without oscillations.

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In a constant electric field the current due to electron tunneling from a metal is described by the commonly used Fowler-Nordheim (FN) equations [1]. They are modified to include arbitrary strong fields, see e.g. [2] and [3], though for practical needs the low field approximation made in [1] are usually sufficient. The triangular potential barrier used in these articles was corrected for image forces by Schottky [4]. They are important in constant electric fields, but an adequate method for treating them is not clear under laser radiation. Their physical origin is based on the rearrangement of the electron distribution inside the emitters; a time dependent process whose duration is not known well. To simplify the problem (and losing some precision) the Schottky term will not be included here.

In experiments the constant electric fields are often supplemented by short laser pulses with electric component of amplitude F orthogonal to the metal surface. Here we carry out computations for a simple one-dimensional model of the emitting surface in order to explore a practically important [5], [6] situation when F can be treated adiabatically. This is possible under suitable conditions on the strengths of the constant field E and laser field F, frequency ω of the electromagnetic oscillations, and the emitter properties. A pulse of duration T is

*Also Department of Physics

applied at $t = 0$. The pulse shape is assumed smooth enough and its amplitude F significantly lower than E. Our goal is to find the current gain in the interval $0 < t < T$.

Principal approximation

The laser assisted field emission is an important method to increase tunneling current and have flexible easily manipulated electron sources. The theoretical treatment of this effect is quite difficult because in involves interplay of different processes, such as multi-photon and above barrier emission, FN tunneling, and emitter heating, see for example [5].

Our simple theory disregards emitter heating, the photofield ionization caused by photon absorption from the oscillating field. This can be justified only when the laser field is not strong and/or the photon frequency is relatively low for significant contribution of multi-photon processes. Nevertheless the optical field changes the shape of the potential barrier and therefore [6] the rate of tunneling. Such situation was studied 50 years ago by Keldysh in [7] who introduced the parameter γ which separates the regions where the time of change of the external electric field is longer or shorter than the transition time τ of crossing the potential barrier by electrons, see [7], [8]. The adiabatic regime can be realized when the Keldysh parameter $\gamma = \omega\tau < 0.5$ where ω is the angular frequency of radiation. In the Keldysh derivation this barrier is rectangular. The parameter γ has been refined and specified [8] and we modify its form taking into account that it is triangular in the FN case. A reasonable expression for the time τ of crossing the potential barrier [9], [10] in the presence of a constant field $\mathbf{E}$ is given by the integral

$$\sqrt{\frac{m}{2}} \int_0^q \frac{dx}{\sqrt{V - e\mathbf{E}x}} = \frac{\sqrt{2m}(\sqrt{V} - \sqrt{W})}{e\mathbf{E}}, \tag{1}$$

where V is the total height of the potential barrier, W is the location of the Fermi level at $x < 0$, 0 and $q = (V - W)/eE$ are turning points of tunneling electrons.

Thus the modified Keldysh parameter has the form

$$\gamma = \omega \frac{\sqrt{2m}(\sqrt{V} - \sqrt{W})}{e\mathbf{E}}. \tag{2}$$

We assume that $F(t) = 0$ for $t < 0$, i.e. the electromagnetic radiation starts at $t = 0$ when an electron enters the potential barrier, it goes out of it at $x = q$, therefore 0 and q are the limits of integration. When $\gamma < 0.5$ the tunneling can be safely approximated as an adiabatic process, i.e. by taking for $\mathbf{E}$ in (2) its instantaneous value of the time dependent sum $\mathbf{E}(t) = E + F(t)$. $q = V/eE$ is evaluated by Eq(8A) in Appendix for a constant E.

Disregarding the relatively small F (compared with E) we finally consider here only the regime when

$$\omega < \frac{eE}{2\sqrt{2m}(\sqrt{V} - \sqrt{W})}. \tag{3}$$

As an illustration we show the results assuming $V = 2W$ and study two cases for the work function $\chi = 5$, $\chi = 2.1$ eV [9]. By denoting $F/E = \beta < 1$ and treating β as a small quantity Eq.(3) can be presented in two forms

$$\omega < p \cdot 10^{14} E, \text{ or } \lambda > \frac{h \cdot 10^3}{E}, \tag{4}$$

where ω, is in sec^{-1}, λ in nm, and E in V/nm. Shown in Fig.1 are two curves of laser wavelengths $\lambda(E)$ ($\beta = 0$ in Fig.4 for simplification). For λ above the curves the adiabatic approximation is valid. The parameters are $p \approx 1.6$, $h \approx 1.18$ for Au, Cu, Ni, W, and $p \approx 2.5$, $h \approx 0.75$ for Cs.

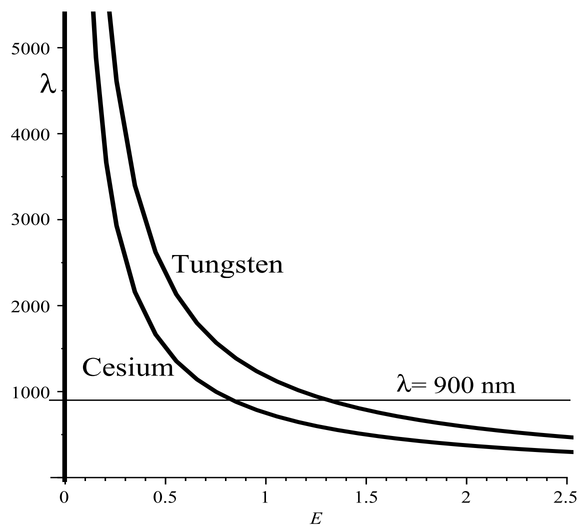

FIG.1. Laser wavelengths λ above corresponding curves can be treated adiabatically

Results in Fig.1 are approximate because the work functions are not the same for different crystal orientations. Nevertheless Fig.1 gives a general picture of permissible regions for treating tunneling by the adiabatic method when emitters are irradiated by lasers of frequencies estimated in our work.

The wavelength of short laser pulses [10], [11], [12] frequently used in experiments and theory is around $\lambda = 800 - 900$ nm, see Fig.1. Note also that in strong fields E about $50 - 100$ V/nm the FN approximate Eq.(9A) for the tunneling probability P becomes incorrect [2].

Results of calculations

1. Rectangular laser pulse

On the interval $0 < t < T$ the total field has the form

$$\mathbf{E}(t) = E + F\cos(\omega t), \quad E, F > 0, \quad 0 < t < T, \tag{5}$$

we simplify our computations by evaluating the tunneling probability $P(t) = P(\mathbf{E}(t))$ and treating $\mathbf{E}(t)$ as a fixed field in (5) in spirit of adiabatic approximation. Then we evaluate $\tilde{P}$ as the average of $P(t)$ on $(0, T)$ when both fields E and $F(t)$ are turned on

$$\tilde{P} = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} P(t) dt. \tag{6}$$

We use here the Eq.(5) which allows to compute $\tilde{P}$ on a shorter time interval of a single period of laser field when the pulse is rectangular. The entries in Table 1 are $\tilde{P}/P_E$, where P_E is the exact tunneling probability in the constant field E given in Eq.(8A). Note that E is equal to the time average of $\mathbf{E}(t)$. The gain of electron tunneling flow during the pulse period is multiplied by the number of oscillations in a single pulse which will correspond to the rectangular laser pulse of duration T. In the case of other pulse shapes such calculation is longer because it should be extended on the whole pulse duration.

Fig.3 in Appendix implies that for the adiabatic treatment we always are quite far from the maximum point of $P(t)$, i.e. the same results can be found using Fowler-Nordheim formula (9A). In Table 1 are shown the results for Tungsten and Cesium in the electric fields permissible for the adiabatic regimes. The entries in Table 1 exhibit the average current gain in time of the pulse action compared with the case when β and F are zero. Note that we use everywhere in this work the CGS unit system.

TABLE 1

Tungsten	$E = 1.5$	$E = 2.0$	$E = 3$	$E = 5$	$E = 8$
$\beta = 0.1$	20.50	7.468	2.917	1.542	1.183
$\beta = 0.2$	745.7	102.6	14.99	3.550	1.750
Cesium	$E = 1.0$	$E = 1.5$	$E = 2.0$	$E = 3.0$	$E = 5.0$
$\beta = 0.1$	2.137	1.432	1.223	1.086	1.021
$\beta = 0.2$	7.612	2.948	1.926	1.341	1.083

One can see that the lower is the constant background electric field the stronger is the effect of laser radiation caused by its negative half-waves. This can be easily justified by using Eq.(9A) for weaker electric fields and evaluating the ratio

$$\frac{1}{P_{FN}} \frac{dP_{FN}}{dE} = \frac{4\sqrt{2m}}{3eE^2\hbar} \chi^{3/2}, \tag{7}$$

which grows rapidly when E is relatively small and decreasing. This property depends strongly on the shape of the pulses.

2. Laser pulse with envelopes $sin(t\pi/T)$ and triangular one

We consider first the laser pulse $F\sin(t\pi/T)\cos(\omega t)$ for $0 < t < T$ where $T = 16\pi/\omega$, which means that the pulse consists of 8 periods. As the average pulse amplitude is lower than of the rectangular one the gain is smaller too but substantial anyway when E is low. Table 2 below is similar to Table 1 and in both of them the tunneling current grows from 10^{-22} to 10^{-4} for Tungsten and from 10^{-9} to 0.03 for Cesium when E increases, but the laser pulse effect gets smaller.

TABLE 2

Tungsten	$E = 1.5$	$E = 2.0$	$E = 3$	$E = 5$	$E = 8$
$\beta = 0.1$	8.733	3.811	1.899	1.263	1.089
$\beta = 0.2$	230.3	37.13	6.871	2.204	1.367
Cesium	$E = 1.0$	$E = 1.5$	$E = 2.0$	$E = 3.0$	$E = 5.0$
$\beta = 0.1$	1.544	1.210	1.108	1.041	1.021
$\beta = 0.2$	3.943	1.931	1.452	1.167	1.010

The shape of laser pulse envelopes is often close to a triangle. In Table 3 we show the results for isosceles triangular pulses symmetric about the vertical axis of the same maximum amplitudes as above only for a Tungsten emitter.

TABLE 3

Tungsten	$E = 1.5$	$E = 2.0$	$E = 3$	$E = 5$	$E = 8$
$\beta = 0.1$	5.685	2.785	1.594	1.178	1.061
$\beta = 0.2$	125.2	21.69	4.652	1.797	1.249

Similar results can be easily obtained for any envelope of electromagnetic pulses.

The electric field amplitude F_0 of the laser radiation for computations [13] can be evaluated by using the Pointing vector

$$\vec{S} = \vec{E} \times \vec{H} = \frac{1}{2} c \epsilon_0 E_0^2 \vec{k}, \tag{8}$$

where c is the speed of light, ϵ_0 - vacuum permittivity, and $\vec{k}$ gives the direction of the beam with rectangular cross section. Using the well known constants the amplitude of the field E_0 can be presented as

$$E_0(V/m) = 27.42 \sqrt{\frac{Beam\ power\ (watt)}{Beam\ crossection\ (m^2)}}. \tag{9}$$

Here units are given in round parentheses. After adding the constant field this equation allows to find the optical field.

When $\gamma > 10$ the main contribution to electron emission comes from multiphoton processes and emitter heating [10],[11], the case $1 < \gamma < 10$ is more difficult. Under conditions given here one can use the adiabatic technique, which in our work is implemented to show that the electromagnetic radiation can very significantly increase the emission, see Tables 1-3.

APPENDIX

We supplement the results for evaluating the time independent tunneling in an arbitrary strong electric field derived in [1] with some additional details. Let us consider a metallic block, which is placed to the left of $x = 0$, an electron with kinetic energy corresponding the Fermi level $W = \hbar^2 k^2 / 2m$ enters the triangular potential field whose shape is determined by the constant electric fields E on $-\infty < x < \infty$ and $-Fx$ on the beam $x > 0$. The governing Schrödinger equation for the electron wave function $\psi(x,t)$ on the infinite interval $-\infty < x < \infty$ can be written in the following form

$$i\hbar \frac{\partial \psi}{\partial t}(x,t) = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2}(x,t) + (V - eEx - eFx \cos \omega t) \psi(x,t), \ E, F > 0. \tag{1A}$$

We neglect here the Schottky term, here $e, \ m$ are the electron mass and charge, F is the amplitude of the dipole field, which assumed to be orthogonal to the emitter surface. Heating of a bulk emitter is neglected here.

The tunneling is described by the stationary equation

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2}(x) + (V - eEx)\psi(x) = W\psi(x), \tag{2A}$$

whose exact solution in the Bessel functions, found in [1], can be written in the form

$$\psi(x) = \left\{ \begin{array}{rll} \exp(ikx) + a_1 \exp(-ikx) & \text{for} & x < 0, \\ a_2 \sqrt{y} I_{1/3}(2y^{3/2}/3) + a_3 \sqrt{y} K_{1/3}(2y^{3/2}/3) & \text{for} & 0 < x < q, \\ a_4 \sqrt{z} H_{1/3}^{(1)}(2z^{3/2}/3) & \text{for} & x > q. \end{array} \right\} \tag{3A}$$

where $q = (V - W)/eE$. The variables y and z are non-negative in their domains and they are

$$y = (V - W - eEx)(2m)^{1/3}(eE\hbar)^{-2/3}, \quad z = (eEx + W - V)(2m)^{1/3}(eE\hbar)^{-2/3}. \tag{4A}$$

Inside the emitter the tunneling electrons with the Fermi energy $W = (\hbar k)^2/2m$ and wave their function it is represented by the first term in Eq.(3), where e^{ikx} creates the incoming current $j_- = \hbar k/m$ toward the surface from its left side. The coefficients $a_1, ..., a_4$, calculated in [2] using the continuity of $\psi(x)$ and its derivatives at $x = 0$ and $x = q$ are:

$$a_2 = -2a_4 e^{\pi i/3}, \quad a_3 = \frac{2a_4}{\pi i}, \tag{5A}$$

$$a_1 = a_4 \rho^{1/3} \left\{ -e^{\frac{\pi i}{3}} I_{1/3}(\rho) - iK_{1/3}(\rho)/\pi + i\sigma \left[e^{\frac{\pi i}{3}} I_{2/3}(\rho) + \frac{e^{\frac{\pi i}{6}}}{\pi} K_{2/3}(\rho) \right] \right\},$$

where

$$a_4 = \frac{-2(2/3)^{1/3}\rho^{-1/3}}{e^{\pi i/3} I_{1/3}(\rho) + iK_{1/3}(\rho)/\pi + i\sigma e^{\pi i/3}[I_{2/3}(\rho) + e^{-\pi i/6} K_{2/3}(\rho)/\pi]}. \tag{6A}$$

Here

$$\rho = \frac{2\sqrt{2m}\chi^{3/2}}{3eE\hbar}, \quad \sigma = \sqrt{\frac{\chi}{W}}, \quad \chi = V - W. \tag{7A}$$

The transversal time of crossing the potential barrier was estimated in [5], [6] and used below, χ is the work function. We emphasize here that the wave function inside the barrier where $0 \le y \le q$ is monotonically decreasing without oscillations, as this can be seen in Fig.2.

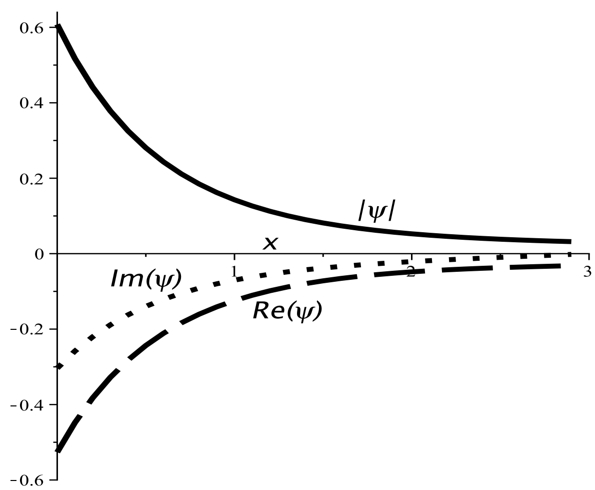

FIG. 2. Wave function $\psi(x)$ inside the barrier

Here the x-scale given in units proportional to the real one, the amplitude of ψ in Fig.2 should be multiplied by $40a_4$ to correspond Eq.(3). The general behavior of curves corresponding different parameters σ, ρ is similar to the ones in Fig.2.

The Hankel function $H^{(1)}$ allows to find the moving to the right current j_+ of escaped electrons proportional to $|a_4|^2$. Thus using Eqs.(5-7) the stationary tunneling probability for $t < 0$ when $F = 0$ is

$$P = \frac{j_+}{j_-} = \frac{2\sigma}{\pi\rho\left|e^{\frac{\pi i}{3}} I_{1/3}(\rho) + iK_{1/3}(\rho)/\pi + \sigma[e^{\frac{5\pi i}{6}} I_{2/3}(\rho) + e^{\frac{2\pi i}{3}} K_{2/3}(\rho)/\pi]\right|^2}. \tag{8A}$$

In practice the usual approximate equation, derived in [1] almost a century ago, with a constant electric field,

$$P_{FN} = \frac{4\sqrt{W\chi}}{V} \exp\left(-\frac{4\sqrt{2m}}{3eE\hbar}\chi^{3/2}\right) = \frac{4e^{-2\rho}}{\sigma + \sigma^{-1}}, \tag{9A}$$

provides the same results for P as the exact Eq.(8) for $\rho^{1/3} > 1$, which corresponds to all $E < 60$ V/nm for Copper with $\chi \approx 5$ eV and not very different for other metals (but Cs, where $\chi = 2.1$). It was shown in [1] that in very strong fields E the tunneling probability P is very close to $P_S = 5.44\sigma^{-1}\rho^{1/3}$ which helps to construct an interpolation equation

$$P_{int} = \frac{1}{1/P_{FN} + 1/P_S}. \tag{10A}$$

The results provided by Eq.(10A) give decent approximations [1] with a slightly higher maximum than the exact Eq.(8A), including the location of the maximum of P in a wide range of problem parameters.

Below are shown the exact tunneling probability P and its approximations P_S, P_{FN} as functions of the dimensionless parameter $r = \rho^{1/3}$ in a case when $\sigma = 1$, i.e. $V = 2W$. These approximate functions become unphysical in the region around the maximum of P, which they cannot describe.

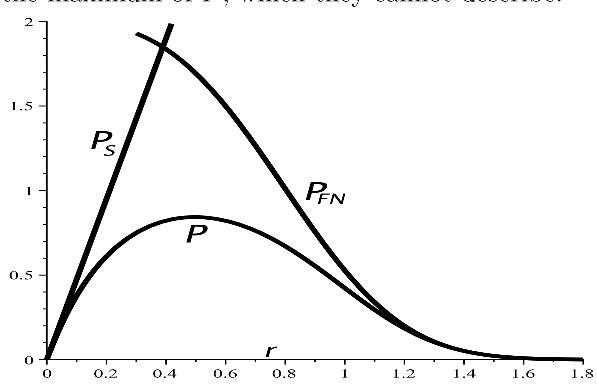

FIG. 3. Tunneling probability P and its approximations

Parameter r depends on E and the work function χ, therefore the horizontal scale corresponds to different electric fields for different materials.

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References

[1] R.H.Fowler and L.W.Nordheim, Proc.Roy.Soc. Lond. A **119**, 173 (1928); T.E.Stern, B.S.Gossling and R.H.Fowler, *ibid* **124**, 699 (1929).

[2] A.Rokhlenko, J. Phys. A: Math. Theor. **44**, 055302 (2011).

[3] I.W.Gadzuk and E.W.Plummer, Rev. Mod. Phys., **45**, 487 (1973).

[4] W.Schottky, Physik Z. **15** 872 (1914).

[5] P.Zhang, Y.Y.Lau, L.K.Ang, D.Shiffler, K.L.Jensen, R.M.Gilgenbach, 57 Annual Meeting of APS Division of Plasma Physics **60**, No.4 (2015).

[6] P.Hommelhof, Y.Sortais, A.Anoush-Talesh, and M.A.Kasevich, Phys. Rev.Let. **96**, 077401 (2006).

[7] L.V.Keldysh, Sov.Phys.JETP, **20**, 1307 (1965).

[8] V.S.Popov, Physics-Uspekhi, **47**, 855 (2004).

[9] M.Büttiker and R.Landauer, Phys.Rev.Lett. **49**, 1739 (1982).

[10] M.J.Hagmann, J.Vac.Sci.Technol.B, **13**, 403 (1995).

[11] W.B.Pearson, *Handbook of Lattice Spacings and Structure of Metals and Alloys* (Pergamon Press, New York 1958).

[12] P.Zhang and Y.Y.Lau, Scientific reports, **6**, 19894 (2016).

[13] L.K.Ang and M.Pant, Physics of Plasmas, **20**, 056705 (2013).

[14] M.Pant and L.K.Ang, Phys. Rew. B, **88**, 195434 (2013).

[15] P.Lorrain and D.Corson, *Electromagnetic Fields and Waves* (Freeman, New York 1970).