

High-Fugacity Expansion, Lee–Yang Zeros, and Order–Disorder Transitions in Hard-Core Lattice Systems

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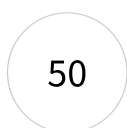
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Abstract

We establish existence of order–disorder phase transitions for a class of “non-sliding” hard-core lattice particle systems on a lattice in two or more dimensions. All particles have the same shape and can be made to cover the lattice perfectly in a finite number of ways. We also show that the pressure and correlation functions have a convergent expansion in powers of the inverse of the fugacity. This implies that the Lee–Yang zeros lie in an annulus with finite positive radii.

Communicated by H. Duminil-Copin

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