Dynamics of Fermi-Ulam pingpong.

Dmitry Dolgopyat

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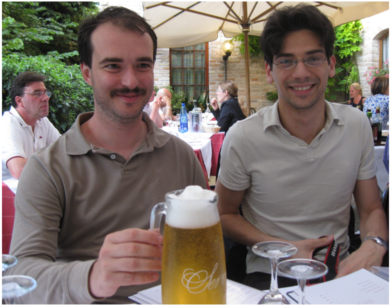
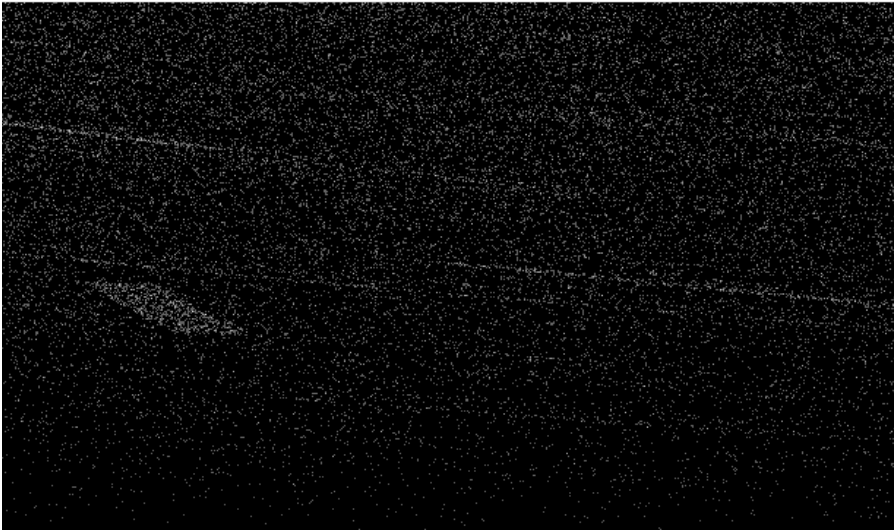
Joint work with Jacopo de Simoi.



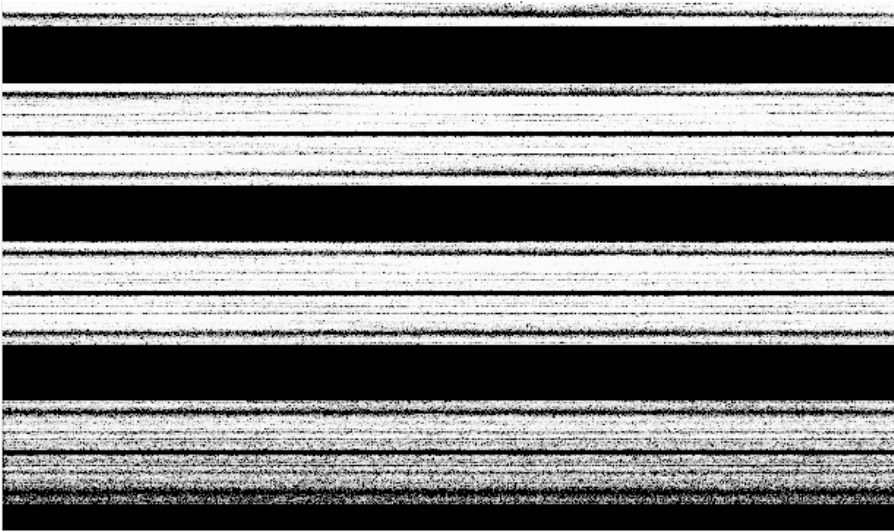
Ideas coming from

joint work with Bassam Fayad and with Nikolai Chernov and papers of Yakov Sinai, Leonid Bunimovich and Maciej Wojtkowski. . .

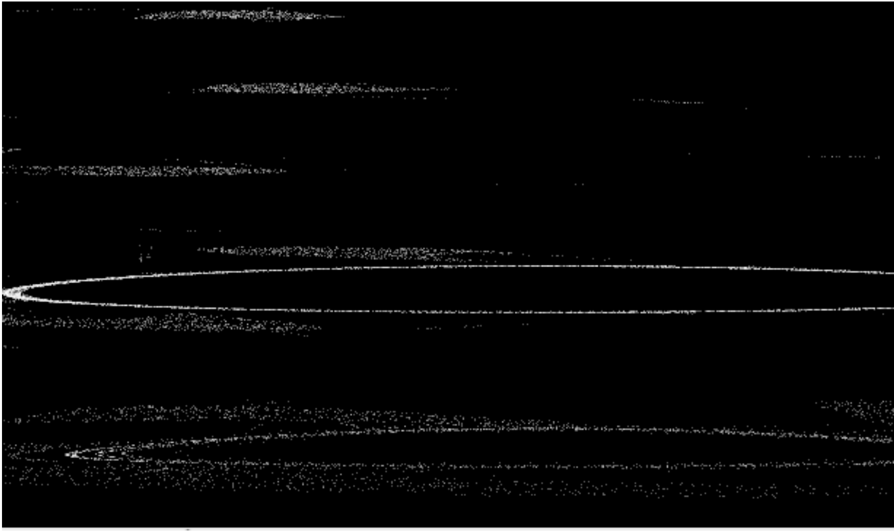
a=1.25g



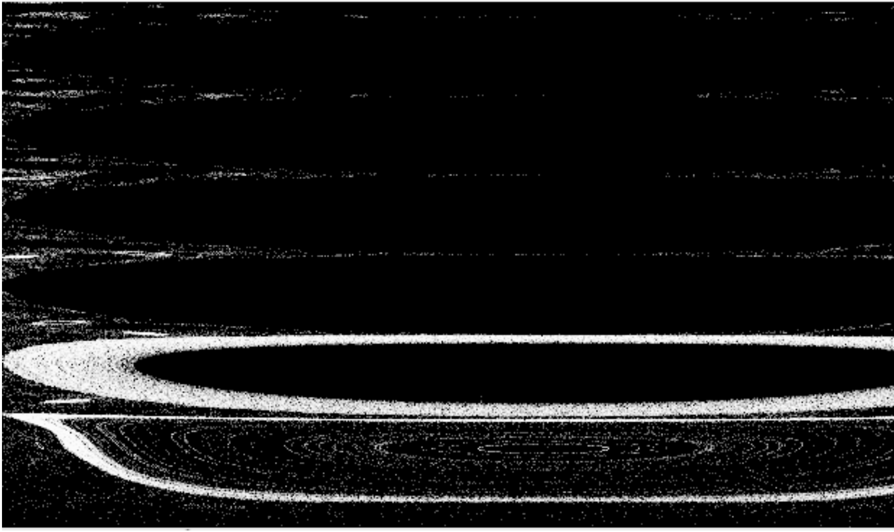
a=g



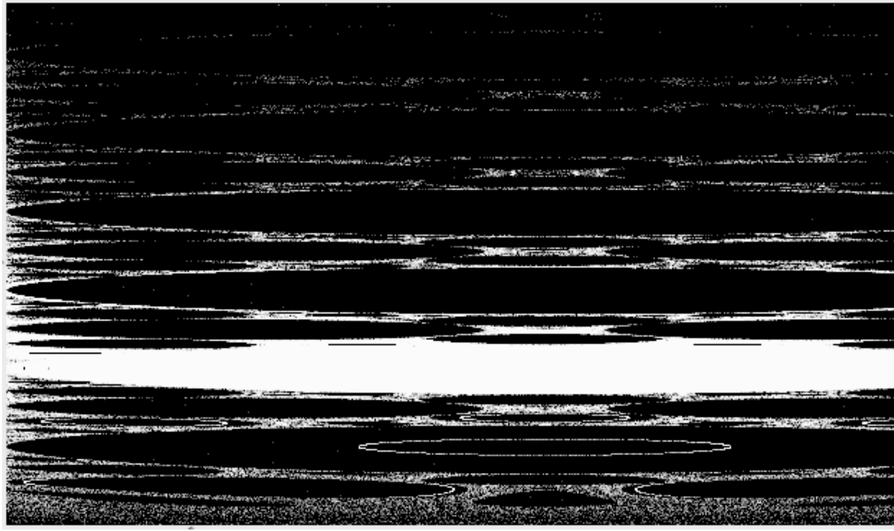
$a = 0.9g$



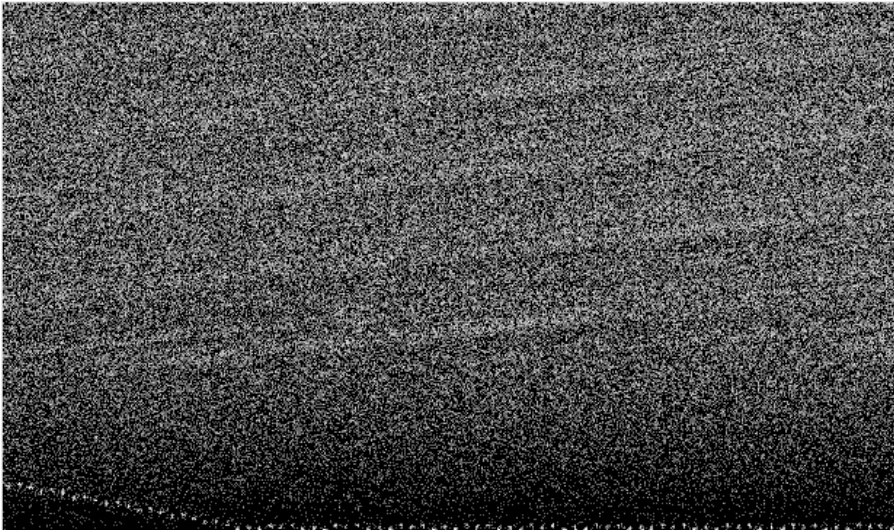
a=0.55g



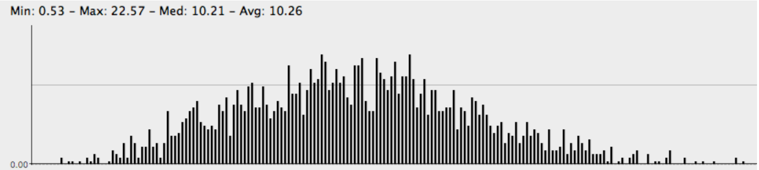
a=0.1g

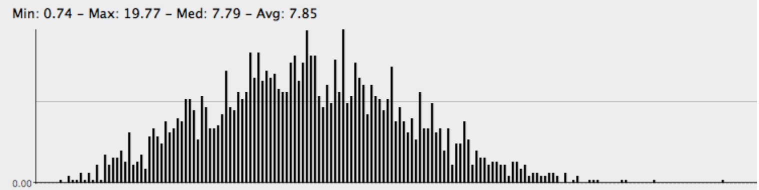


$a = -0.3g$

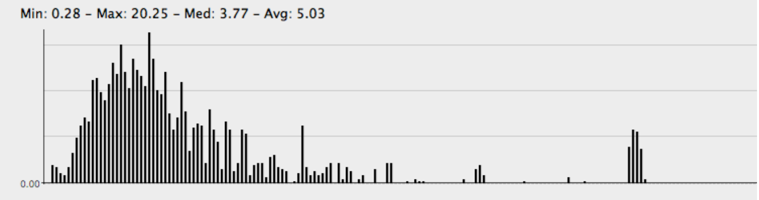


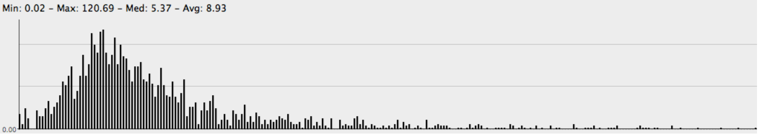
$a = 1.25g$ and $a = -0.3g$



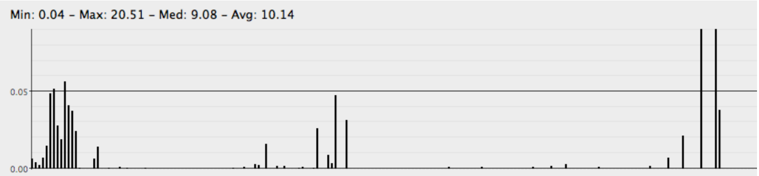


$a = g$


Min: 0.28 – Max: 20.25 – Med: 3.77 – Avg: 5.03
0.00

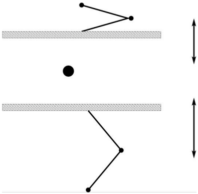

Min: 0.02 – Max: 120.69 – Med: 5.37 – Avg: 8.93
0.00

$a = 0.9g$ and $a = 0.55g$



The model

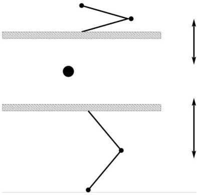
A ball bounces between two infinitely heavy walls performing a periodic motion.
Motion between collisions is free, collisions with the walls are elastic.



Let $\ell(t)$ denote the distance between the walls at time t.

Histrory

First studied numerically by Ulam and Wells (reported in Ulam, S. M. *On some statistical properties of dynamical systems,* Proc. 4th Berkeley Sympos. Math. Statist. and Prob., Vol. III pp. 315320 Univ. California Press, Berkeley, CA, 1961)
as a simplified model for appearance of fast particles in cosmic rays.



To speed up the computations it was assumed that

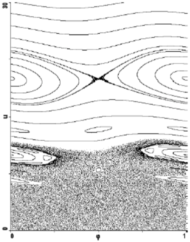
- One of the walls is fixed,
- Either $\ell(t)$ or $\dot{\ell}(t)$ is piecewise linear.

Motivations.

- Presence of singularities (universality!)
- Easy to study numerically (see
  B. Chirikov and G. Zaslavsky (1964),
  A. Brahic (1971),
  M. A. Lieberman and A. J. Lichtenberg (1972))

Previous results

Theorem (Pustylnikov (1977), R. Douady (1988), Laederich–Levi (1991)) If the motion of the wall is smooth then all ping-pong trajectories are bounded.



Theorem (Zharnitsky (1998)) If wall's velocity jump is large enough then there are unbounded trajectories.

Piecewise Linear Wall Velocity.

$$\ell_{a,b}(t) = b + a\left((t \mod 1) - 0.5\right)^2.$$

We take $b = 1$ then $\ell(t) \geq 0$ for all t iff $a > -4$.

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Three regimes.

(I) **Dispersive.** $a > 0$.

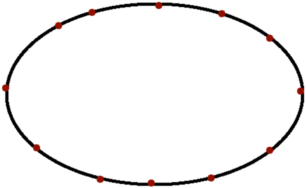
(II) **Focusing.** $a \in (a_c, 0)$ where $a_c \approx -2.77972$

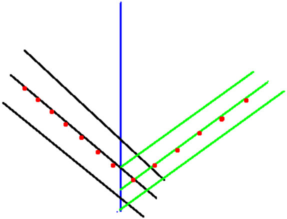
(III) **Defocusing.** $a \in (-4, a_c)$.

	Dispersive	Focusing	Defocusing
Recurrence	Yes	No (?)	Yes
Decceleration	Yes	No	Yes
Ergodicity	Yes	No	No
Typical Vel Growth	CLT(*)	linear (?)	CLT(*)
High Velocity	stochastic	quasiperiodic	stochastic
Low Velocity	stochastic	mixed	mixed

High Velocity.

Smooth vs nonsmooth case.





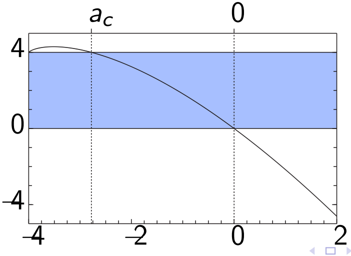
Measure of stochasticity: $\Delta =$Twist $\times$ Kick.

Universality for one velocity jump.

$$\Delta(a) = \ell(0)\left[\dot{\ell}(0+) - \dot{\ell}(0-)\right]\int_0^1 \frac{ds}{\ell^2(s)}$$

$$\ell_a(t) = 1 + a\left((t \mod 1) - 0.5\right)^2 \Rightarrow \Delta(a) = -2a(1 + a/4)J(a)$$

$$J(a) = \frac{2}{a+4} + \begin{cases} (|a|^{-1/2}/2)\log\frac{2+|a|^{1/2}}{2-|a|^{1/2}} & \text{if } -4 < a \leq 0 \\ |a|^{-1/2}\arctan(|a|^{1/2}/2) & \text{if } a > 0. \end{cases}$$



Hyperbolic and elliptic regions

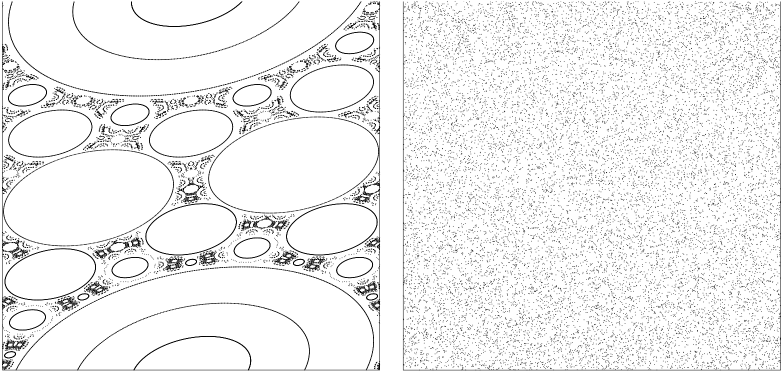


Figure: Left: selected orbits of the first return map for $\Delta = 0.32$. Right: a single orbit of the first return map for $\Delta = -0.3$.

Low velocity.

Low velocity dynamics is determined by the local geometry of the wall motion.

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Billiard analogy.

z-height, v-velocity.

$$\begin{array}{lll} \textsc{Free motion:} & \delta z(t) = \delta z(0) + t\delta v, & \delta v(t) = \delta v(0) \\ \textsc{Collision:} & \delta z^+ = -\delta z^-, & \delta v^+ = -\mathcal{R}\delta z^- - \delta v^- \end{array}$$

where $\mathcal{R} = \frac{2\kappa}{w}$ and $\kappa = \ddot{\ell}(t)$, w-relative velocity.

For billiards: $\mathcal{R} = \frac{2\kappa}{\cos\phi}$ where κ is the curvature of the boundary and ϕ angle with the normal vector.