Global Phase Space of Coherence and Entanglement in a Double-Well BEC

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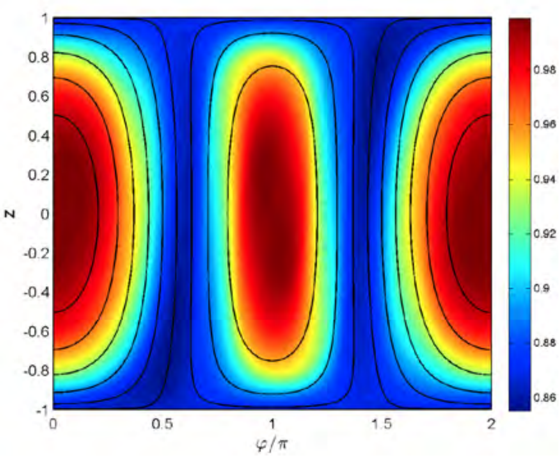
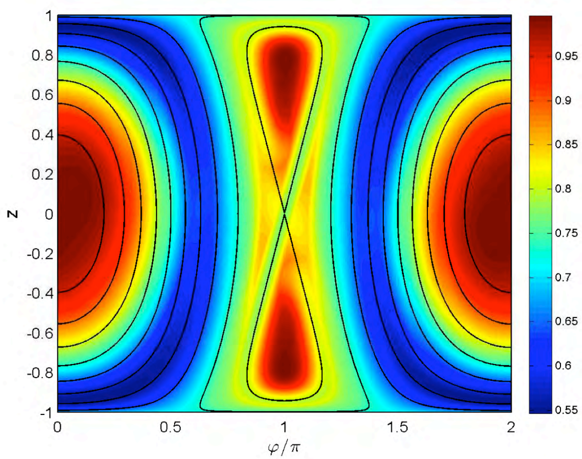
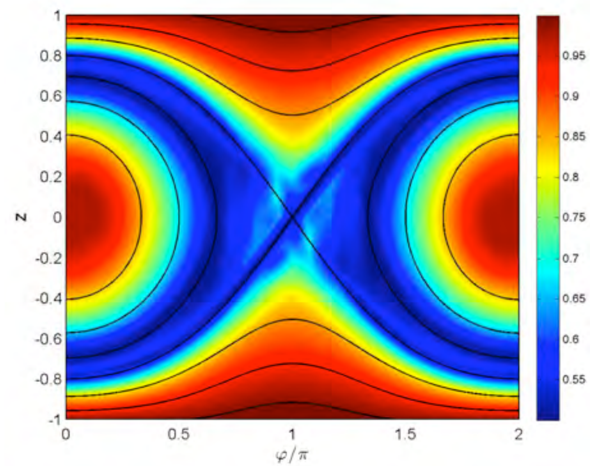


Λ = 0.5



Λ=1.5



Λ=5

* With Holger Hennig (Harvard), Ted Pudlik (BU) and Dirk Witthaut (MPIDS-Göttingen)

Bottom Line

- The issue of temporal decoherence of BEC is critical for potential applications. For a simple system (*BEC dimer ~ Bosonic Josephson Junction=BJJ*) we show that *decoherence is suppressed near fixed points of classical analog system, including self-trapped regions but is enhanced near separatrix*
- We do this by introducing "global phase space" (GPS) portraits of quantum observables including 1) the "condensate fraction"; 2) entanglement; and 3) "spin squeezing" as functions of time for different values of the key parameters of the system. We show that much of the observed behavior can be understood by studying the phase space of a related classical nonlinear dynamical system, BUT
- We predict some novel quantum effects, which should be observable in near-term experiments.

Physical Context and Models

- Single coherent BEC in a “double well” potential or two internal (hyperfine) states with resonant interactions in single well.
  - Analogous to two-site “dimer” system and therefore called “BEC dimer” but also to Josephson Junction therefore called Bosonic Josephson Junction (BJJ).
  - Parameters: number of particles in BEC (N) and in each of two “wells”/states (N_1 and N_2), $z = N_1 - N_2$, phase difference between two condensates ($\phi = \phi_1 - \phi_2$), resonant coupling/tunneling between two wells (J), interaction of Bosons within condensate (U); single parameter in simple models $\Lambda = (NU)/J$
- Three Models:
  - Bose-Hubbard Hamiltonian (*BHH*) : fully quantum *
  - Liouville Dynamics (*LD*) : semi-classical in this system
  - Gross-Pitaevskii equation (*GPE*): corresponds to integrable classical dynamical system* in (z, ϕ) for this case

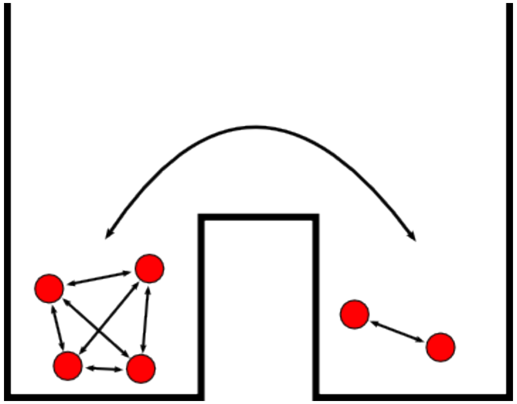
* NB: Quantum dimer also intergrable by Bethe Ansatz: solution not useful for calculating physical observables

Bose-Hubbard dimer

$$H = -J(a_1^\dagger a_2 + a_2^\dagger a_1) + \frac{U}{2}(a_1^\dagger a_1^\dagger a_1 a_1 + a_2^\dagger a_2^\dagger a_2 a_2)$$

- hopping
- repulsive interaction

- Atoms in a double-well optical trap
- Two spin states of atoms trapped in one well



- Observables of interest:
  - Single-particle density matrix (condensate purity)

$$\rho = \frac{1}{N}\begin{pmatrix} \langle a_1^\dagger a_1 \rangle & \langle a_1^\dagger a_2 \rangle \\ \langle a_2^\dagger a_1 \rangle & \langle a_2^\dagger a_2 \rangle \end{pmatrix}$$

  - Entanglement measures

$$\mathrm{EPR} = \langle a_1^\dagger a_2 \rangle \langle a_2^\dagger a_1 \rangle - \langle a_1^\dagger a_1 a_2^\dagger a_2 \rangle$$

Semiclassical approximation

$$|\Psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle, \quad \langle\Psi|a_i|\Psi\rangle = \psi_i$$

$$\langle\Psi|H|\Psi\rangle = -J(\psi_1^*\psi_2 + \psi_2^*\psi_1) + \frac{U}{2}\left(|\psi_1|^2(|\psi_1|^2 - 1) + |\psi_2|^2(|\psi_2|^2 - 1)\right)$$

- Operators replaced with c-numbers

$$\psi_i = \sqrt{N(1 \pm z)/2}\, \mathrm{e}^{\imath\theta \pm \imath\phi/2}$$

$$\begin{cases} z \\ \phi \end{cases}$$

- population imbalance
- phase difference

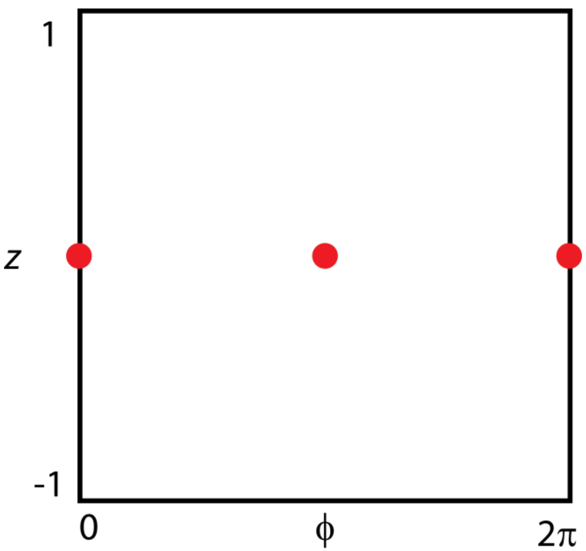
$$\mathcal{H} = \frac{\Lambda}{2} z^2 - \sqrt{1 - z^2} \cos\phi$$

$$\Lambda = \frac{U(N-1)}{2J}$$

Classical Nonlinear Dynamics

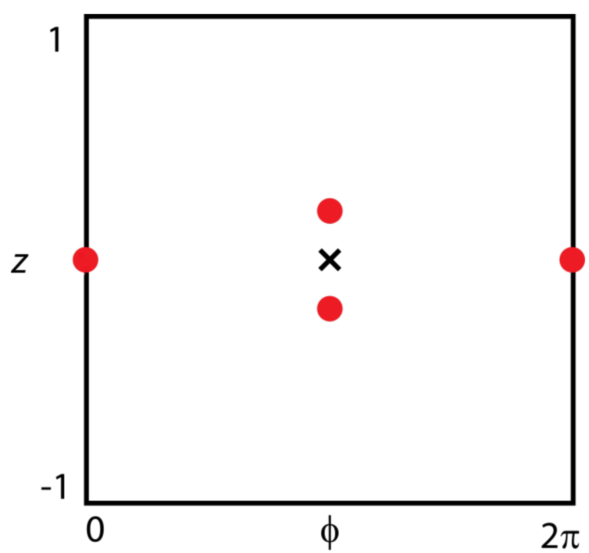
$$\mathcal{H} = \frac{\Lambda}{2} z^2 - \sqrt{1 - z^2} \cos \phi$$

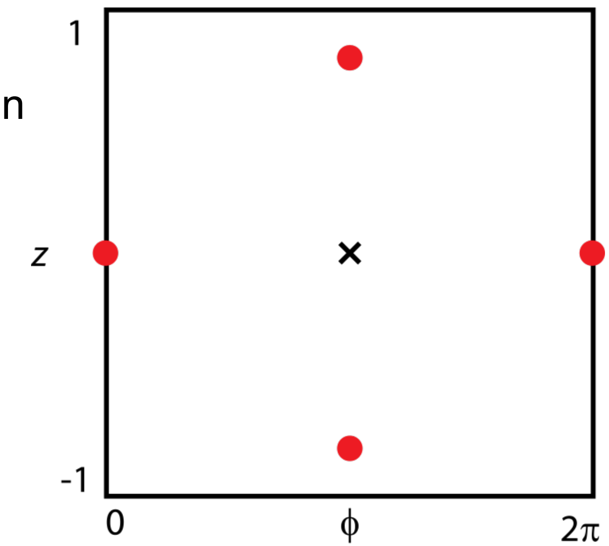
- Supercritical pitchfork bifurcation



$$\Lambda < 1$$

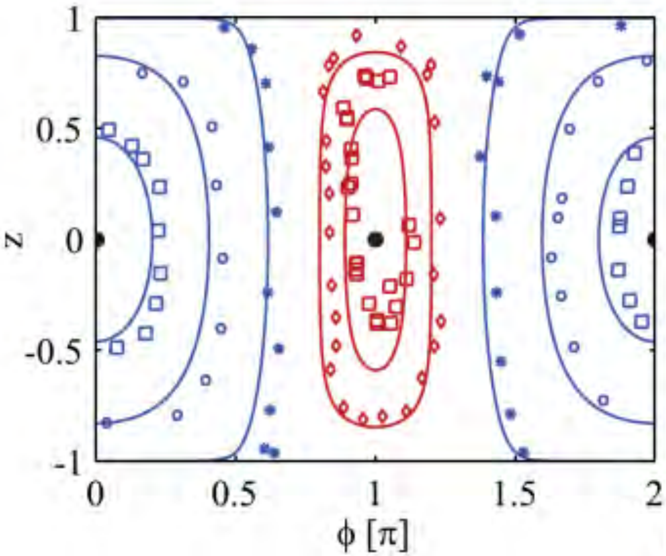
$$\Lambda \gtrsim 1$$





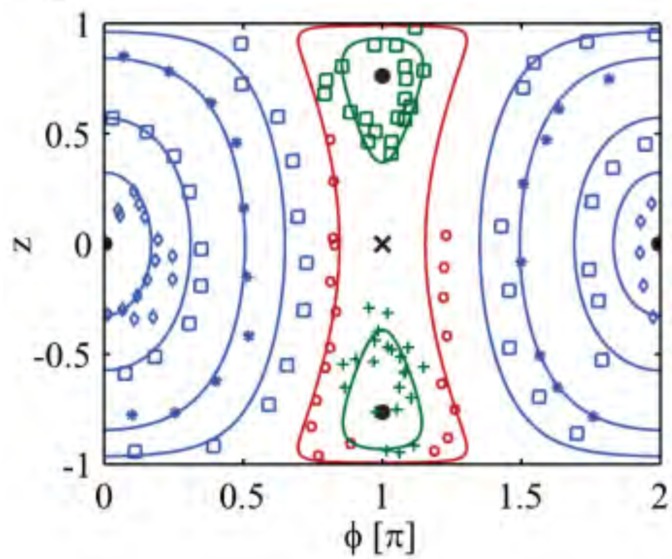
$$\Lambda \gg 1$$

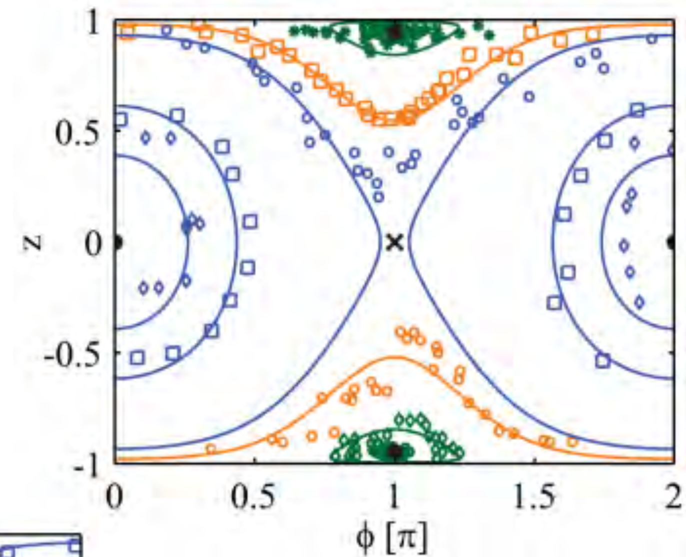
Relevant experiments



$\Lambda = 0.78$

$\Lambda = 1.55$



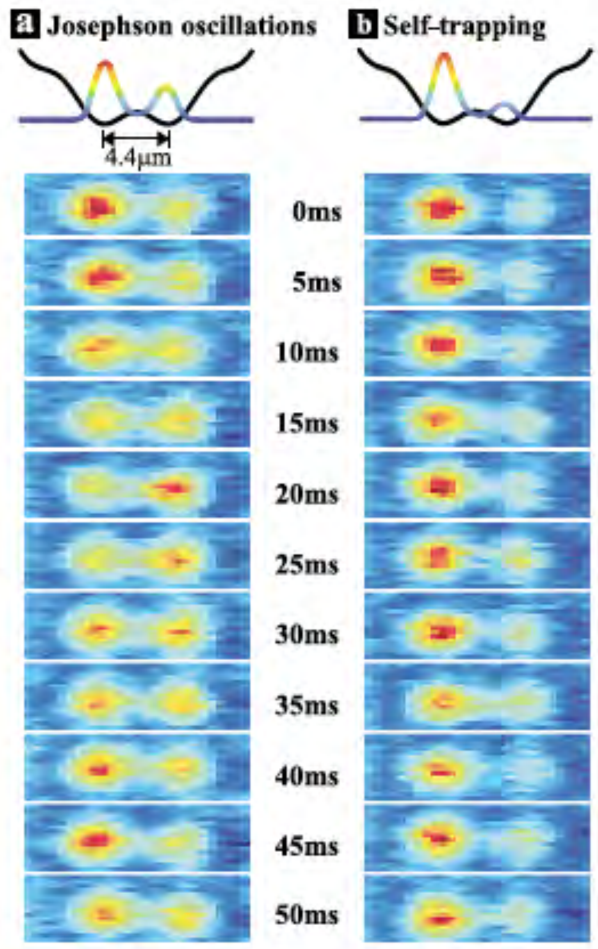


$\Lambda = 3.1$

- Zibold et al., PRL **105** 204101 (2010)

Experimental Observations

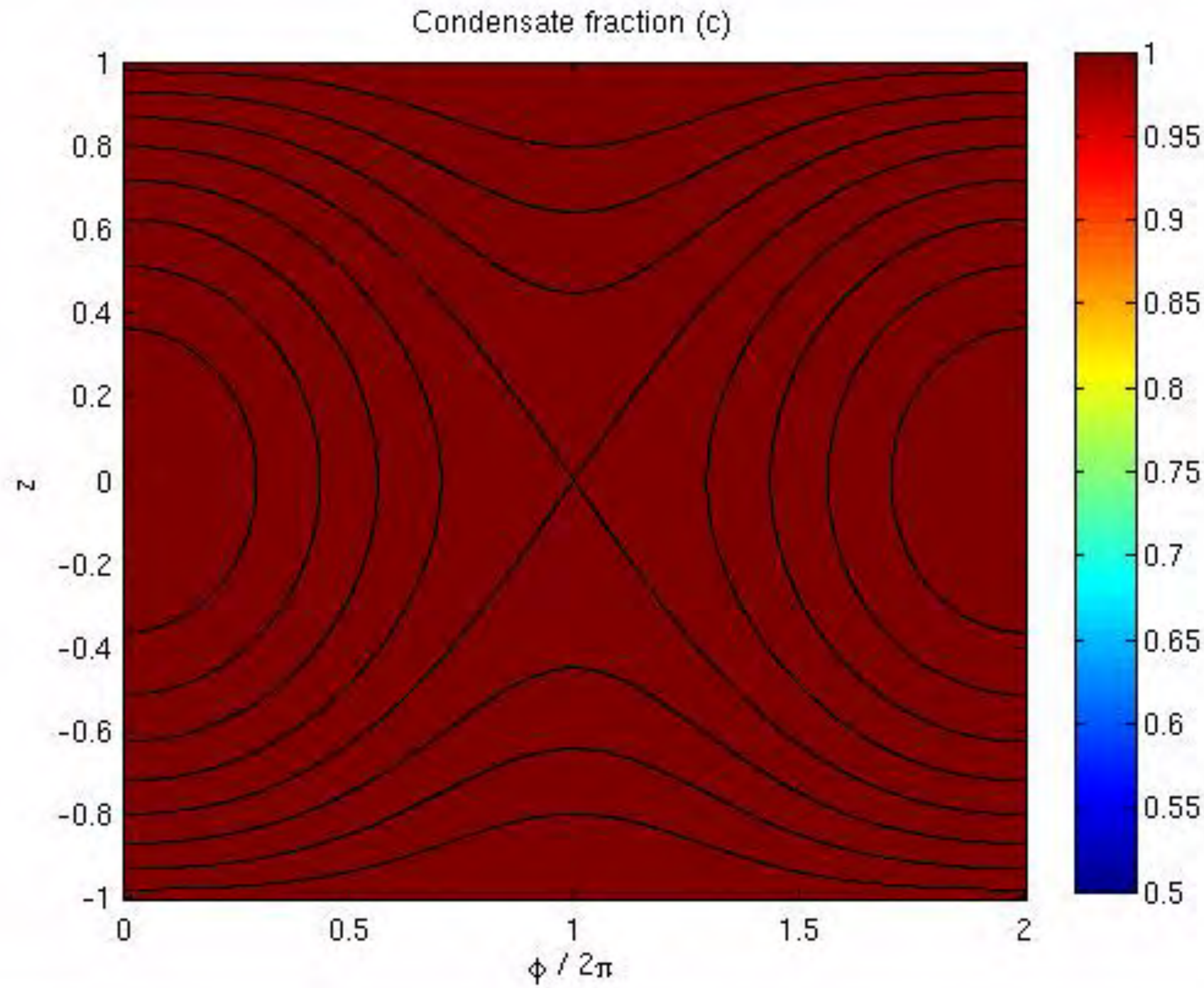
Λ just above 1



Λ much greater than 1

M. Albiez et al, Phys. Rev. Lett. 95 010402 (2005)

Classical phase space, quantum dynamics



$$\Lambda = 5$$

- Hennig et al., PRA **86**, 051604(R) (2012)

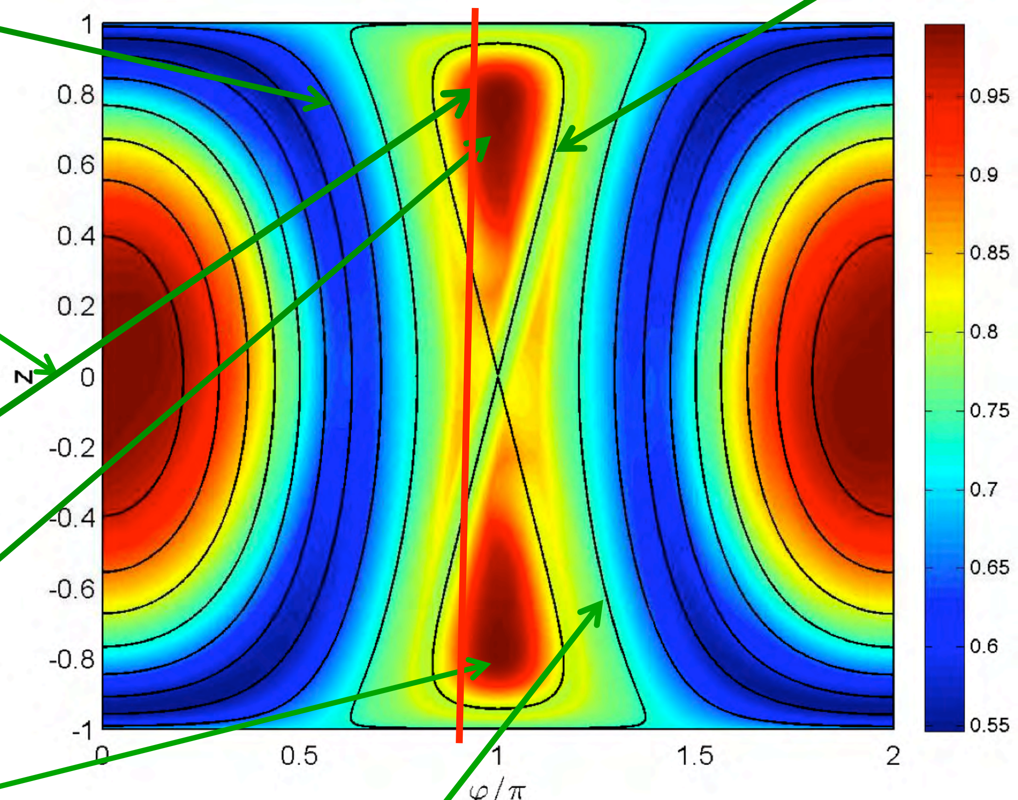
Condensate Fraction at T=1 sec Λ = 1.5

Note considerable variation in $c(z,\phi,T=1)$ and how it generally follows contours of classical dynamics

Constant energy contours

Separatrix

Orbits around FP (0,0): have $\langle z\rangle=\langle\phi\rangle=0$



z=>$-z$ symmetry broken

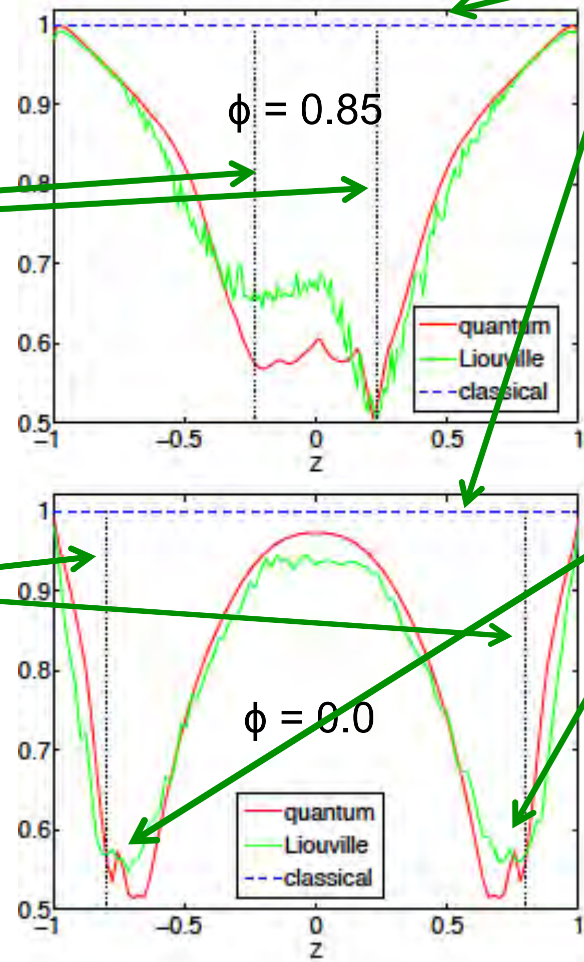
Orbits around FP_+ and FP_- are macroscopic self-trapped states, $\langle z\rangle\neq 0$, $\langle\phi\rangle=\pi$

"π-phase" orbits still exist, $\langle z\rangle=0$, $\langle\phi\rangle=\pi$

Results for "Purity" of Dimer Λ = 5.0

GPE="classical" gives *p=1*

Separatrix, purity falls rapidly nearby: decoherence enhanced



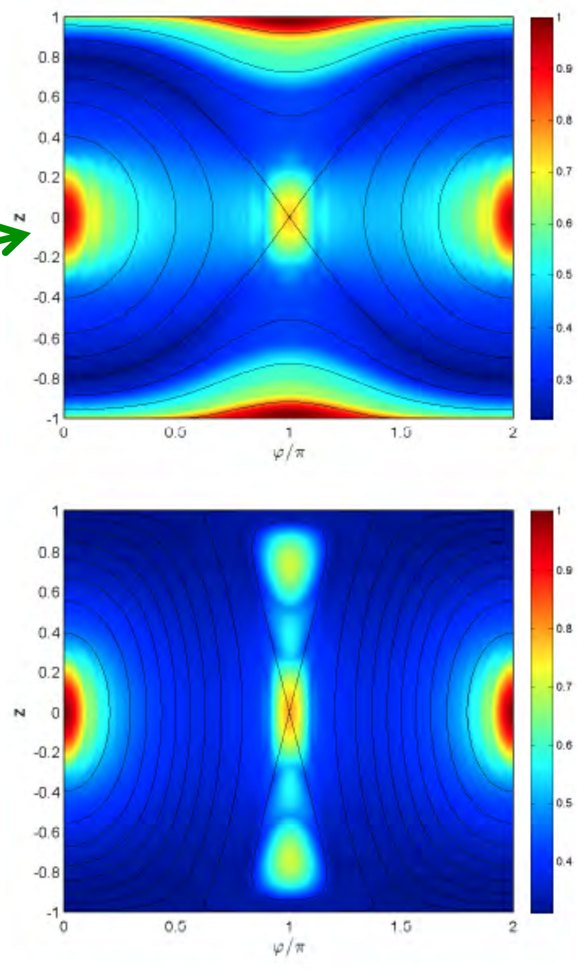
"Ridge" of increased *c*, pure quantum effect

Separatrix, purity falls, then recovers as FP(0,0) "controls" dynamics

Initial State overlap with Eigenfunctions

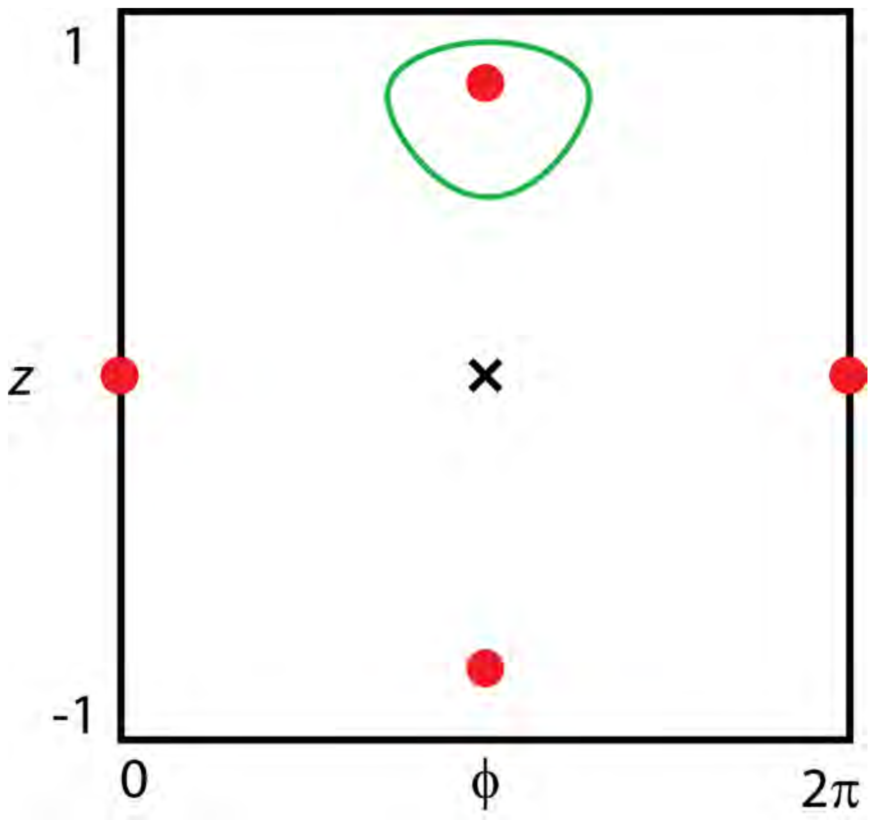
Color code shows $A(\phi,z)\cong \max_n <\phi,z|E_n>$, where E_n is an eigenstate of the BH dimer

$\Lambda = 5.0$: $\Lambda = 1.5$



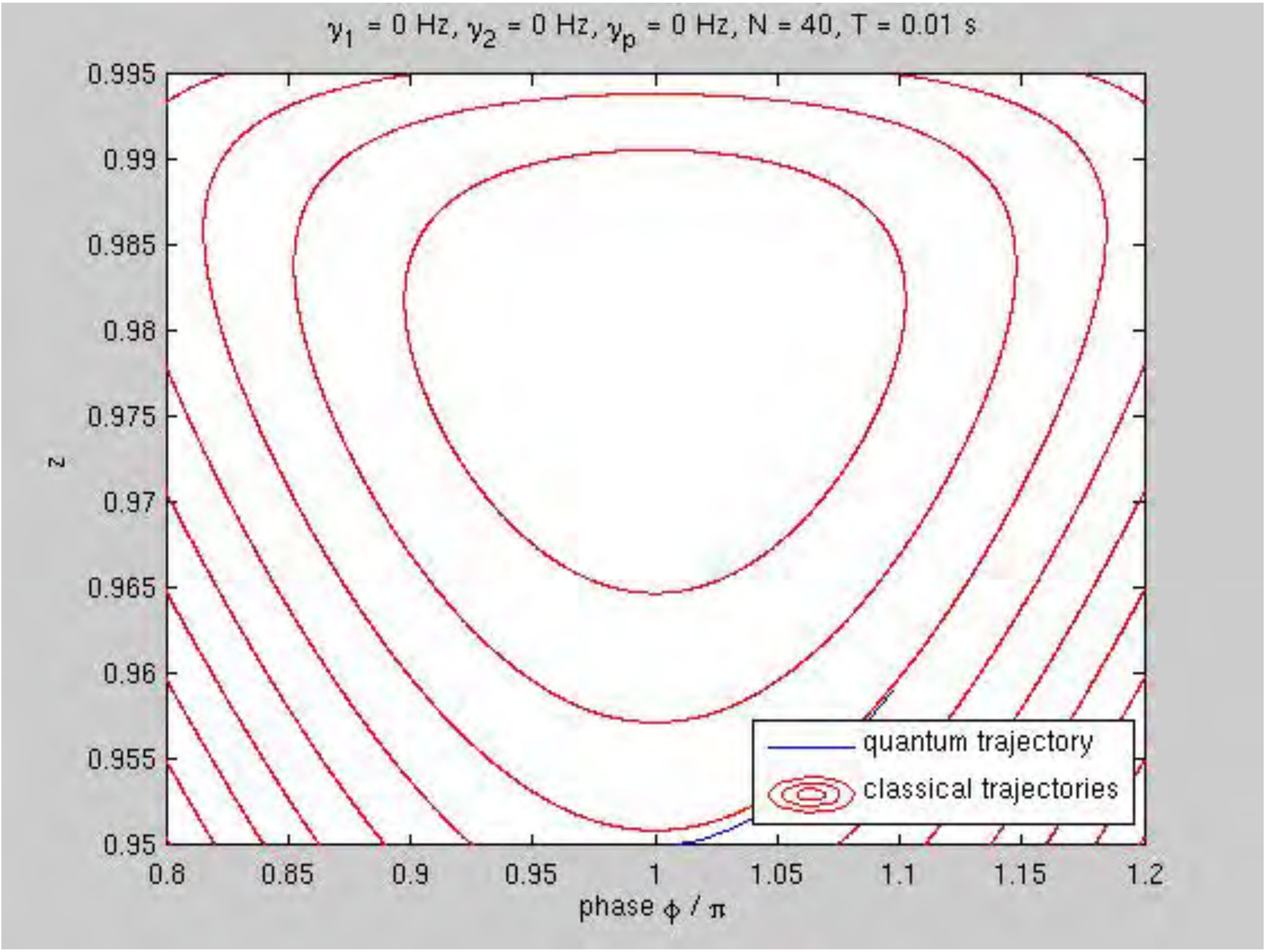
Conclusion: localization in phase space (FPs) maintains coherence, delocalization (separatrix) induces decoherence.

What do quantum orbits really look like?



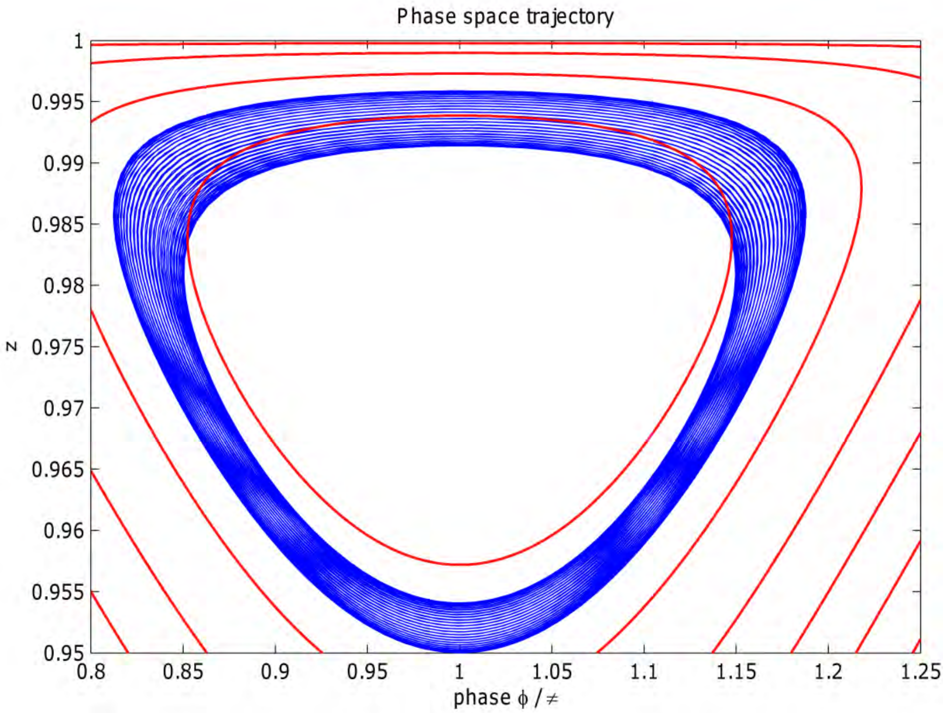
- Large Λ, near fixed point, green is classical orbit

Quantum Dynamics



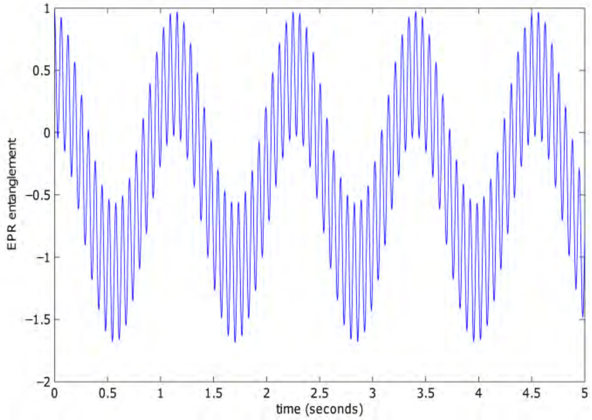
Quantum „orbits," interesting features

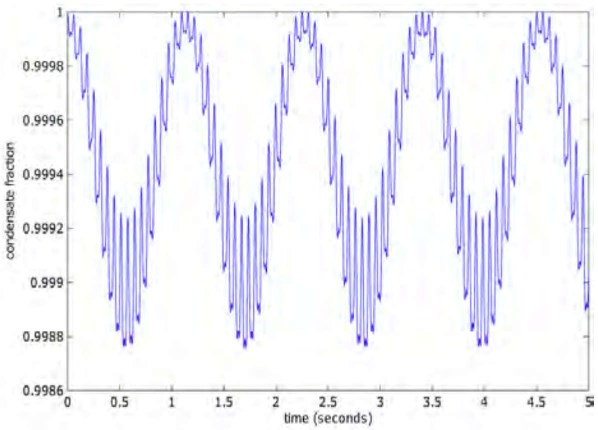
- „thick" orbits



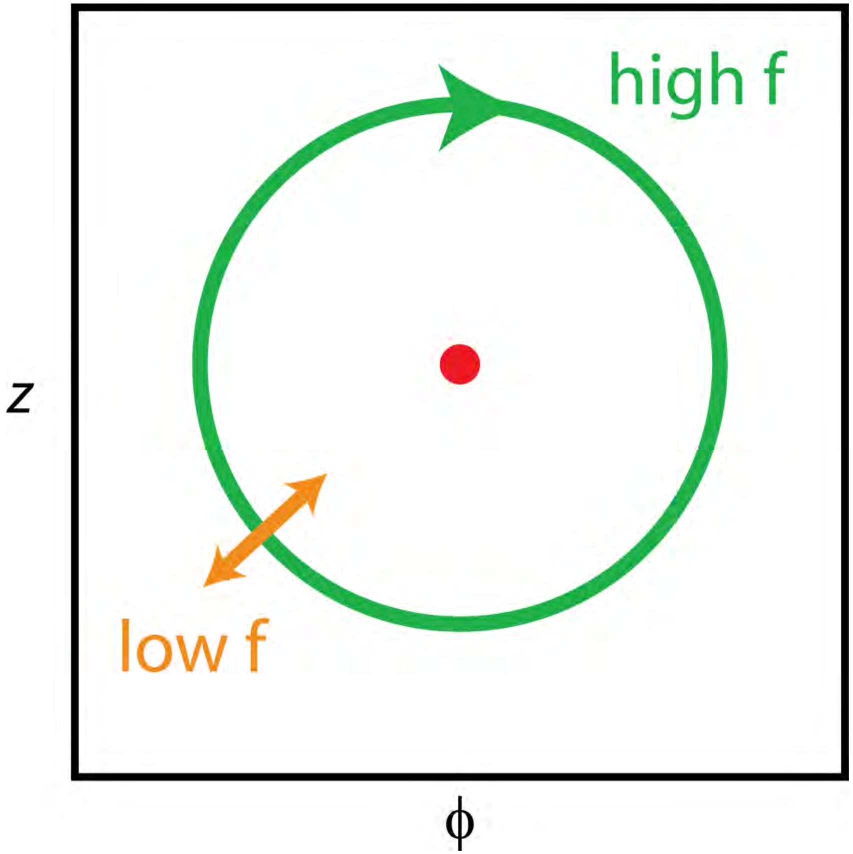
$$\Lambda = 5$$

- *two frequencies* in EPR, condensate fraction





Explain two frequencies: High frequency



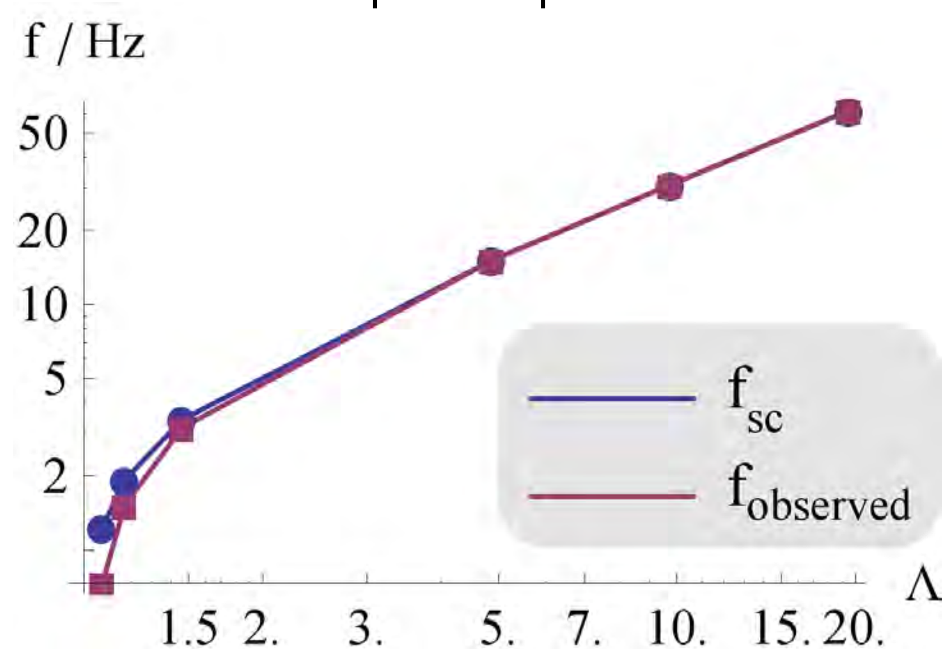
$$\Lambda = \frac{U(N-1)}{2J}$$

- High frequency is mean-field (semiclassical)

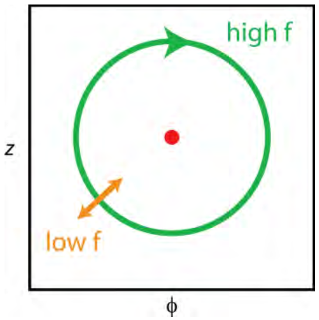
$$f_{sc} = \frac{\sqrt{\Lambda^2 - 1}}{\pi}\frac{J}{\hbar}$$

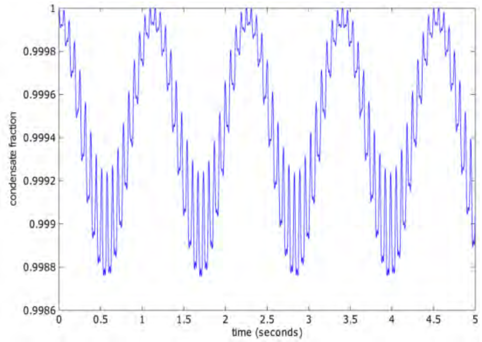
- Conversion to seconds

- Data from power spectrum of EPR:



Low-frequency oscillations





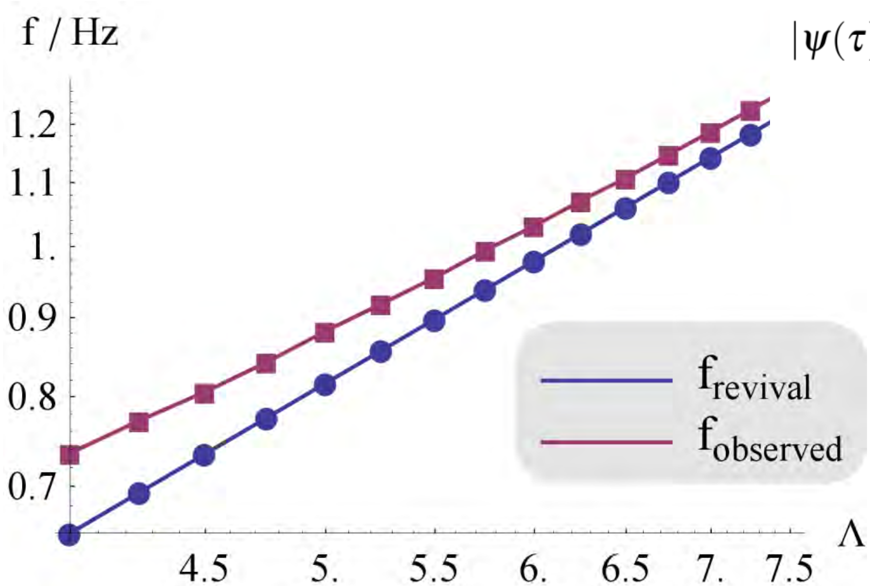
- Low frequency quantum revival

- In the limit J/U = 0, for any state $\tau \equiv \frac{\pi\hbar}{U}$

and $\mathrm{EPR}(t=\tau) = \mathrm{EPR}(t=0)$

$$\max \mathrm{eig}\, \rho(t=\tau) = \max \mathrm{eig}\, \rho(t=0)$$

$$|\psi(\tau)\rangle = \begin{cases} |\psi(0)\rangle & \text{for } N = 1+4p \text{ with } p \in \mathbb{Z}_{\geq 0}, \\ -\sum_{n=0}^{N}(-1)^n a_n |E_n\rangle & \text{for } N = 2+4p \text{ with } p \in \mathbb{Z}_{\geq 0}, \\ -|\psi(0)\rangle & \text{for } N = 3+4p \text{ with } p \in \mathbb{Z}_{\geq 0}, \\ \sum_{n=0}^{N}(-1)^n a_n |E_n\rangle & \text{for } N = 4+4p \text{ with } p \in \mathbb{Z}_{\geq 0}. \end{cases}$$

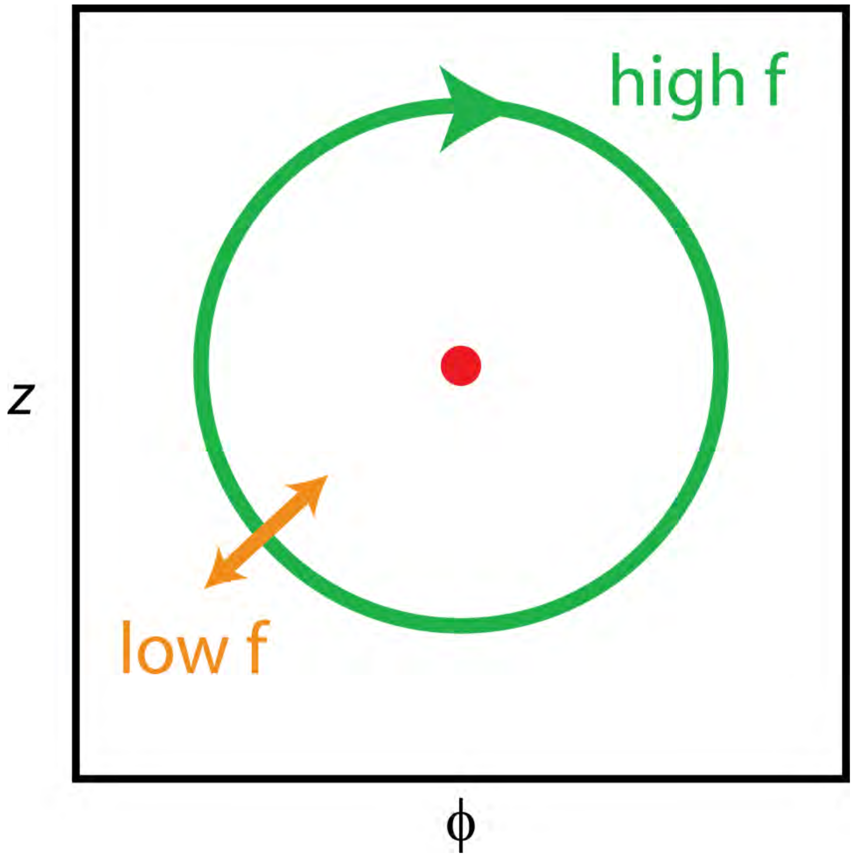


- But revivals seen also for J/U > 0 (or, Λ < ∞)

$$\Lambda = \frac{U(N-1)}{2J}$$

- Greiner et al., Nature **419** 51 (2002)

Summary: Dynamics near the FP



$$\Lambda = \frac{U(N-1)}{2J}$$

- High frequency is mean-field (semiclassical)

$$f_{sc} = \frac{\sqrt{\Lambda^2 - 1}}{\pi} \frac{J}{\hbar}$$

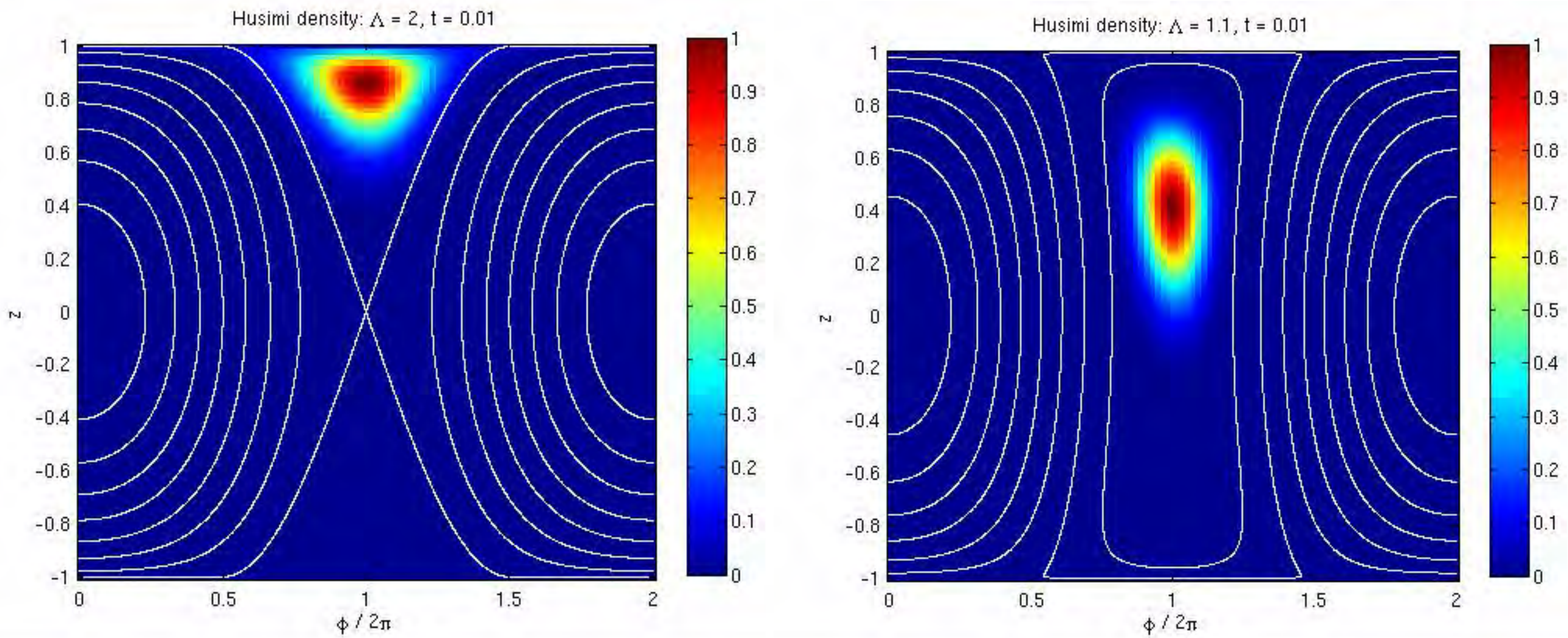
- Low frequency is a quantum revival:

$$f_G = \frac{U}{\pi\hbar}$$

- But how far from the fixed point is this a good picture?

Low-Λ anomaly: Husimi density

$$Q(z,\phi,t) = |\langle z,\, \phi|\psi(t)\rangle|^2$$



- Classical approximation poor for Λ close to 1; see quantum tunneling as observed in experiment

Summary of Results

- Results for 1) "condensate fraction"—$C(z,\phi) \cong \lambda_{max}/N \leq 1$, λ_{max} = largest eigenvalue of single particle density matrix and "entanglement"—
  $E \cong |\langle a_1^* a_2 \rangle|^2 - \langle a_1^* a1 a_2^* a_2 \rangle$, E > 0 for entangled state
  - Both $C(z,\phi)$ and E generally "track" classical orbits in GPS
  - $C(z,\phi)$ decreases dramatically near classical separatrix, remains large at classical fixed point and especially in self-trapped region=> regular motion enhances coherence, chaotic motion destroys coherence
  - Symmetry *z* => - *z* of classical equations is broken by quantum dynamics
  - "Ridge" of enhanced coherence exists in quantum but not in LD or GPE (classical) or dynamics
  - Overlap of initial coherent state with eigenfunctions of BHH is minimal along separatrix, high near fixed points
  - Quantum "orbits" show two frequencies: one semiclassical, the other quantum revival
  - See Josephson tunneling at small Λ, as in experiments

Bottom Line Redux and Outlook

- The issue of temporal decoherence of BEC is critical for potential applications. For a simple system (*BEC dimer ~ Bosonic Josephson Junction=BJJ*) we show that *decoherence is suppressed near fixed points of classical analog system, including self-trapped regions but is enhanced near separatrix*
- We do this by introducing “global phase space” portraits of quantum observables including 1) the “condensate fraction”; 2) entanglement; and 3) “spin squeezing” as functions of time for different values of the key parameters of the system. We show that much of this effect can be understood by studying the phase space of a related classical nonlinear dynamical system.
- Incredible control of experimental situation will allow for detailed testing these predictions in near-term experiments.

Beyond the Dimer

For spatially extended systems, “Intrinsic Localized Modes” play role of self-trapped states: may serve as means of maintain quantum coherence in, *e.g.*, macromolecules.

QUESTIONS ?