

Eddie Cohen, long time tails, and  
FM quantum phase transitions

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- $\eta = \int_0^\infty C(t)$

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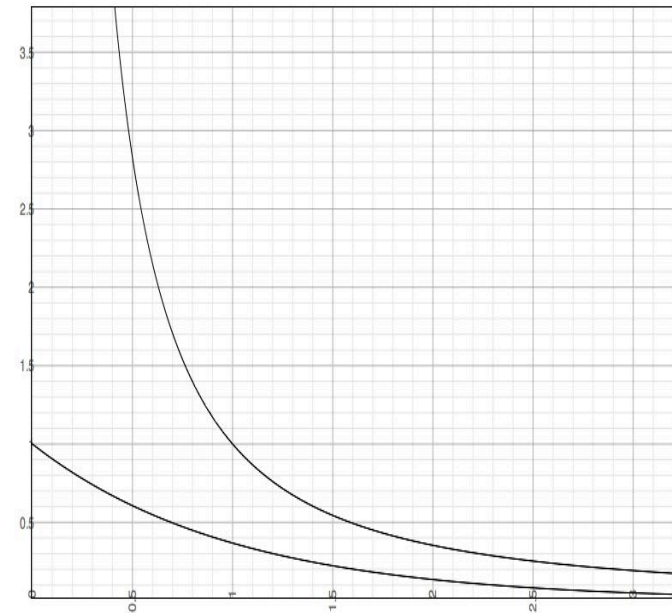
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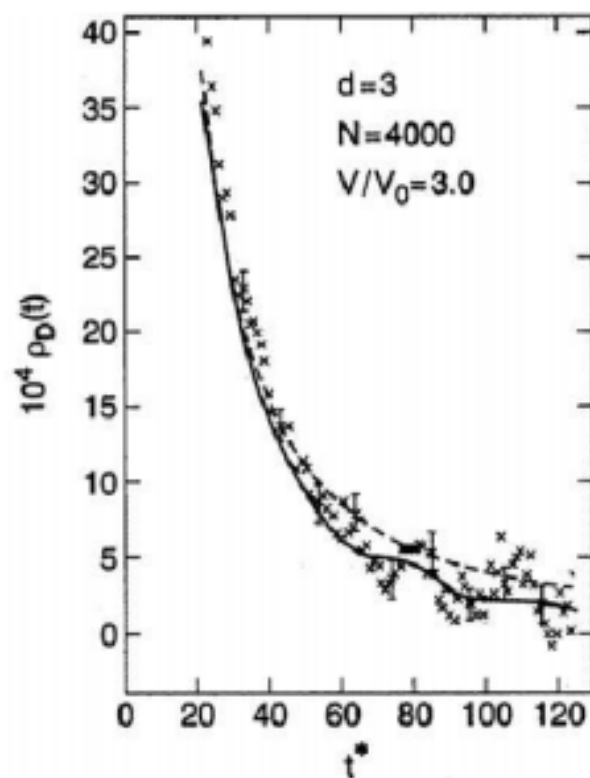


FIG. 3 Normalized velocity autocorrelation function  $\rho_D(t) = C_D(t)/\langle v^2(0) \rangle$  as a function of the dimensionless time  $t^* = t/t_0$ , where  $t_0$  is the mean-free time. The crosses indicate computer results obtained by Wood and Erpenbeck (1975) for a system of 4000 hard spheres at a reduced density corresponding to  $V/V_0 = 3$ , where  $V$  is the actual volume and  $V_0$  is the close-packing volume. The dashed curve represents the theoretical curve  $\rho_D(t) = \alpha_D (t^*)^{-3/2}$ . The solid curve represents a more complete evaluation of the mode-coupling formula with contributions from all possible hydrodynamic modes and with finite-size corrections included (Dorfman, 1981). From Dorfman *et al.* (1994).

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Universal effects in liquids due to soft or slow modes coupling to.....stuff.

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- $\langle |\delta\sigma(k)|^2 \rangle \sim \text{const} + \sum \langle |\pi(k-q)|^2 \rangle \langle |\pi(q)|^2 \rangle \sim 1/k^{4-d} \rightarrow \text{singular in all } d < 4$

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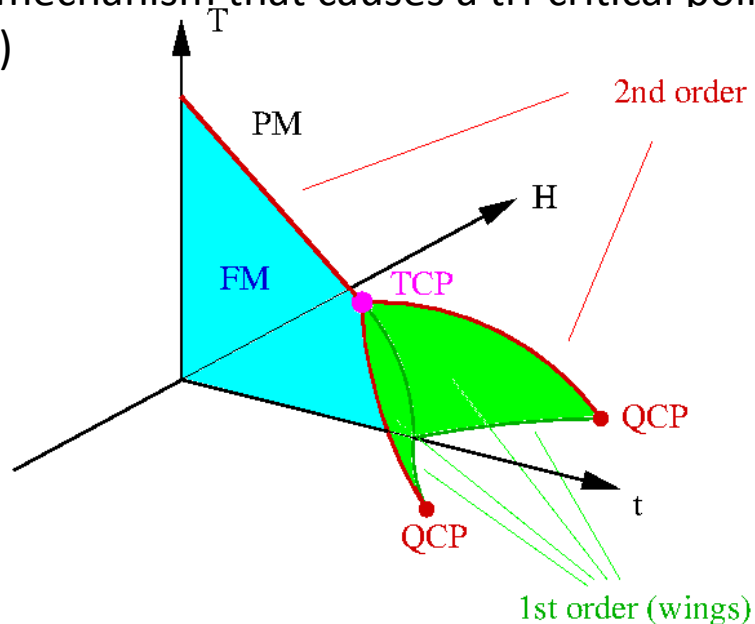
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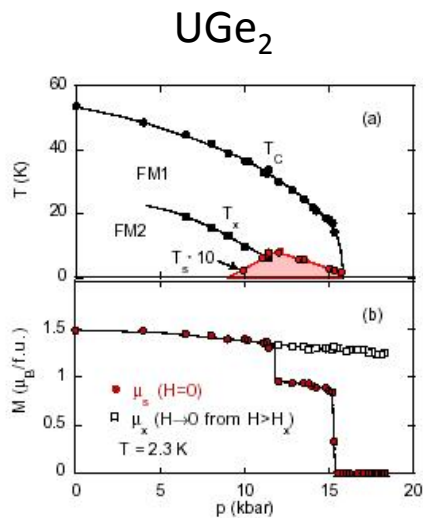
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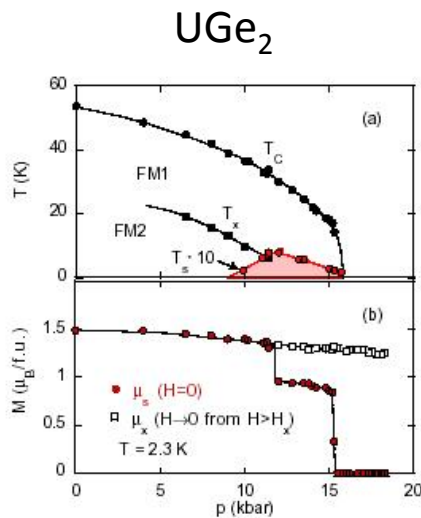
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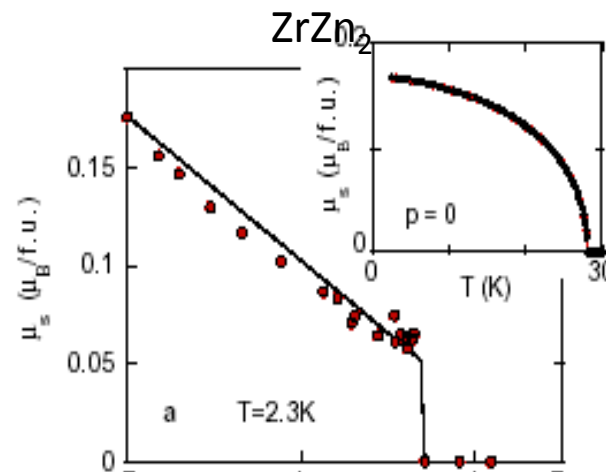
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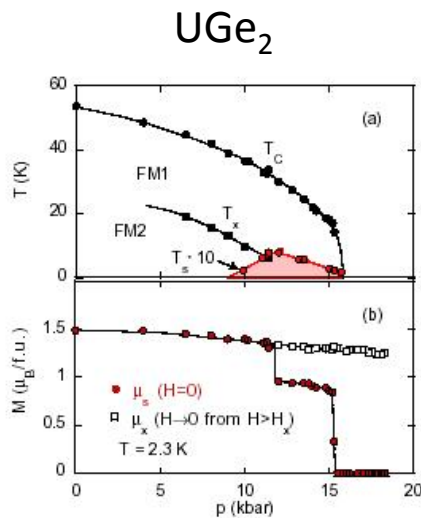
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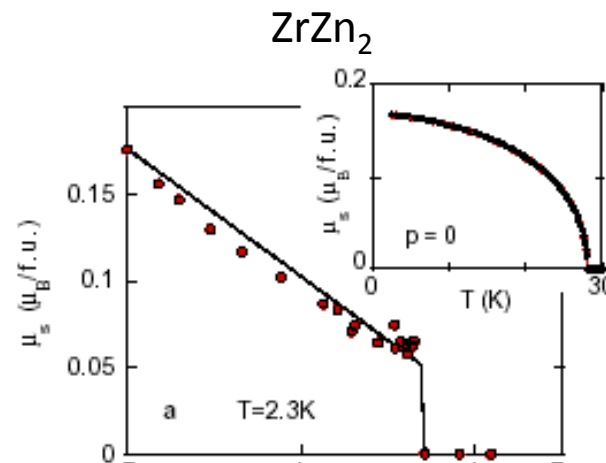
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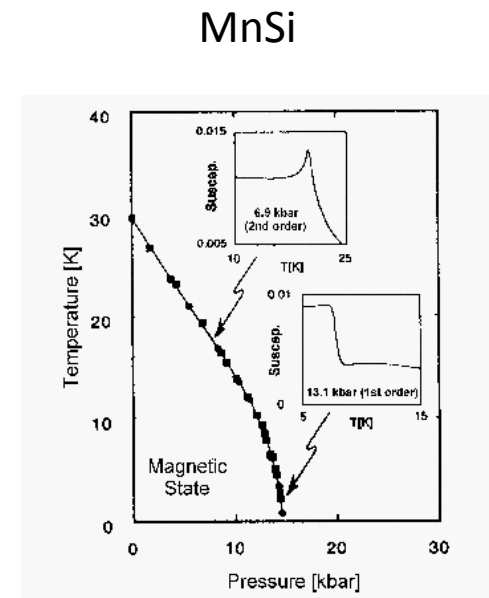
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[\(Pfleiderer et al 1997\)](#)

TABLE I: Systems with low- $T$  ferromagnetic transitions and their properties.  $T_c$  = Curie temperature,  $T_{ic}$  = tricritical temperature.  $\rho_0$  = residual resistivity. FM = ferromagnet, SC = superconductor. N/A = not applicable; n.a. = not available.

System <sup>a</sup>	Order of Transition <sup>c</sup>	$T_c/K$ <sup>b</sup>	magnetic moment/ $\mu_B$ <sup>d</sup>	tuning parameter	$T_{ic}/K$	wings observed	Disorder ( $\rho_0/\mu\Omega\text{cm}$ ) <sup>e</sup>	Comments
MnSi <sup>27</sup>	1st <sup>18</sup>	29.5 <sup>28</sup>	0.4 <sup>28</sup>	hydrostatic pressure <sup>18</sup>	$\approx 10$ <sup>18</sup>	yes <sup>25</sup>	0.33 <sup>25</sup>	weak helimagnet <sup>17</sup> exotic phases <sup>25,26</sup>
ZrZn <sub>2</sub> <sup>27</sup>	1st <sup>29</sup>	28.5 <sup>29</sup>	0.17 <sup>29</sup>	hydrostatic pressure <sup>29</sup>	$\approx 5$ <sup>29</sup>	yes <sup>29</sup>	$\geq 0.31$ <sup>30</sup>	confusing history, see Ref. 27
Sr <sub>3</sub> Ru <sub>2</sub> O <sub>7</sub>	1st <sup>f</sup>	0 <sup>g</sup>	0 <sup>g</sup>	pressure <sup>g</sup>	n.a.	yes <sup>31</sup>	$< 0.5$ <sup>31</sup>	foliated wing tips, nematic phase <sup>31</sup>
UGe <sub>2</sub> <sup>33</sup>	1st <sup>34</sup>	52 <sup>35</sup>	1.5 <sup>35</sup>	hydrostatic pressure <sup>22,35</sup>	24 <sup>36</sup>	yes <sup>35,36</sup>	0.2 <sup>22</sup>	easy-axis FM coexisting FM+SC <sup>22</sup>
URhGe <sup>33</sup>	1st <sup>37</sup>	9.5 <sup>23</sup>	0.42 <sup>23</sup>	transverse $B$ -field <sup>37,39</sup>	$\approx 1$ <sup>37</sup>	yes <sup>37</sup>	8 <sup>38</sup>	easy-plane FM coexisting FM+SC <sup>23</sup>
UCoGe <sup>33</sup>	1st <sup>40</sup>	2.5 <sup>40</sup>	0.03 <sup>24</sup>	none	$> 2.5?$ <sup>h</sup>	no	12 <sup>24</sup>	coexisting FM+SC <sup>24</sup>
CoS <sub>2</sub>	1st <sup>41</sup>	122 <sup>41</sup>	0.84 <sup>41</sup>	hydrostatic pressure <sup>41</sup>	$\approx 120$ <sup>41</sup>	no	0.7 <sup>41</sup>	rather high $T_c$
La <sub>1-x</sub> Ce <sub>x</sub> In <sub>2</sub>	1st <sup>42</sup>	22 – 19.5 <sup>42 i</sup>	n.a.	composition <sup>42</sup>	$> 22?$ <sup>j</sup>	no	n.a.	third phase between FM and PM? <sup>42</sup>
Ni <sub>3</sub> Al <sup>27</sup>	(1st) <sup>k</sup>	41 – 15 <sup>l</sup>	0.075 <sup>m</sup>	hydrostatic pressure <sup>43</sup>	n.a.	no	0.84 <sup>44</sup>	order of transition uncertain
YbIr <sub>2</sub> Si <sub>2</sub> <sup>n</sup>	1st <sup>45</sup>	1.3 – 2.3 <sup>o</sup>	n.a.	hydrostatic pressure <sup>45</sup>	n.a.	no	$\approx 22$ <sup>p</sup>	FM nature of ordered phase suspected <sup>45</sup>
YbCu <sub>2</sub> Si <sub>2</sub> <sup>n</sup>	n.a.	4 – 6 <sup>46 q</sup>	n.a.	hydrostatic pressure <sup>46</sup>	n.a.	no	n.a.	nature of magnetic order unclear
URu <sub>2-x</sub> Re <sub>x</sub> Si <sub>2</sub>	2nd <sup>47,48</sup>	25 – 2 <sup>r</sup>	0.4 – 0.03 <sup>48</sup>	composition <sup>47</sup>	N/A	N/A	$\approx 100$ <sup>s</sup>	strongly disordered
Ni <sub>2</sub> Pd <sub>1-x</sub>	2nd <sup>50</sup>	600 – 7 <sup>t</sup>	n.a.	composition <sup>50</sup>	N/A	N/A	n.a.	disordered, lowest $T_c$ rather high
YbNi <sub>4</sub> P <sub>2</sub>	2nd <sup>51</sup>	0.17 <sup>51</sup>	$\approx 0.05$ <sup>51</sup>	none	N/A	N/A	2.6 <sup>51</sup>	quasi-1d, disordered

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- Conclusion: **Conventional theory not viable**

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- $\langle |q(k, \omega)|^2 \rangle \sim [k + |\omega|]^{-1}$

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- $Q_{nn}(x) \sim \psi_{\alpha}^{\dagger}(x, n) \sigma_{\alpha\beta} \psi_{\beta}(x, n)$
- → Due to mmc
- $\delta S \sim \int dx \, m(x) \cdot q(x) q^{\dagger}(x)$
- Using the soft electronic CF (structurally like)
- $\langle |q(k, \omega)|^2 \rangle \sim [k + |\omega|]^{-1}$
- Integrate out soft electronic modes to obtain a generalized MFT,



## 2. Renormalized mean-field theory.

■ In general, conventional theory misses effects of fermion soft modes:

- Contribution to  $f_0$ :

$$\int d\mathbf{k} \int d\Omega \ln((k + \Omega)^2 + m^2)$$

- Contribution to eq. of state:

- Renormalized mean-field equation of state:

$$\frac{d}{dm} \int d\mathbf{k} \int d\Omega \ln[(k + \Omega)^2 + m^2] \sim m \begin{cases} \text{const.} - m^{d-1} & (1 < d < 3) \\ \text{const.} + m^2 \ln m & (d = 3) \end{cases}$$

→  $h = tm + vm^3 \ln m + um^3$  (clean,  $d=3$ ,  $T=0$ )

- $v > 0$

Transition is generically 1<sup>st</sup> order! (TRK, T Vojta, DB 1999)

Physics?-Free energy gain by making soft fluctuations massive.

→ Coleman-Weinberg mechanism

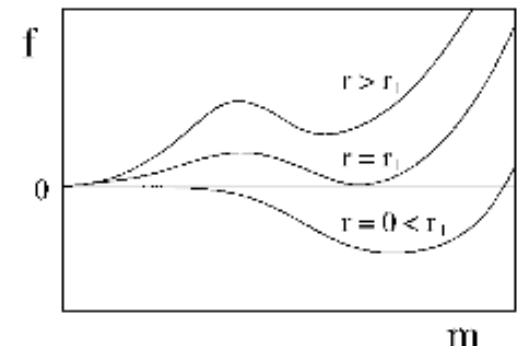
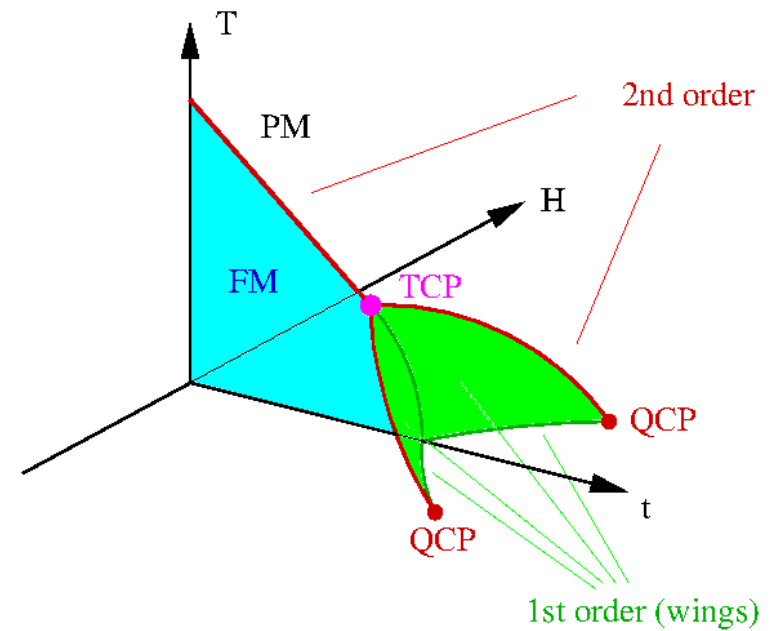


FIG. 2: Schematic sketch of the free energy for three values of the parameter  $r$ . The first-order transition occurs at  $r = r_1 > 0$ . It pre-empt the second-order transition of Landau theory which would occur at  $r = 0$ .

# II. Quantum Ferromagnetic Transitions: Theory

## 2. Renormalized mean-field theory

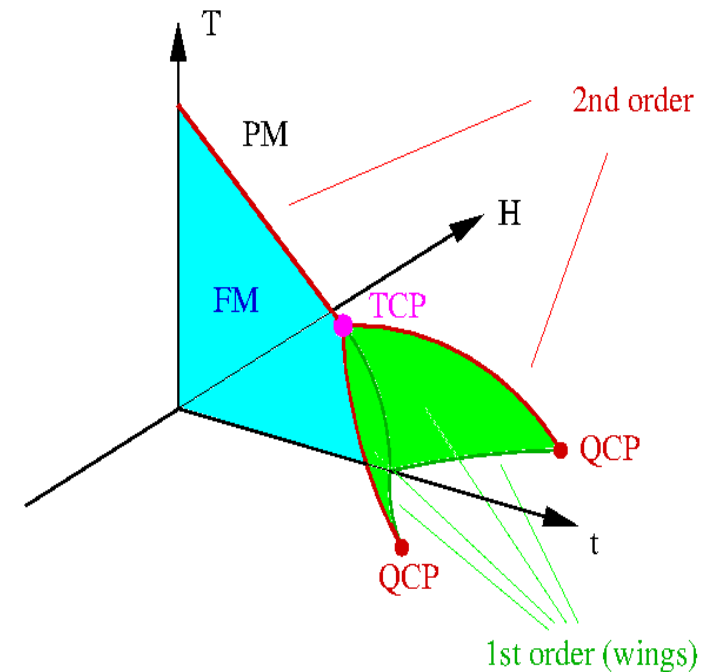
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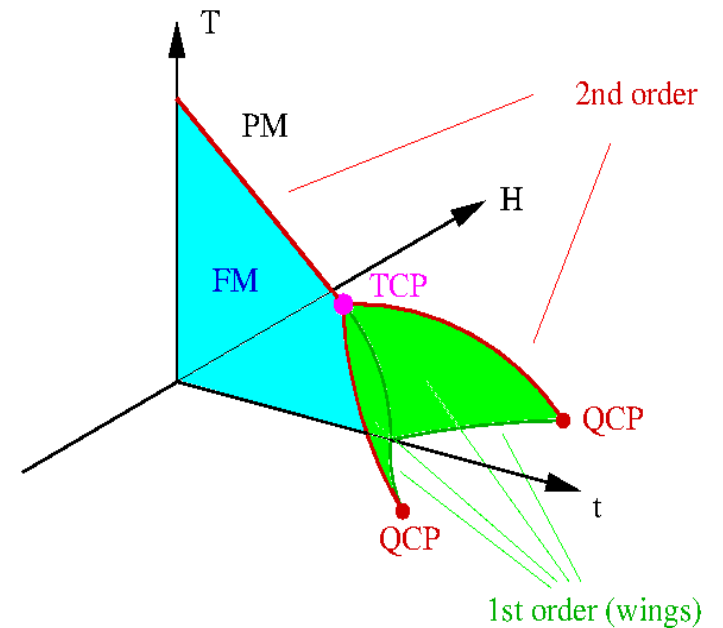
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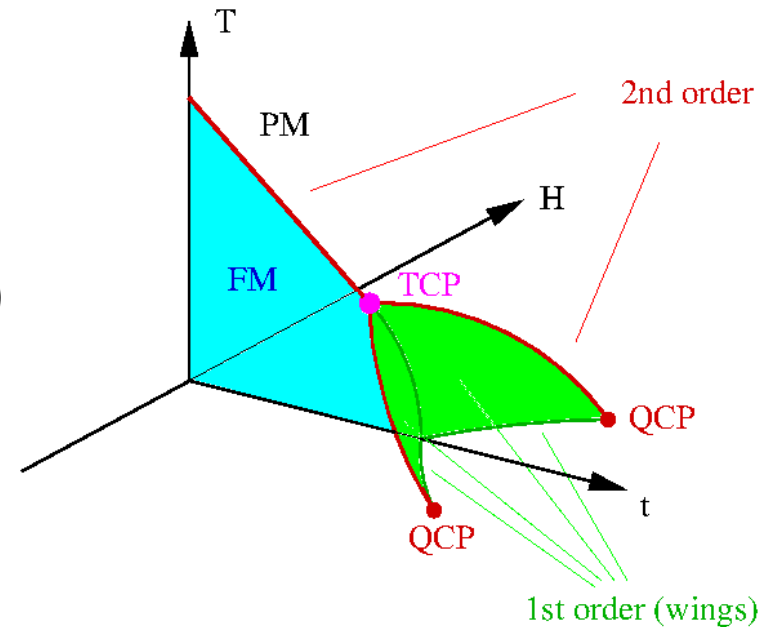
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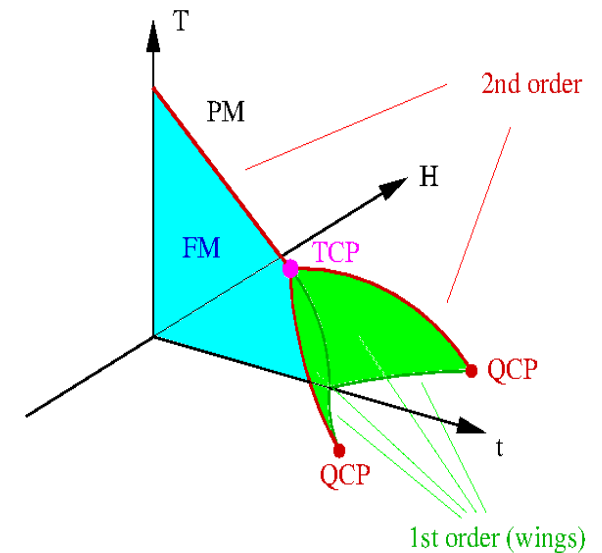
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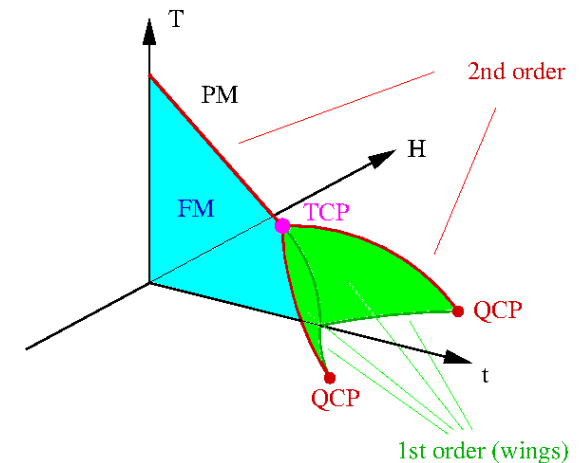
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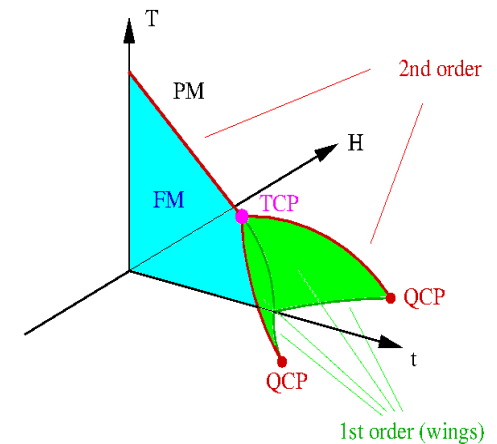
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- So far no OP fluctuations have been considered
- More generally, Hertz theory works if field conjugate the OP does not change the soft-mode spectrum (DB, TRK, T Vojta 2002)





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- Theory explains existence of generic tri-critical point and magnetic field dependence of phase diagram.**

- Sufficiently strong non-magnetic disorder drives transition 2<sup>nd</sup> order. Also understood.**

TABLE I: Systems with low- $T$  ferromagnetic transitions and their properties.  $T_c$  = Curie temperature,  $T_{ic}$  = tricritical temperature.  $\rho_0$  = residual resistivity. FM = ferromagnet, SC = superconductor. N/A = not applicable; n.a. = not available.

System <sup>a</sup>	Order of Transition <sup>c</sup>	$T_c/K$ <sup>b</sup>	magnetic moment/ $\mu_B$ <sup>d</sup>	tuning parameter	$T_{ic}/K$	wings observed	Disorder ( $\rho_0/\mu\Omega\text{cm}$ ) <sup>e</sup>	Comments
MnSi <sup>27</sup>	1st <sup>18</sup>	29.5 <sup>28</sup>	0.4 <sup>28</sup>	hydrostatic pressure <sup>18</sup>	$\approx 10$ <sup>18</sup>	yes <sup>25</sup>	0.33 <sup>25</sup>	weak helimagnet <sup>17</sup> exotic phases <sup>25,26</sup>
ZrZn <sub>2</sub> <sup>27</sup>	1st <sup>29</sup>	28.5 <sup>29</sup>	0.17 <sup>29</sup>	hydrostatic pressure <sup>29</sup>	$\approx 5$ <sup>29</sup>	yes <sup>29</sup>	$\geq 0.31$ <sup>30</sup>	confusing history, see Ref. 27
Sr <sub>3</sub> Ru <sub>2</sub> O <sub>7</sub>	1st <sup>f</sup>	0 <sup>g</sup>	0 <sup>g</sup>	pressure <sup>g</sup>	n.a.	yes <sup>31</sup>	$< 0.5$ <sup>31</sup>	foliated wing tips, nematic phase <sup>31</sup>
UGe <sub>2</sub> <sup>33</sup>	1st <sup>34</sup>	52 <sup>35</sup>	1.5 <sup>35</sup>	hydrostatic pressure <sup>22,35</sup>	24 <sup>36</sup>	yes <sup>35,36</sup>	0.2 <sup>22</sup>	easy-axis FM coexisting FM+SC <sup>22</sup>
URhGe <sup>33</sup>	1st <sup>37</sup>	9.5 <sup>23</sup>	0.42 <sup>23</sup>	transverse $B$ -field <sup>37,39</sup>	$\approx 1$ <sup>37</sup>	yes <sup>37</sup>	8 <sup>38</sup>	easy-plane FM coexisting FM+SC <sup>23</sup>
UCoGe <sup>33</sup>	1st <sup>40</sup>	2.5 <sup>40</sup>	0.03 <sup>24</sup>	none	$> 2.5?$ <sup>h</sup>	no	12 <sup>24</sup>	coexisting FM+SC <sup>24</sup>
CoS <sub>2</sub>	1st <sup>41</sup>	122 <sup>41</sup>	0.84 <sup>41</sup>	hydrostatic pressure <sup>41</sup>	$\approx 120$ <sup>41</sup>	no	0.7 <sup>41</sup>	rather high $T_c$
La <sub>1-x</sub> Ce <sub>x</sub> In <sub>2</sub>	1st <sup>42</sup>	22 – 19.5 <sup>42 i</sup>	n.a.	composition <sup>42</sup>	$> 22?$ <sup>j</sup>	no	n.a.	third phase between FM and PM? <sup>42</sup>
Ni <sub>3</sub> Al <sup>27</sup>	(1st) <sup>k</sup>	41 – 15 <sup>l</sup>	0.075 <sup>m</sup>	hydrostatic pressure <sup>43</sup>	n.a.	no	0.84 <sup>44</sup>	order of transition uncertain
YbIr <sub>2</sub> Si <sub>2</sub> <sup>n</sup>	1st <sup>45</sup>	1.3 – 2.3 <sup>o</sup>	n.a.	hydrostatic pressure <sup>45</sup>	n.a.	no	$\approx 22$ <sup>p</sup>	FM nature of ordered phase suspected <sup>45</sup>
YbCu <sub>2</sub> Si <sub>2</sub> <sup>n</sup>	n.a.	4 – 6 <sup>46 q</sup>	n.a.	hydrostatic pressure <sup>46</sup>	n.a.	no	n.a.	nature of magnetic order unclear
URu <sub>2-x</sub> Re <sub>x</sub> Si <sub>2</sub>	2nd <sup>47,48</sup>	25 – 2 <sup>r</sup>	0.4 – 0.03 <sup>48</sup>	composition <sup>47</sup>	N/A	N/A	$\approx 100$ <sup>s</sup>	strongly disordered
Ni <sub>2</sub> Pd <sub>1-x</sub>	2nd <sup>50</sup>	600 – 7 <sup>t</sup>	n.a.	composition <sup>50</sup>	N/A	N/A	n.a.	disordered, lowest $T_c$ rather high
YbNi <sub>4</sub> P <sub>2</sub>	2nd <sup>51</sup>	0.17 <sup>51</sup>	$\approx 0.05$ <sup>51</sup>	none	N/A	N/A	2.6 <sup>51</sup>	quasi-1d, disordered



**Many More BDs Eddie!!!**

**Helena and Ted**