Multi-species simple exclusion processes with two-way traffic and overtaking

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As shown by Karimipour[1] the Matrix Product Ansatz method for solving the totally asymmetric exclusion process can be generalized to a system of several species of particles. For this to be possible the following conditions must hold:
1) Define the free velocity v_i of a particle of species i as plus or minus the exchange rate with an empty site to its right respectively to its left depending on the sign of v_i. Then the exchange rate of a particle of type i with a particle of type j to its right must be $v_i - v_j$ if $v_i - v_j > 0$ and zero otherwise.
2) Define α_i and γ_i as the injection respectively extraction rate for species i, with injection on the left and extraction on the right for species of positive velocity and the other way around for species of negative velocity. Then the extraction rates have to satisfy the conditions $v_i - v_j = \gamma_i - \gamma_j$ if v_i and v_j have the same sign (in contrast to what is stated in [1] no conditions have to be imposed on the injection rates).
In a grand ensemble formalism one can postulate a general form for the partition function, with a few parameters that can be identified easily. From this one immediately obtains all the currents and densities as functions of the exchange, injection and extraction rates. Like in the 1-species case, depending on these rates there can be two types of phases, some that are dominated by the density of one of the species (corresponding to the high and low density phases), and others resulting from singularities in a Jacobian (corresponding to the maximum current phase in the one-species system).

1) V. Karimipour, Phys. Rev. E **59** (1999) 205; M. Khorrami and V. Karimipour, J. Stat. Phys. **100** (2000)