Electromagnetic cloaking and near cloaking at all frequencies

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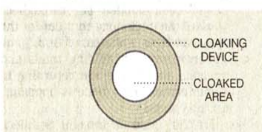
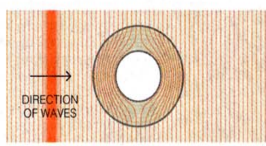
Light Fantastic

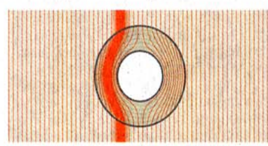
Flirting with invisibility: New materials can bend some wavelengths of light around an object (as in this idealized simulation) to make it vanish.



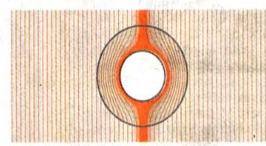
The cloaking device is a cylinder designed to bend microwaves of a specific wavelength.



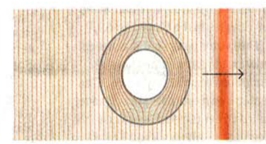
The device is placed in an electromagnetic field. Above, microwaves travel left to right.



A wavefront entering the cloaking device is bent and diverted.



The wave splits, flowing around the cloaked area and reforming almost seamlessly.



Since the device does not significantly reflect or disturb the wave, it is effectively invisible.

The New York Times; image courtesy of American Physical Society and Steven A. Cummer, Duke University

By KENNETH CHANG

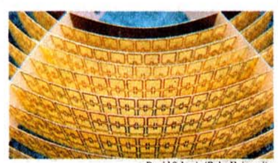
Increasingly, physicists are constructing materials that bend light the "wrong" way, an optical trick that could lead to sharper-than-ever lenses or maybe even make objects disappear.

Last October, scientists at Duke demonstrated a working cloaking device, hiding whatever was placed inside, although it worked only for microwaves.

In the experiment, a beam of microwave light split in two as it flowed around a specially designed cylinder and then almost seamlessly merged back together on the other side. That meant that an object placed inside the cylinder was effectively invisible. No light waves bounced off the object, and someone looking at it would have seen only what was behind it.

The cloak was not perfect. An alien with microwave vision would not have seen the object, but might have noticed something odd. "You'd see a darkened spot," said David R. Smith, a professor of electrical and computer engineering at Duke. "You'd see some distortion, and you'd see some shadowing, and you would see some reflection."

A much greater limitation was that this particular cloak worked for just one particular "color," or wavelength, of microwave light, limiting its usefulness as a hiding place. Making a cloak that works at the much shorter wavelengths of visible light or one that works over a wide range of colors is an even harder, perhaps impossible, task.



David Schurig/Duke University

NOW YOU SEE IT Duke researchers built a simplified version of their cloaking device out of copper rings and wires patterned onto fiberglass sheets and demonstrated that it successfully diverted microwaves.

Nonetheless, the demonstration showed the newfound ability of scientists to manipulate light through structures they call "metamaterials."

Obviously the military would be interested in any material that could be used to hide vehicles or other equipment. But such materials could also be useful in new types of microscopes and antennae. So far, scientists have written down the underlying equations, performed computer simulations and conducted some proof-of-principle experiments like the one at Duke. They still need to determine the practical limitations of how far they can bend light to their will.

The method is not magic, nor are the materials novel. Physicists are taking ordinary substances like fiberglass and copper to build metamaterials that look like mosaics of repeating tiles. The metamaterials interact with the electric and magnetic fields in light waves, manipulating a quantity known as the index of refraction to bend the light in a way that no natural material does.

"There are some things that chemistry can't do on its own," said John B. Pendry, a

Continued on Page 4

Time harmonic solutions to the Maxwell Equations with TM symmetry:

$$\begin{cases} \nabla \cdot (\sigma \nabla u) + \omega^2 q u = 0 & \text{in } \Omega \\ u = f & \text{on } \partial\Omega. \end{cases}$$

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GOAL: To construct a subset D of Ω and material parameters σ , q in $\Omega \setminus D$ such that independent of what one puts inside D the observable data are the same as for $\sigma = q = 1$. In that case we shall say that the subset D is perfectly cloaked.

For simplicity we now take $\omega = 0$.

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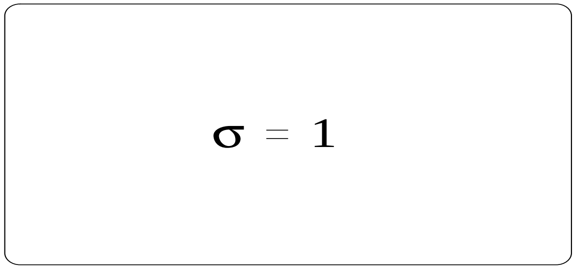
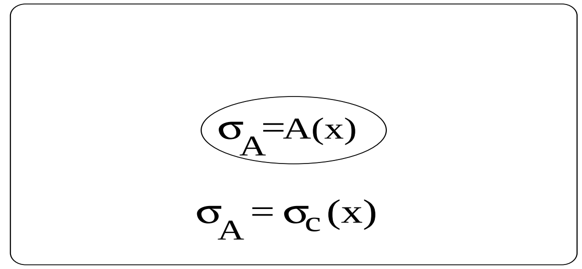


voltage f implies current flux g



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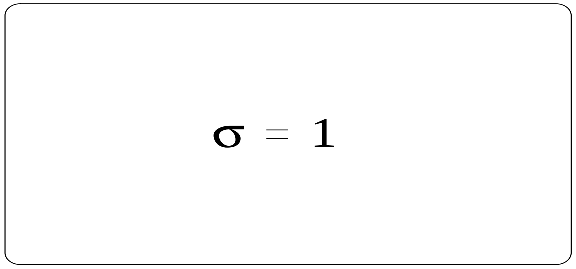

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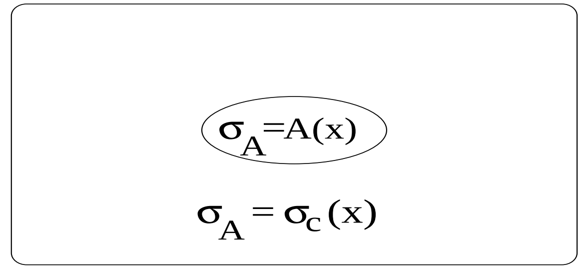

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Suppose σ_c cloaks $D \subset \Omega$ in the sense of giving same Dirichlet-to -Neumann map on $\partial\Omega$ (for any A inside D) as $\sigma = 1$ in all of Ω.

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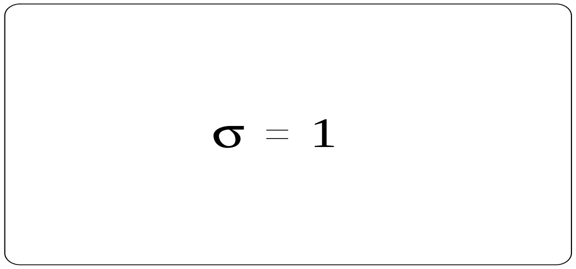

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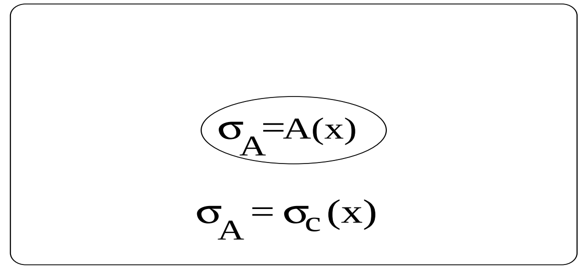

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Let $\Omega \subset \Omega'$. Then the Dirichlet-to-Neumann map of

$$\sigma(x) = \begin{cases} A(x) & \text{for } x \in D \\ \sigma_c(x) & \text{for } x \in \Omega \setminus D \\ 1 & \text{for } x \in \Omega' \setminus \Omega \end{cases}$$

is independent of A, and identical to that of the domain Ω' with constant conductivity $\sigma = 1$.

A "mapping technique" to construct the perfect cloak σ_c.

Set $\Omega = B_2$ and $D = B_1$ (concentric balls of radius 2 and 1) and define

$$F(x) = (1 + \frac{1}{2}|x|)\frac{x}{|x|} \qquad , \quad B_2 \setminus \{0\} \ \rightarrow B_2 \setminus \overline{B_1}$$

Notice: $\quad F|_{\partial B_2} = \text{identity} \quad$ and

$$\int_{B_2} < \sigma \nabla u \ , \nabla u > \ dx = \int_{B_2 \setminus \overline{B_1}} < F_* \sigma \nabla v \ , \nabla v > \ dx$$

with

$$F_*\sigma(x) = \frac{DF \sigma DF^t}{|det DF|} \circ F^{-1}(x) \text{ and } \quad v(x) = u \circ F^{-1}(x) \quad .$$

This makes

$$\Lambda_\sigma = \Lambda_{F_* \sigma}$$

independent of what we put inside B_1 and so as a "perfect" cloak we may use

$$\begin{aligned}\sigma_c &= F_*1\\ &= \frac{2^n}{(2+|z|)^{n-1}}\left[(\frac{1}{4}|z|^{n-1}+|z|^{n-2}+|z|^{n-3})(I-\hat{x}\hat{x}^t)+\frac{1}{4}|z|^{n-1}\hat{x}\hat{x}^t\right]\end{aligned}$$

with

$$|z| = |F^{-1}(x)| = 2(|x|-1) \quad , \quad \hat{x} = \frac{x}{|x|} \quad .$$

A "mapping technique" to construct the approximate cloak $\sigma_c^{(\rho)}$.

Define

$$F_\rho(x) = \begin{cases} \frac{x}{\rho} & \text{for } x \in B_\rho \\ (\frac{2-2\rho}{2-\rho} + \frac{1}{2-\rho}|x|)\frac{x}{|x|} & \text{for } x \in B_2 \setminus B_\rho \end{cases}$$

and

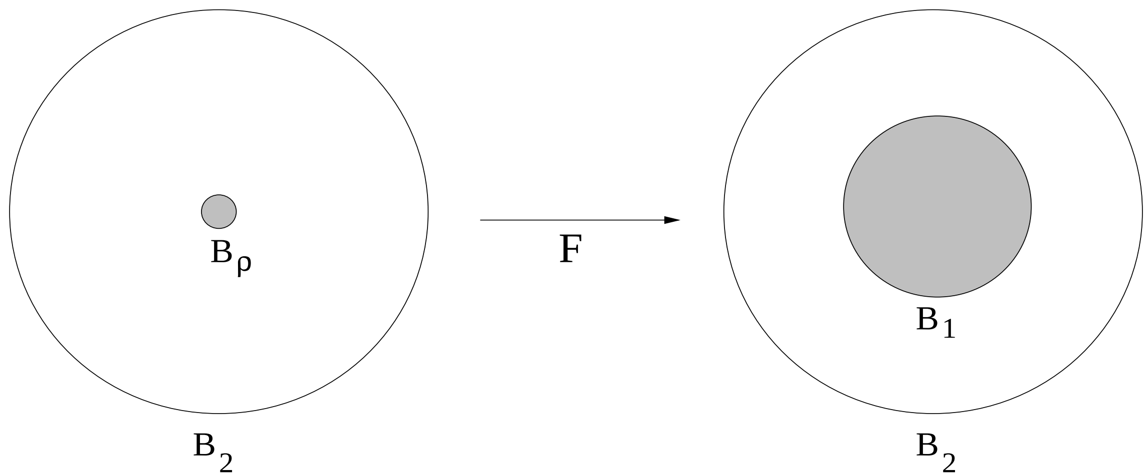
$$\sigma_c^{(\rho)} = (F_\rho)_*1$$

$$\sigma_\rho(x) \ :$$

$$\begin{cases} (F_\rho^{-1})_* A(x) & \text{for } x \in B_\rho \\ 1 & \text{for } x \in B_2 \setminus B_\rho \end{cases}$$

$$\sigma_A(x) \ :$$

$$\begin{cases} A(x) & \text{for } x \in B_1 \\ \sigma_c^{(\rho)}(x) & \text{for } x \in B_2 \setminus B_1 \end{cases}$$



$$\sigma_\rho(x) \ : \qquad\qquad \sigma_A(x) \ :$$

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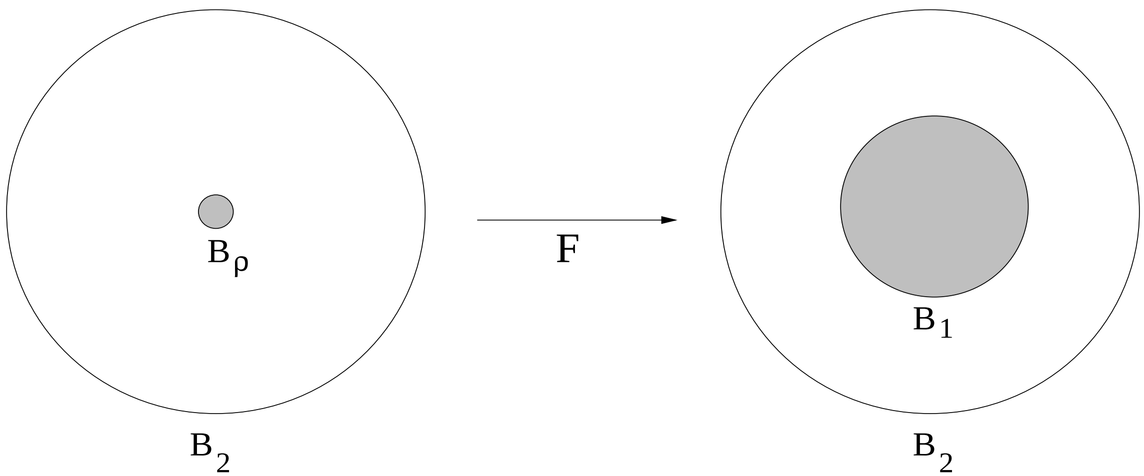

Then $\Lambda_{\sigma_A} = \Lambda_{\sigma_\rho}$ and so

$$\|\Lambda_{\sigma_A} - \Lambda_1\| = \|\Lambda_{\sigma_\rho} - \Lambda_1\| \leq C\rho^n$$

This asserts that "near cloaking" may be achieved at any prescribed level

– the essential fact being that the constant C is independent of A!!

For the Helmholtz equation (non-zero fixed frequency) we similarly obtain

$$\|\Lambda_{\sigma_A, q_B} - \Lambda_{1,1}\| = \|\Lambda_{\sigma_\rho, q_\rho} - \Lambda_{1,1}\| \leq \frac{C}{|\log \rho|} \text{ for } n = 2$$

For $n = 3$ the similar bound becomes $C\rho^1$. The constant C is independent of A and B (the coefficients we are attempting to cloak).

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An additional (absorbing) layer of conductivity $\approx \rho^{-2}$, and occupying $B_\rho \setminus \overline{B_{\rho/2}}$, is necessary. The "cloaked" region then becomes $B_{1/2}$.

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We have studied the uniformity of C with respect to frequency ω. To avoid some of the eigenvalue issue we have conducted this study in the context of the scattering problem ($\Omega = I\!R^n$). We have considered only incident plane waves:

$$u_{\rho,s}(x) = u_\rho(x) - e^{i\omega x\cdot\eta}$$

With

$$\begin{cases} \sigma_\rho = q_\rho = 1 & \text{in } \Omega \setminus B_\rho \\ \sigma_\rho = 1,\ q_\rho = 1 + \frac{i}{\omega\rho\lambda} & \text{in } B_\rho \setminus B_{\rho/2} \\ \sigma_\rho, q_\rho \text{ arbitrary, real} & \text{in } B_{\rho/2} \end{cases},$$

and $u_{\rho,s}$ satisfying the outgoing radiation condition,

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Let u and u_c be the solutions to the $3d$ wave equations:

$$\begin{cases} \partial_{tt}^2 u - \Delta u = f, \\ u(t=0) = u_0, \\ \partial_t u(t=0) = u_1. \end{cases} \tag{1}$$

and

$$\begin{cases} q_B \partial_{tt}^2 u_c - \nabla \cdot (\sigma_A \nabla u_c) + \gamma \partial_t u_c = f, \\ u_c(t=0) = u_0, \\ \partial_t u_c(t=0) = u_1. \end{cases} \tag{2}$$

Here

$$\sigma_A, q_B, \gamma = \begin{cases} F_{\rho_*}I, F_{\rho_*}1, 0 & \text{in } I\!R^3 \setminus B_1, \\ F_{\rho_*}I, F_{\rho_*}1, F_{\rho_*}\Big(1/\rho^{2+\alpha}\Big) & \text{in } B_1 \setminus B_{1/2}, \\ A, B, 0 & \text{in } B_{1/2}. \end{cases}$$

We will assume that $\text{supp} f \subset [0,T] \times (B_4 \setminus B_2)$ for some $T > 0$, $\text{supp} u_0,\ \text{supp} u_1 \subset B_4 \setminus B_2,\ f, u_0, u_1$ are smooth.

For $\alpha > 1/2$ there exists a positive constant C depending on the range of A and B such that

$$\sup_{t>0} \|u_c(t) - u(t)\|_{L^2(K)} \leq C\rho\Big(\|f\| + \|u_0\| + \|u_1\|\Big).$$

K is a compact subset of $I\!R^n \setminus \overline{B_2}$. Here

$$\|f\| = \|f\|_{C^2}, \quad \|u_0\| = \|u_0\|_{C^2}, \quad \text{and} \quad \|u_1\| = \|u_1\|_{C^1}.$$