Coupling and damage spreading in Markov chains

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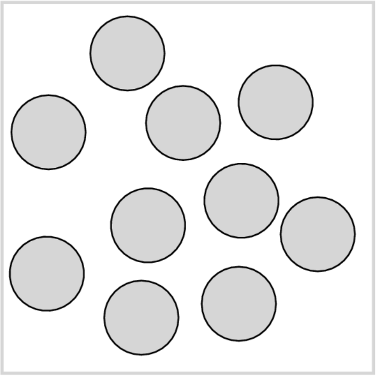
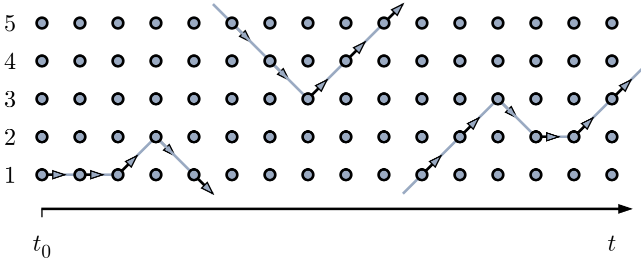
Faster algorithms



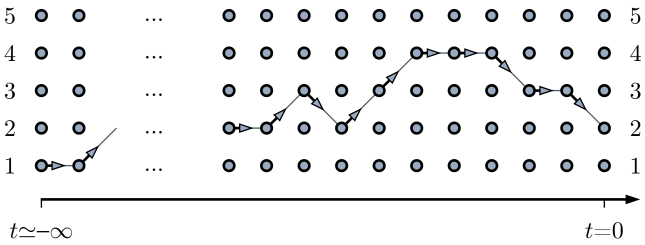
- Bernard, Krauth, Wilson (2009)
- much faster than previous methods . . .
- . . . but is it fast enough?

One-d convergence



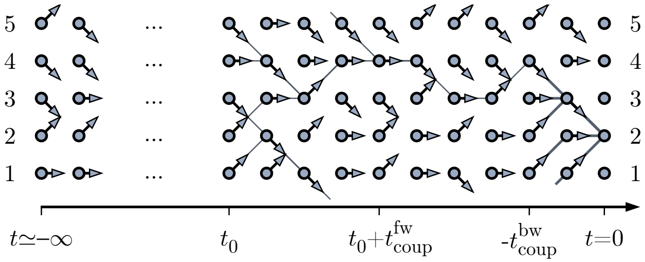
- Markov-chain Monte Carlo algorithm on 5 sites...
- ...converges as $\exp[-t/\tau]$ with finite correlation time τ ...

One-d calculation that finishes on time!



- ...start earlier and earlier ...
- ...get done on time ...
- ...Propp, Wilson (1995).

Correlations and coupling (from the past)



- Simulation starts really early (at time $t \simeq -\infty$) ...
- ... At time $t = 0$, we are done ...
- ... infinite simulation.

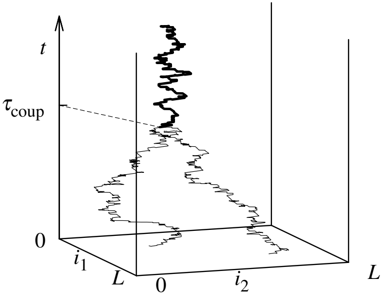
Higher-dimensional random walk

- configuration $i = \{i_0, \dots, i_{N-1}\}$ with $i_k \in \{0, \dots, L-1\}$

$$p(i \to j) = \begin{cases} \frac{1}{3N} & \text{for } j = i \pm \delta_k \\ \frac{1}{3} & \text{for } j = i \\ 0 & \text{otherwise} \end{cases} .$$

- Random walk in N dimensions, lattice with L^N sites.
- N non-interacting particles on a one-d lattice of length L.
- N non-interacting Potts spins with L states.

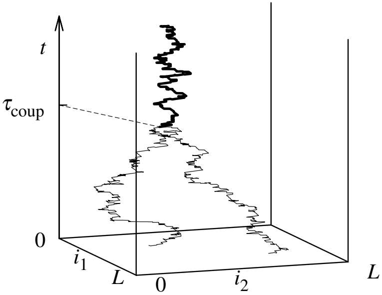
Coupling in higher dimensions: product ansatz



$$\tilde{p}(i \to j, i' \to j') = \begin{cases} p(i \to j)p(i' \to j') & \text{if } i \neq i' \\ p(i \to j) & \text{if } i = i', j = j' \\ 0 & \text{otherwise} \end{cases},$$

- coupling time $\propto L^N \gg$ correlation time

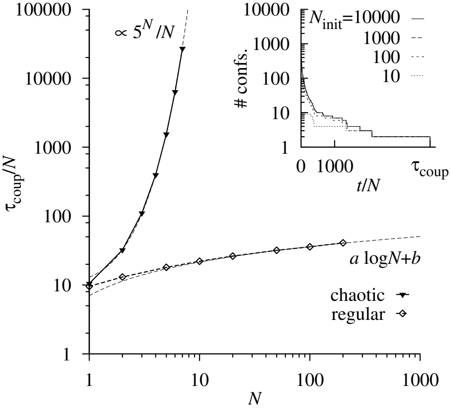
Coupling in higher dimensions: component ansatz



$$\tilde{p}(i_k \to j_k, i'_k \to j'_k) = \begin{cases} p(i_k \to j_k)p(i'_k \to j'_k) & \text{if } i_k \neq i'_k \\ p(i_k \to j_k) & \text{if } i_k = i'_k, j_k = j'_k \\ 0 & \text{otherwise} \end{cases}$$

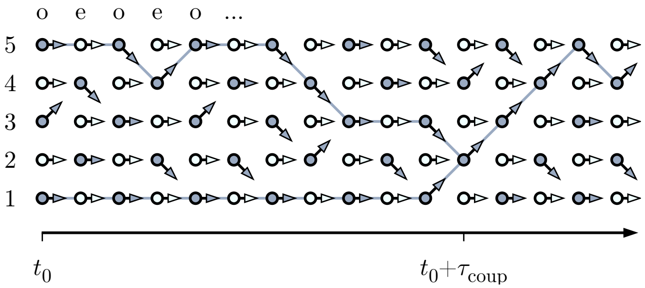
- coupling time $\propto L^2 \sim$ correlation time

Random walk: coupling time



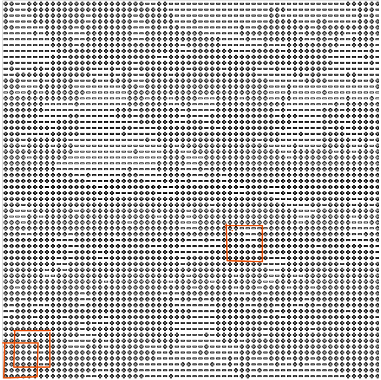
- NB: N-dimensional lattice of length L
- regular dynamics: component ansatz
- chaotic dynamics: product ansatz

Survey problem in the random walk (with walls)



- partial order of configurations (walls) . . .
- . . . preserved by Monte Carlo dynamics
- partial order $\implies$ coupling time $\sim$ correlation time (Propp & Wilson)

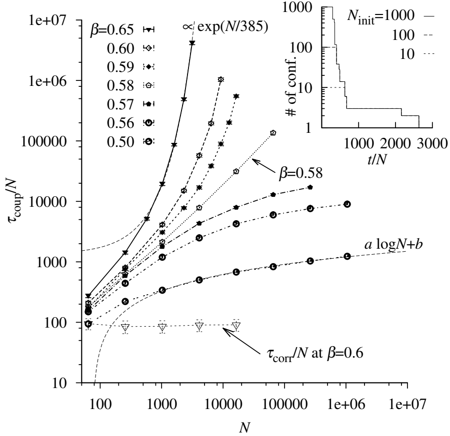
Survey problem in spin glasses



- Chanal and Krauth (2008, 2010)

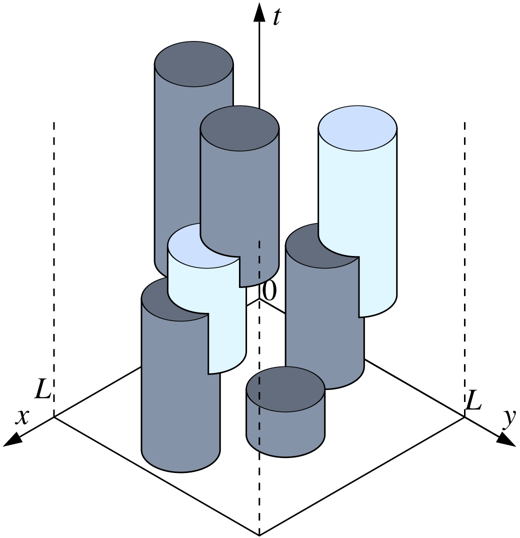
- 64×64 Ising spin glass has $2^{32\times 64} \sim 3 \times 10^{616}$ states.
- We have solved the survey problem using an exact block-spin renormalization procedure

Spin glass coupling time

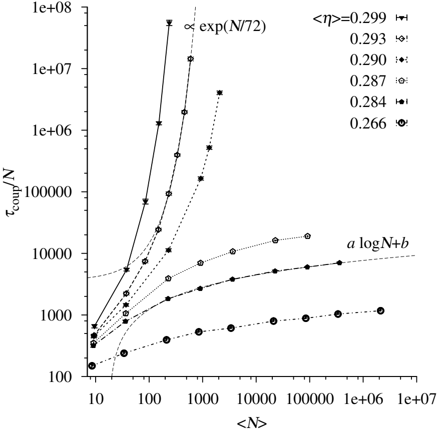


- ... this is the "Damage spreading" transition (Kauffman (1969) and $c \sim 10^3$ papers since

Survey problem for hard spheres

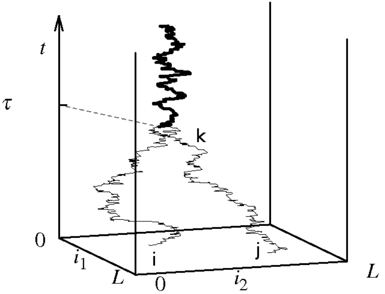


Hard sphere coupling time



- ... this is again the Damage spreading transition

Maximal Coupling: Griffeath (1975)



- the coupling probability must satisfy
  $\tilde{p}(i \to k, j \to k, \tau) \leq \min(p(i \to k, \tau), p(j \to k, \tau))$
- Griffeath constructs a coupling that satisfies the bound.

Conclusion

References

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- W. Krauth ‘Statistical Mechanics: Algorithms and Computations’ (Oxford University Press, 2006)
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- E. P. Bernard, C. Chanal, W. Krauth ‘Damage spreading and coupling in Markov chains’ EPL to appear

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