Weakly interacting Bose gas in disordered environment*

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Outline

Ideal Bose gas in random potential

Weakly repulsive gas in a random potential

Bosons in one dimension

Bosons in traps

Introduction

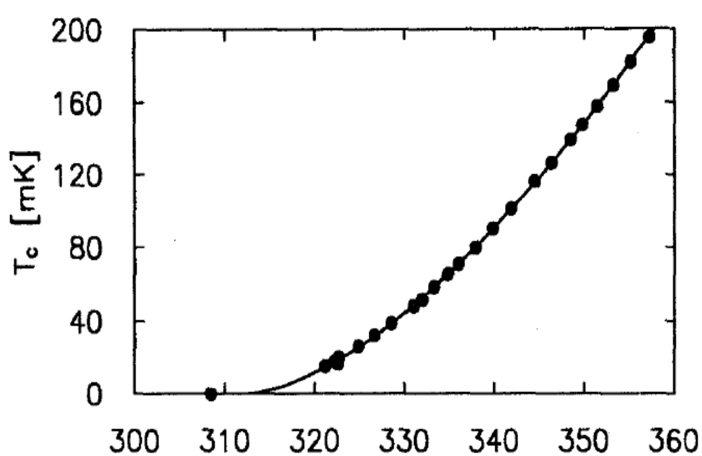
BEC : finite part of atoms in the state with minimal energy.

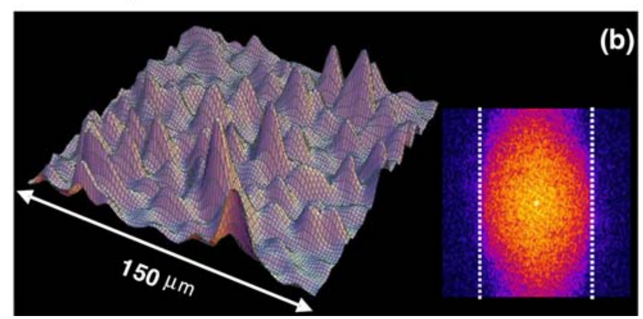
Examples: Superfluid ^4He, laser cooled atoms in a trap

Disorder: Superfluid He in porous media (J.D. Reppy et al '92)
Cold atoms in speckle potential (R.G. Hulet et al. '08)

→ Breakdown of superfluidity at strong disorder

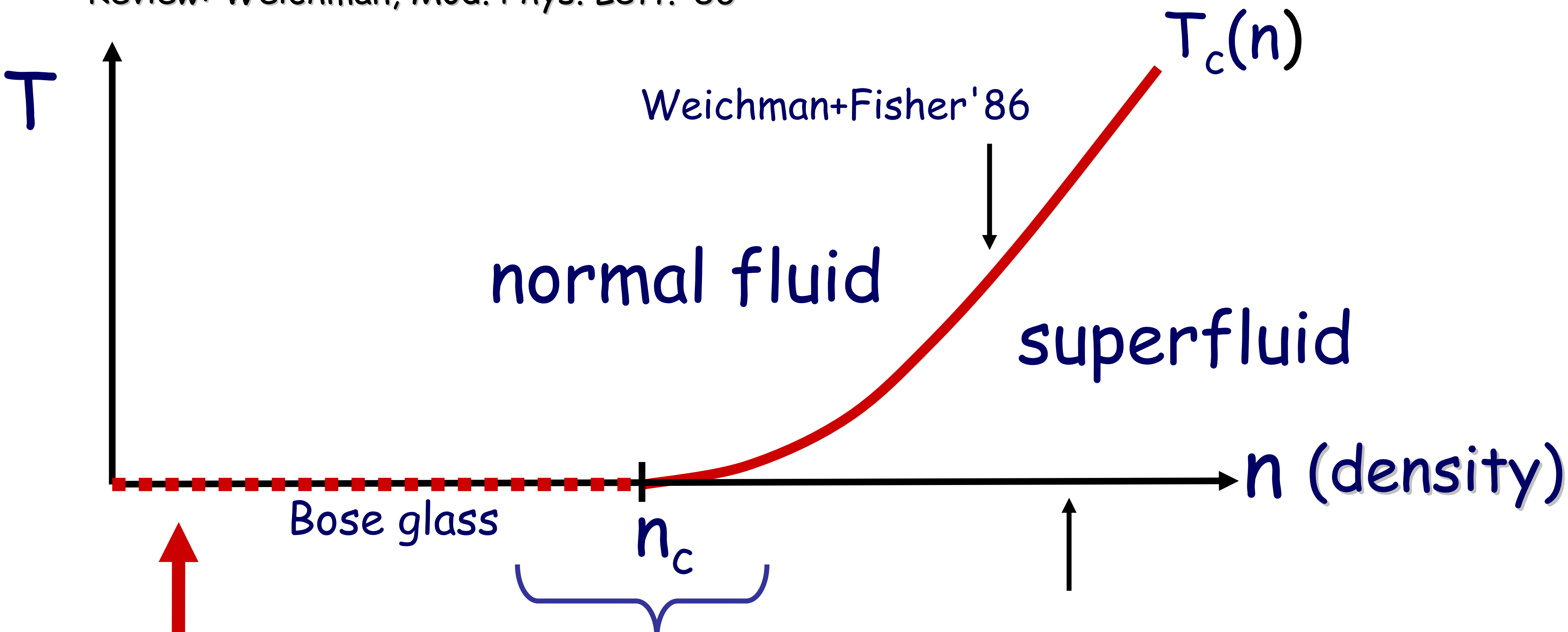
Onset of Superfluidity in ^{4}He Films





Bosons in disordered environment:

Review: Weichman, Mod. Phys. Lett. '86



this work

Shklovskii '08
Babichenko² '08

Halperin, Lee+ Ma'86

Giamarchi+Schulz '87 (d=1)

Fisher,Weichman,
Grinstein+Fisher '89
$\rightarrow$ z=d

Bogoliubov theory
Huang+Meng '92

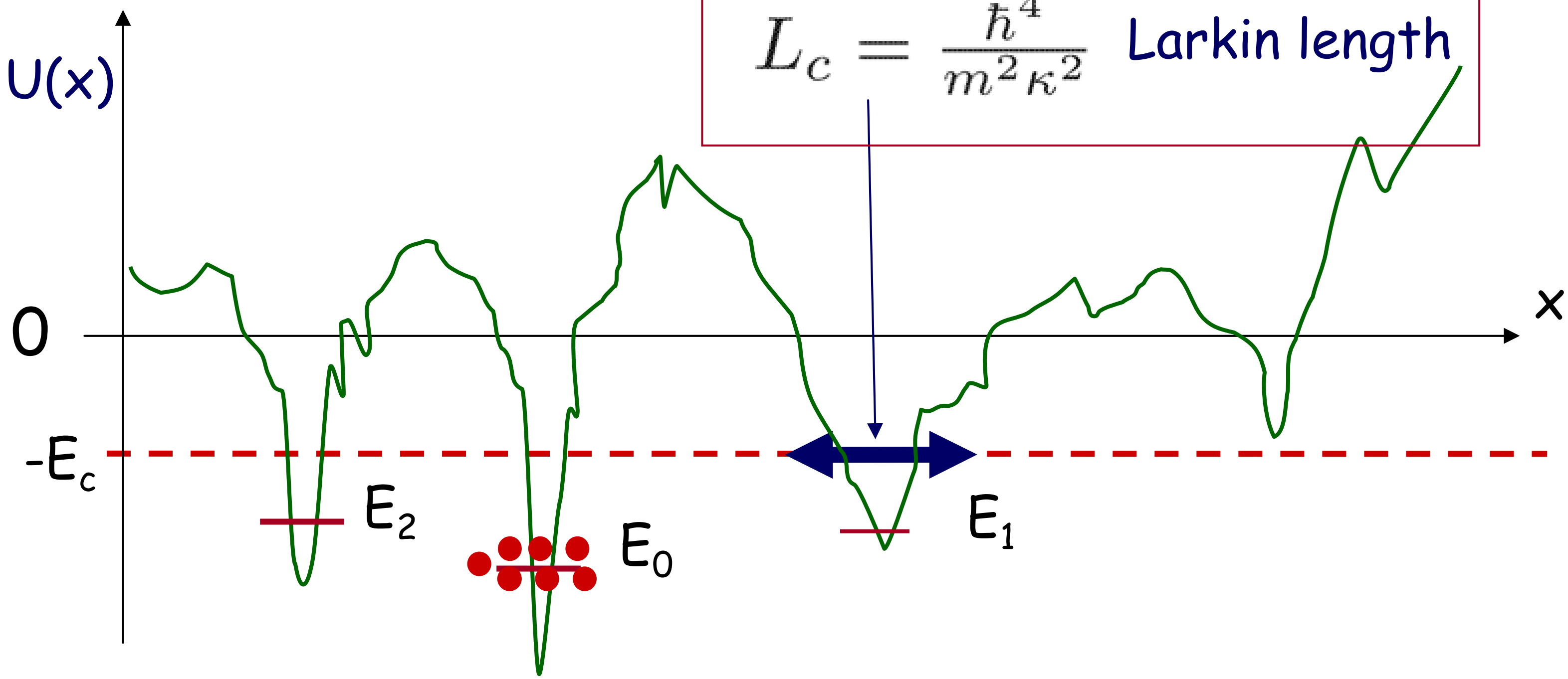
$$\rho_s \approx \rho_0[1 - c_1\sqrt{n_c/n}]$$

Cross-Pitaevskii equation

Ideal 3d Bose gas in random potential

$$\frac{\hbar^2}{2m}\nabla^2\psi + \left(E - U\left(\mathbf{x}\right)\right)\psi = 0 \qquad \langle U(\mathbf{x})U(\mathbf{x}')\rangle = \kappa^2\delta(\mathbf{x}-\mathbf{x}')$$

$$L_c = \frac{\hbar^4}{m^2\kappa^2}$$ Larkin length



$$E_c = \frac{\hbar^2}{2mL_c{}^2}$$ Larkin energy

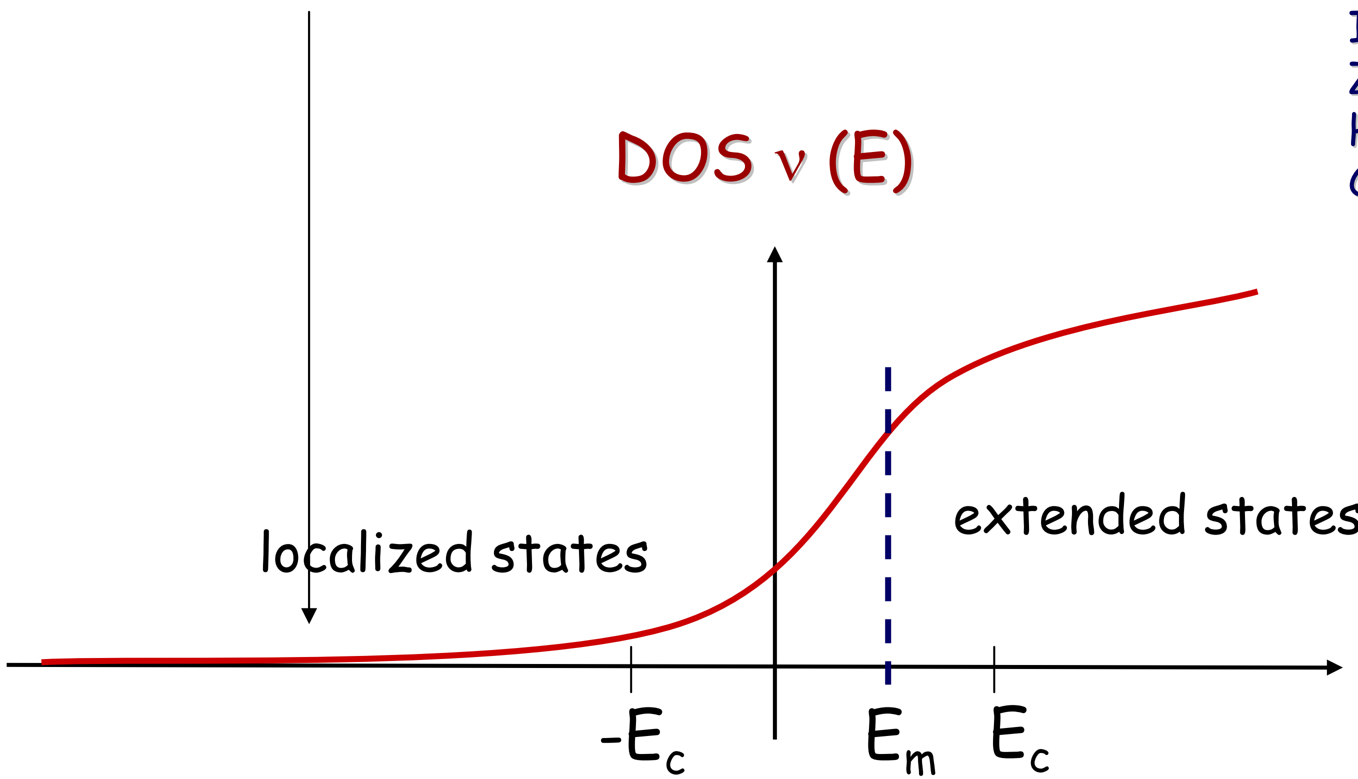
T=0: All particles in ground state $E_0 \approx -E_c \ln^2(L_0/L_c)$

Ideal Bose gas in random potential

DOS for E << - E_c dominated by wells of width $R \sim \hbar/\sqrt{m|E|} \ll L_c$

$$\nu(E) = \frac{1}{V}\langle\delta(E - E[U(\mathbf{x})])\rangle \sim |E|^{3/2} e^{-\sqrt{|E|/E_c}}$$

I.M. Lifshitz '66,
Zittartz and Langer '66,
Halperin and Lax,'66
Cardy '78



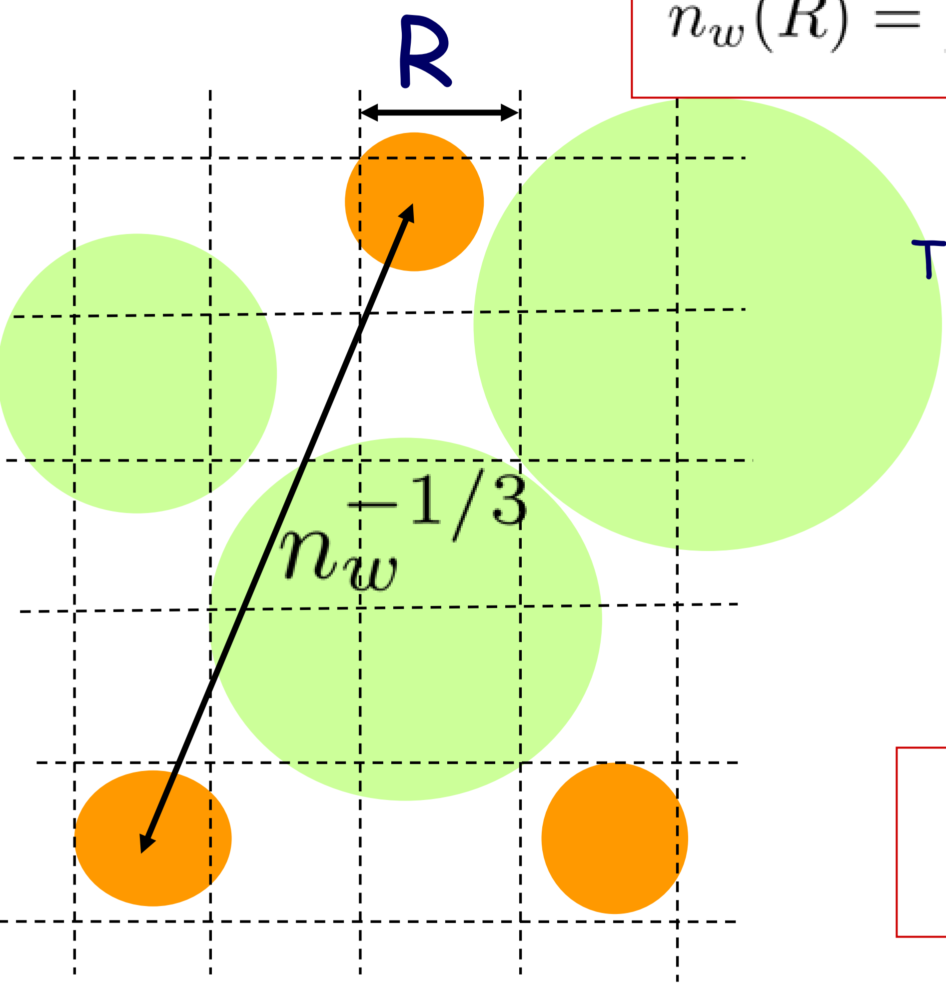
Width of well

$E \sim -\hbar^2/mR^2$

Ideal Bose gas in random potential

Spatial density $n_W(R)$ of wells with radius $< R \ll L_c$ $\left(E < -\hbar^2/(2mR^2) \ll E_c \right)$

$$n_w(R) = \int_{-\infty}^{-\frac{\hbar^2}{2mR^2}} dE\ \nu(E) \sim \frac{L_c}{R^4} e^{-L_c/R}$$



Tunneling amplitude t(R) between wells with radius < R :

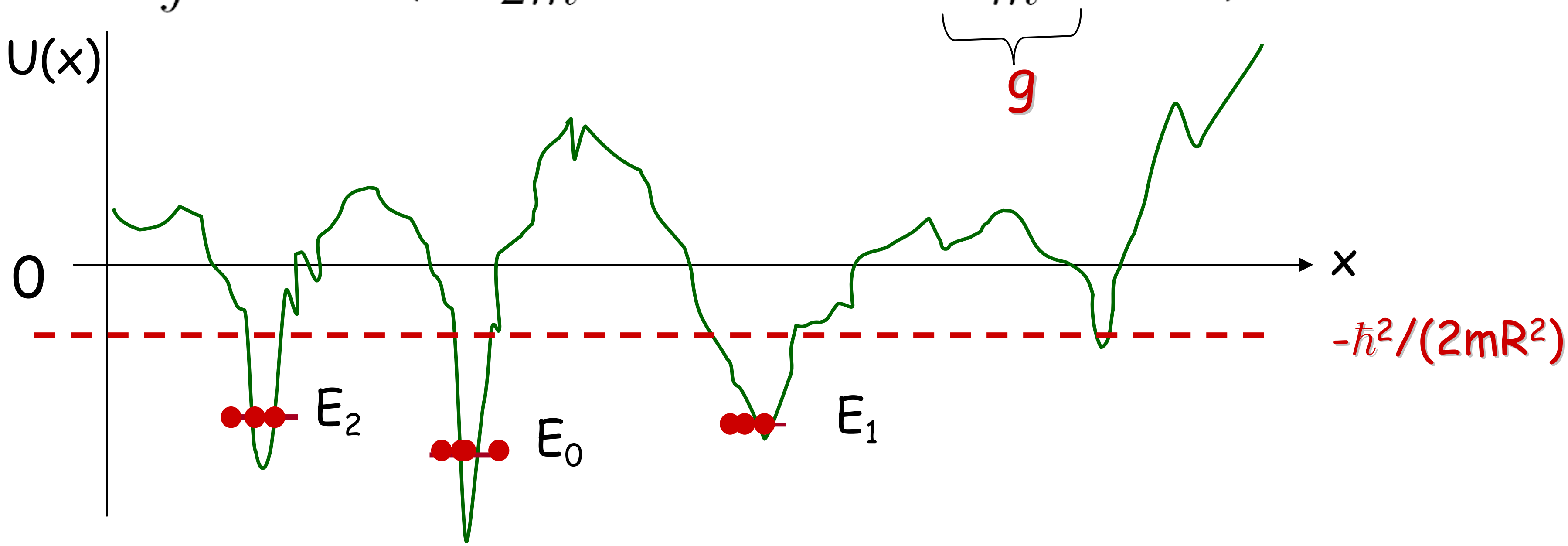
$$t\,(R)\ =\ \exp\left(-\frac{1}{\hbar}\int |p|\ dl\right)$$

$$\frac{1}{\hbar}\int |p| dl \approx n_w^{-1/3}/R \sim e^{L_c/3R}$$

$$t(R) \sim e^{-\left(\frac{R}{L_c} e^{L_c/R}\right)^{1/3}}$$

Weakly repulsive bosons in a random potential

$$\mathcal{H} = \int d^3x \Psi^\dagger \Big(-\frac{\hbar^2}{2m}\nabla^2 + U(\mathbf{x}) + \underbrace{\frac{2\pi\hbar^2 a}{m}}_{g} \Psi^\dagger\Psi \Big)\Psi$$



Assume that all potential wells with radii up to R are filled:

⇒ number of particles per well of size R : $N_w(R) = n/n_w(R) \gg 1$

⇒ repulsion energy per particle: $E_g(R) \approx g\, N_w/R^3 \sim g\, n\, e^{L_c/R}$

⇒ total energy per particle: $\mu(R) = -\hbar^2/(2mR^2) + E_g(R)$

Weakly repulsive bosons in a random potential

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Mininization over R: $\Rightarrow$ $R(n)=L_c/\ln(n_c/n)$, $n \ll n_c \approx 1/(3L_c^2 a)$

$$\mu(n) = -\frac{\hbar^2}{2mR^2(n)} = -\frac{1}{2}E_c(\ln\frac{n_c}{n})^2$$

$$\frac{n_c}{n} = \frac{\xi^2}{{L_c}^2}$$

(Fermions: $L_c \rightarrow a$)

Preliminary conclusions

$\Rightarrow$ At $n \ll n_c$ Bose gas decays into fragments, particle density in fragments each of density $n_c \sim 1/(aL_c^2)$

$\Rightarrow$ tunneling exponentially suppressed: $t(n) \sim e^{-c(n_c/n)^{1/3}}$

$\Rightarrow$ particle number in fragments $N_w = L_c / \left[3a(\ln \frac{n_c}{n})^3\right]$ well defined

$\Rightarrow$ phase uncertain, no phase coherence $\Rightarrow$ no superfluidity

$\Rightarrow$ finite compressibility $\frac{n}{E_c} \ln\left(\frac{n_c}{n}\right)$ „Bose glass"

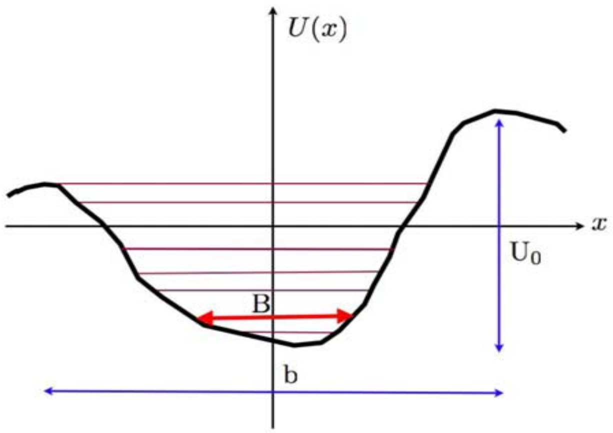
$\Rightarrow$ $$\hat{H}_{\text{eff}} = \sum_j C_j(\hat{N}_j - \langle N_j \rangle)^2 - \sum_{i,j} t_{ij} \cos(\hat{\phi}_i - \hat{\phi}_j)$$

$\Rightarrow$ charged bosons VRH $\sigma(T) \sim e^{-C[E_c n_c/(Tn)]^{1/4}}$

For $n \approx n_c$ i.e. fragments merge $\rightarrow$ transition to superfluid

Correlated disorder

$$\langle U(\mathbf{x})U(\mathbf{x}')\rangle = \frac{U_0^2}{b^3} e^{-|\mathbf{x}-\mathbf{x}'|/b}$$



$\Rightarrow$ 2 length scales b , $B \sim \hbar^2/mU_0))^{1/2}$

b << B $\Rightarrow$ uncorrelated disorder

$$\nu(E) \sim |E|^3 \exp(-E^2/2U_0^2)$$

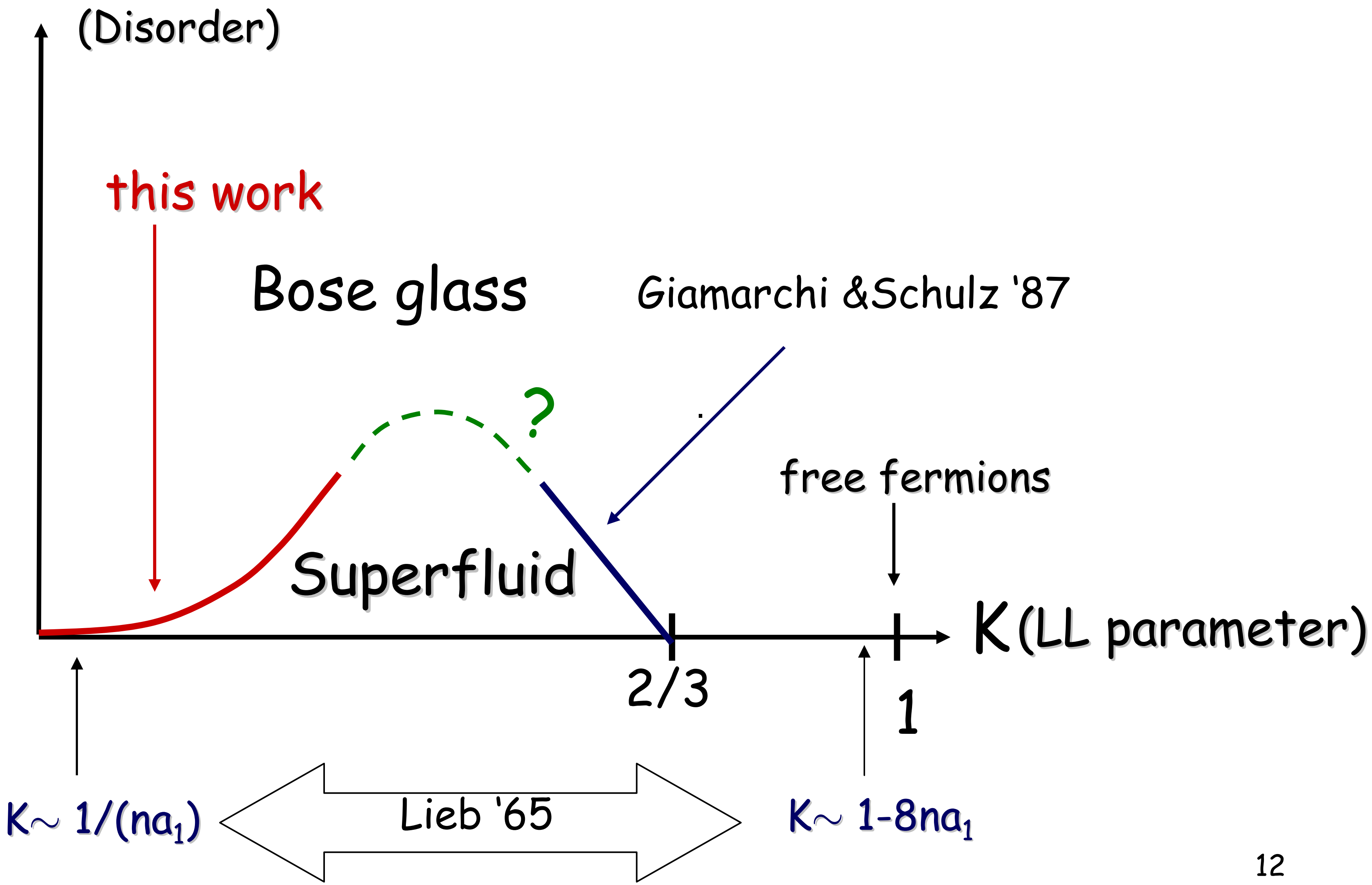
b >> B $\Rightarrow$ new results

Keldysh & Proshko '63
Kane '63
Shklovskii and Efros '70
John & Stephen '84

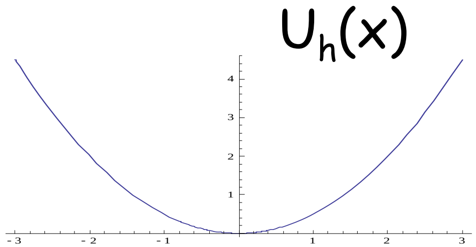
$$\mu(b,n) \approx -U_0\sqrt{2\ln(\frac{n_c}{n})}$$

$n \ll n_c \sim 1/(B^2 a)$

Bose gas in one dimensions

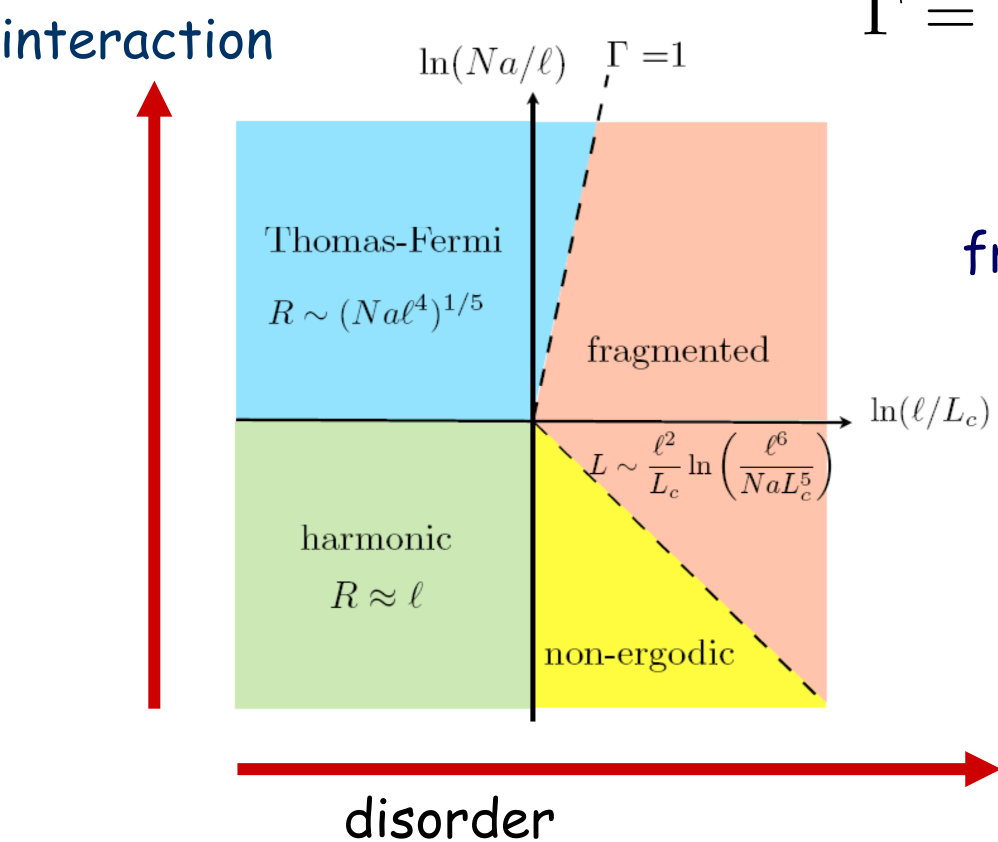


Bosons in traps (uncorrelated disorder)

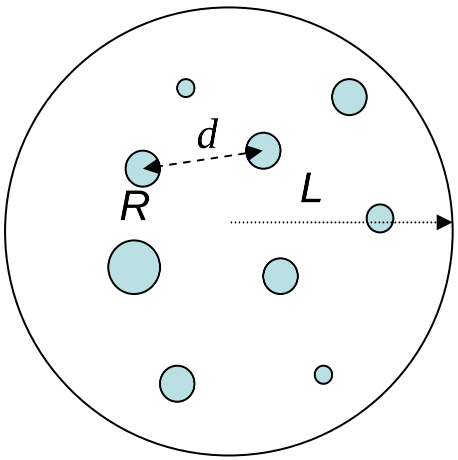


$$\mu(R) = -\frac{\hbar^2}{2mR^2} + E_{int}(R) + \frac{\hbar^2}{2m}\frac{R^2}{\ell^4}$$

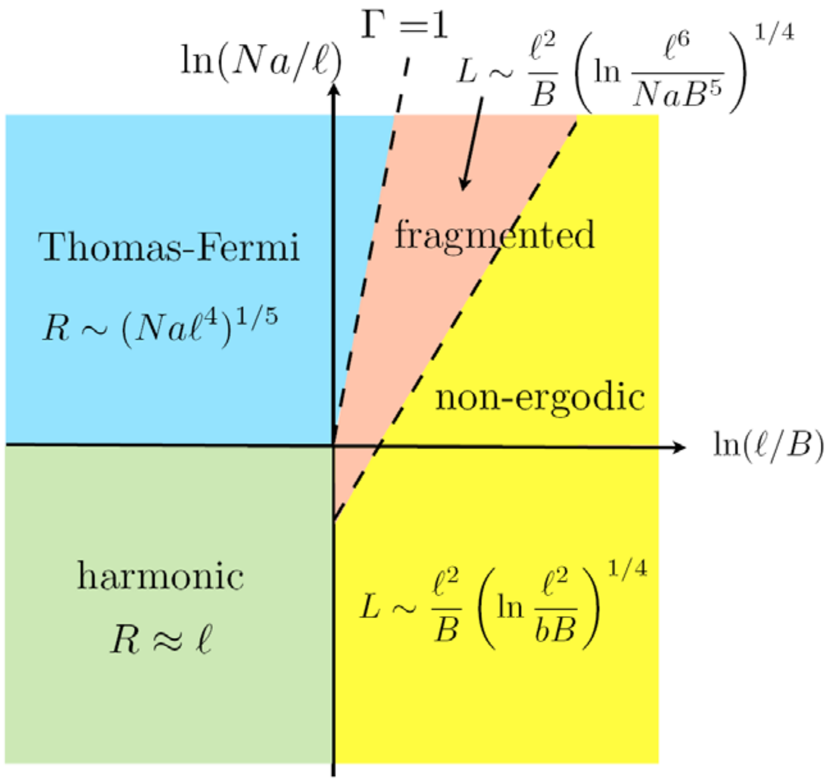
$$\Gamma = \frac{\ell^6}{3Na\mathcal{L}^5}$$



fragmented state

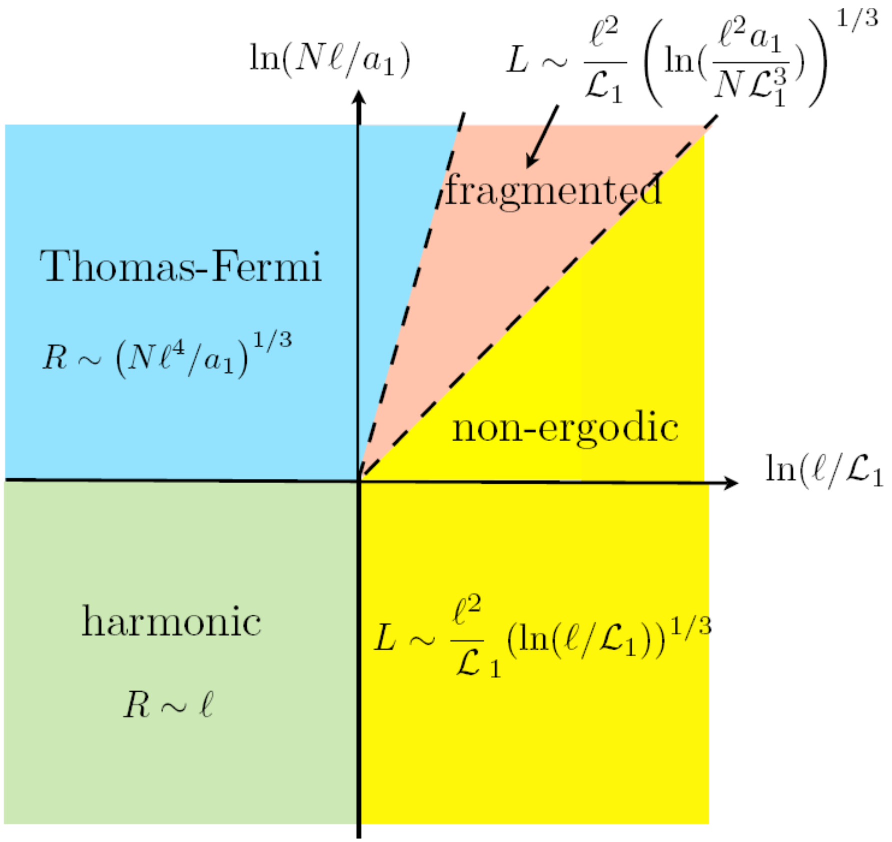


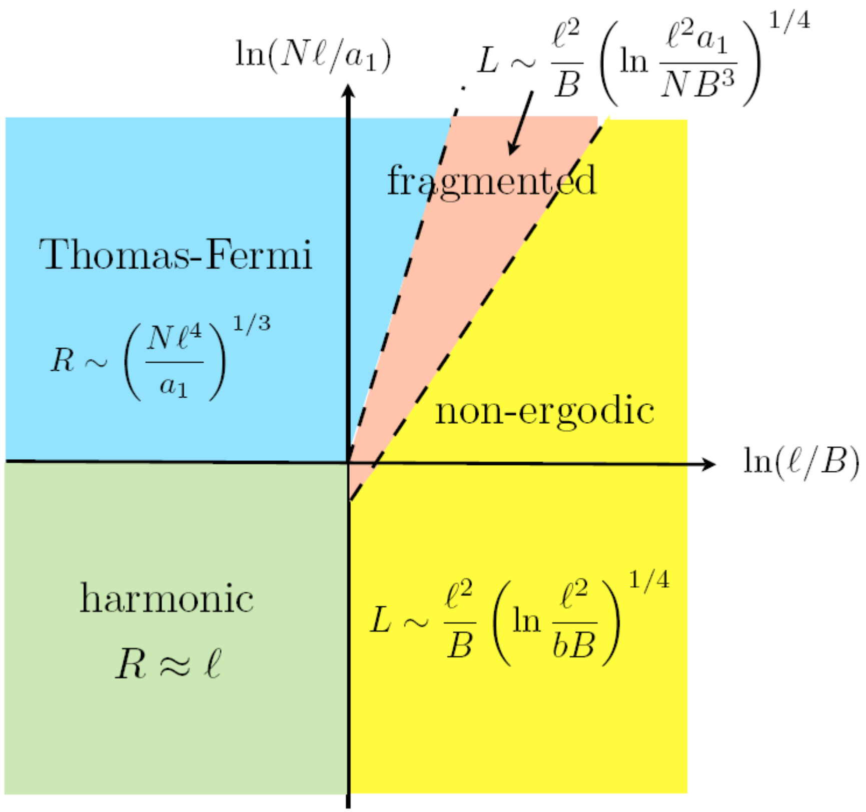
Bosons in traps (correlated disorder, d=3)



$$\Gamma = \frac{\ell^6}{NaB^5}$$

Bose gas in 1 dimensions: parabolic trap





Conclusions

- Semi-quantitative analysis of the phase states of a weakly interacting strongly diluted Bose gas in a random Gaussian potential.
- The system is charcterized by the Larkin length L_c and the scattering length a
- At particle density $n \ll n_c \approx 1/(aL_c^2)$ the Bose particles occupy deep potential wells and exponentially weakly tunnel to other wells. The number of particles in each well is defined, but phases are uncertain.
- At average particle density $n \approx n_c$ the transition to the superfluid proceeds.
- In a trap the oscillator length l appears as a new length scale. Four different regimes are found, depending on the mutual strength of L_c, aN and l , respectively.
- All results can be extended to lower dimensions and to correlated disorder.

Theory

Corrections to the Bogolyubov theory caused by a weak disorder: $n \gg n_c$

K. Huang and H.F. Meng, Phys. Rev. Lett. **69**, 644 (1992);
S. Giorgini, L.P. Pitaevsky and S. Stringari, Phys. Rev. B **49**, 12938 (1994)
A. V. Lopatin and V.M. Vinokur, Phys. Rev. Lett. **88**, 235503 (2002)

Possible Bose-glass state

M.P.A. Fisher et al., Phys. Rev. B **50**, 546 (1989)
R.T. Scalettar et al., Phys. Rev. Lett. **66**, 3144 (1991)
W. Krauth et al., Phys. Rev. Lett. **67**, 2307 (1991)

Spin model, transition from the normal state to superfluid

M. Ma, B.I. Halperin and P.A. Lee, Phys. Rev. B **34**, 3136 (1986)

One-dimensional Hubbard model

T. Giamarchi and H. Schulz, Europhys. Lett. **3**, 1287 (1987) $n \gg n_c$