THE “TUNNELLING TWO-LEVEL SYSTEMS” MODEL OF THE LOW-TEMPERATURE PROPERTIES OF GLASSES: SUCCESSES, PROBLEMS, PROSPECTS*

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**SOME PROPERTIES OF SOLIDS BELOW 1K**

<table>
<thead>
<tr>
<th></th>
<th>Crystals</th>
<th>Glasses</th>
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</thead>
<tbody>
<tr>
<td>Specific heat</td>
<td>$\sim T^3$</td>
<td>$\sim T$</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>$\sim T^2 \times \exp(-a\theta_D/T)$</td>
<td>$\sim T^2$</td>
</tr>
<tr>
<td>Ultrasonic absorption</td>
<td>$\sim \omega^4$</td>
<td>$\sim \omega^2/T$ (for $\omega \ll T$)</td>
</tr>
<tr>
<td>Hysteresis?</td>
<td>no</td>
<td>yes</td>
</tr>
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The (tunnelling) two-level systems (TTLS) model

Intuitively:

\[
\hat{H}_{TLS} = \frac{1}{2} \begin{pmatrix} \epsilon & \Delta \\ \Delta & -\epsilon \end{pmatrix}
\]
Precisely:

a) “TLS” model:

\[
\hat{H} = \hat{H}_{ph} + \hat{H}_{TLS} + \hat{H}_{int}
\]

\[
\hat{H}_{ph} = \sum_{k\alpha} \hbar \omega_{k\alpha} a^{+}_{k\alpha} a_{k\alpha}, \quad \omega_{kd} = c_{\alpha} |k| \quad \left[ a_{k\alpha}, a^{+}_{k'\beta} \right] = \delta_{k k'}, \delta_{\alpha \beta}
\]

i.e “ordinary” phonons

\[
\hat{H}_{TLS} = \sum_{i} E_i b^+_i b_i \quad \{ b_i, b^+_i \} = 1, \quad \left[ b_i, b^+_j \right] = 0 \text{ for } i \neq j
\]

i.e. Pauli operators

\[
\hat{H}_{int} = \sum_{\alpha \beta} \int \hat{e}_{\alpha \beta} \hat{T}_{\alpha \beta}(r) dr \quad \hat{T}_{\alpha \beta}(r) \equiv \sum_{i} g_{\alpha \beta \gamma}^{(i)} \hat{\delta}^{i} \delta(r - r_i)
\]

phonon strain
b) “TTLS” model imposes further constraints:

\[ \hat{H}_i = \frac{1}{2} \begin{pmatrix} \epsilon_i & \Delta_i \\ \Delta_i & -\epsilon_i \end{pmatrix} \left( \epsilon_i \equiv \sqrt{\epsilon_i^2 + \Delta_i^2} \right) \]

\[ \rho(\epsilon, \Delta) = \text{const.}/\Delta \Rightarrow \rho(E) = \text{const.} \ (\equiv \bar{P}_0) \]

\[ g_{\alpha\beta\gamma}^{(i)} = \frac{g_{\alpha\beta}^{(i)}}{E_i} (\epsilon_i \delta_{z\gamma} + \Delta_i \delta_{x\gamma}), (\sigma_z \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \sigma_x \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}) \]

with \( g_{\alpha\beta\gamma}^{(i)} \) having random (e.g. Gaussian) distribution with rms value \( \bar{g} \).

(may be different for L and T phonons)
Note with this form $g^{(i)}_{\alpha\beta\gamma}$ is strongly peaked towards small values.

In this talk, I will define ("weakly interacting") TTLS model by the above assumptions plus the assumption that the correct explanation of any given physical property is given by a calculation to the lowest order in $\bar{g}$ which gives a nontrivial result (e.g. $0^\text{th}$ for $C_v$, $1^\text{st}$ for US absorption and $\kappa$ …)

Some successes of the TTLS model (as so defined):

predicts $C_v(T) \propto T$ (✓) (actually $T^{1+\alpha}$, $\alpha \sim 0 \cdot 1 - 0 \cdot 3$)

``
\kappa(T) \propto T^2 (✓) (actually $T^{2-\beta}$, $\beta \sim 0 \cdot 1 - 0 \cdot 3$)
``
\alpha(\omega, T) \propto \omega \tanh \omega/2T ✓
``
saturation, echoes … ✓
``
log’c dependence of $C_v(t)$ ✓

Moreover, in some amorphous systems (e.g. polyethylene) fairly direct evidence (e.g. from luminescence of embedded organic molecules) for TLS ($\uparrow$: at room temp. not (directly) at $\leq 1K$).
Also oxide Josephson junctions, KBr – KCN …
Prediction very specific to TTLS assumption (Jäckle, 1972): in both high-ω, low T ("resonance") and low- ω, high-T ("relaxation") regimes, Q-factor for (linear) ultrasound absorption is constant:

\[ Q_{\text{res}}^{-1} = \pi C \]
\[ C \equiv \bar{P}_0 \gamma^2 / pc^2 \]
\[ Q_{\text{rel}}^{-1} = \frac{\pi}{2} C \]

Direct measurement of \( Q^{-1} \) subject to considered WWWWWW, but can relate by KK to velocity shift: since \( Q_{\text{res}}^{-1} \) has low-frequency, \( \omega \propto T \) cutoff \( Q_{\text{rel}}^{-1} \) has high-frequency \( \omega \propto T \) cutoff, we get (up to additive constants)

\[ \frac{\delta c}{c} \bigg|_{\text{res}} = C \ln \left( \frac{T}{T_0} \right) \]
\[ \frac{\delta c}{c} \bigg|_{\text{rel}} = -\frac{C}{2} \ln \left( \frac{T}{T_0} \right) \]

Thus:

**Slope ratio 2:-1**

prediction:

This general pattern is indeed seen. But …
Some problems with the TTLS scenario

1. Except in a few very special cases (KBr-KCN, Al$_2$O$_3$ JJ’s …) no clear picture of the nature of the TTLS.

2. Does not by itself explain drastic change in experimental properties of glasses above 1K (eg plateau ~ 1-10 K in thermal cond$^\gamma$.)

3. At least one specific prediction definitely wrong (at least in SiO$_2$, Bk7): in plot of $\delta c/c$ vs ln T, which general shape right, slope ratio is not 2:-1 but 1:-1. No simple modification of TTLS postulates appears able to fix this.

4. Universality of $Q^{-1}$ (measurable by velocity shift and use of KK relation). In TTLS model,

$$Q_{res}^{-1} = \pi C, \quad C = \frac{P_0 \gamma^2}{\rho c^2}$$

In C, 4 factors, each fluctuating between materials by factor ~ 5-10; no verticals nevertheless for ~ 30 different materials

$$Q_{res}^{-1} = 3 \times 10^{-4} \pm \sim 50\%$$
Is TTLS model successful because it is unique, or because it is a special case of a much more general scenario?

“crystals are the anomaly, glasses the norm!”


whatever non-phonon excitations we start with (maybe TTLS?) dominant effect phonon-mediated interaction.
The (generalized) collective scenario:

\[ \hat{H} = \hat{H}_{ph} + \hat{H}_{np} + \hat{H}_{int} \]

\( \hat{H}_{np} \) specified by (possibly random) matrix elements.

Quantity of fundamental interest is (non-phononic) stress tensor

\[ \hat{T}_{ij} \equiv \frac{\partial \hat{H}_{np}}{\partial e_{ij}} \]

\[ \Rightarrow H_{int} = \sum_{ij} \int d\mathbf{r} \, \hat{e}_{ij}(\mathbf{r}) \hat{T}_{ij}(\mathbf{r}) \]

Elimination of phonons leads to effective stress-stress interaction (Joffin & Levelut 1975):

\[ H_{np}^{(eff)} = \sum_{ijkl} \int d\mathbf{r} \int d\mathbf{r'} \frac{\Lambda_{ijkl}(\hat{n}_{rr'})}{|r - r'|^3} \hat{T}_{ij}(\mathbf{r}) T_{kl}(\mathbf{r}') \]

\( \Lambda_{ijkl}(\hat{n}_{rr'}) = \text{nasty 4}^{th}\text{- rank tensor} \)

Conjecture: \( H_{np}^{(eff)} \) dominates over original \( \hat{H}_{np} \)
In view of $|\boldsymbol{r} - \boldsymbol{r}'|^3$ dependence, problem is self-similar \( \Rightarrow \) expect real-space normalization procedure to scale to fixed point.

DC Vural & AJL 2011 (cf. Burin & Kagan 1996): universal value of $Q^{-1}$ due to fact that absorbed entity (phonon) identical to one whose exchange generates effective interaction. Small value of $Q^{-1}$ a result of (a) multiplicity of phonon modes and stress-tensor matrix elements (b) logarithmic factor arising from real-space scaling (indeed, predict that as $L \to \infty$,

\[
Q^{-1} \sim \left( \ln \left( \frac{L}{L_0} \right) \right)^{-1/2} \to 0!
\]

Obvious question: similar effects in electrodynamics of complex media? (note: in many glasses such as SiO$_2$, electric-dipole interactions may be comparable to stress-stress.)
“Smoking – gun” tests?

Problem: alternative scenario at present too generic to make many specific predictions. So as Aunt Sally, choose scenario as different as possible from TTLS while not simply SHO:

\[ \hat{T}_{ij} \text{ is random matrix} \]

1. Temperature dependence of ultrasound absorption:

\[ Q_{TTLS}^{-1}(\omega, T) = \text{const.} \times \tanh(\hbar \omega / 2k_B T) \]

\[ Q_{RM}^{-1}(\omega, T) = \text{const.} \times \tanh(1 - \exp - \hbar \omega / k_B T) \]

2. Low-T properties of amorphous toluene:

fluorescence of organic molecules embedded in eg PET (typical “glass”) seems to reflect TLS characteristics. However, similar experiments on solid amorphous toluene give no evidence for TLS. Thus, if we can measure \( T < 1K \) properties of solid amorphous toluene:

if very different from typical glass, supports TLS hypothesis

if similar to other glasses, suggests TLS model is not the explanation.
Happy birthdays, Phil and Freeman!